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# New measures of entropy for intuitionistic fuzzy sets

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**Abstract**

We propose the new measures of entropy for intuitionistic fuzzy sets. This paper is in a sense a continuation of our previous paper on entropy of intuitionistic fuzzy sets – the inferences are based on the same two types of distances as previously – to the farer and to the nearer crisp elements. But instead of the ratio of these distances we examine their difference. The distances are calculated using the formulas for the normalized Hamming distance, and the normalized Euclidean distance. In the case of the Hamming distance we obtain simpler formulas than in our previous paper. We show some special properties for the formulas when Hamming distance is applied. We also suggest $\pi$-entropy, a function strongly accounting for the lack of knowledge as to the membership and non-membership.
**Keywords:** fuzzy sets, intuitionistic fuzzy sets, entropy, similarity

## 1 Introduction

Fuzziness, a feature of imperfect information, results from the lack of crisp distinction between the elements belonging and not belonging to a set (i.e. the boundaries of a set under consideration are not sharply defined). A measure of fuzziness often used and cited in the literature is the entropy first mentioned by Zadeh [26]). The name entropy was chosen due to an intrinsic similarity of equations to the ones in the Shannon entropy (Jaynes [12]). However, the two functions measure fundamentally different types of uncertainty. Basically, the Shannon entropy measures the average uncertainty in bits associated with the prediction of outcomes in a random experiment.

De Luca and Termini [6] introduced some requirements which capture our intuitive comprehension of a degree of fuzziness. Kaufmann (1975) (cf. [14]) proposed to measure a degree of fuzziness of any fuzzy set $A$ by a metric distance between its membership function and the membership function (charecteristic function) of its nearest crisp set. Another way given by Yager [25] was to view a degree of fuzziness in terms of a lack of distinction between the fuzzy set and its complement. Higashi and Klir [5] extended Yager's concept to a very general class of fuzzy complements. Yager's aproach was also further developed by Hu and Yu [10]. Indeed, it is the lack of distinction between sets and their complements that distinguishes fuzzy sets from crisp sets. The less the fuzzy set differs from its complement, the fuzzier it is. Kosko [13] investigated the fuzzy entropy in relation to a measure of subsethood. Fan at al. [7], [8], [9] generalized Kosko's approach.

Here we propose the measures of fuzziness for intuitionistic fuzzy sets which are generalization of fuzzy sets. The measures of entropy we consider are the results of the substraction of distances to the nearer and to the farer crisp elements. We use two types of distances - the normalized Hamming distance, and the normalized Euclidean distance. The measure which is based on the normalized Hamming distance is simpler in the sense of calculations than the one we have proposed previously (Szmidt and Kacprzyk [20]). We prove that the result is equivalent to the considering of distances between elements and their complements (see Hung [11]). We also show that the measure is closely related to the similarity measure we proposed in (Szmidt and Kacprzyk [23]). Finally, we propose a function called here a $\pi$-entropy which is not an entropy in the sense of the axioms but has some advantages when measuring the fuzziness of an intuitionistic fuzzy set. Unfortunately for the Euclidean distance we can not show that the entropy can be equivalently expressed in the terms of the distances between elements and their complements (like it is for the Hamming distance). All the simplifications (in the sense of calculations) disappear when we use the basic definition with the normalized Euclidean distance.

For different approaches we refer the reader to Burillo and Bustince [3], Cornelis and Kerre [4].

## 2 A brief introduction to intuitionistic fuzzy sets

As opposed to a fuzzy set in $X$ (Zadeh [26]), given by

$$A' = \{< x, \mu_{A'}(x) > | x \in X\} \tag{1}$$

where $\mu_{A'}(x) \in [0,1]$ is the membership function of the fuzzy set $A'$, a so-called intuitionistic fuzzy set (Atanassov [1], [2]) $A$ is given by

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \tag{2}$$

where: $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{3}$$

and $\mu_A(x)$, $\nu_A(x) \in [0,1]$ denote a degree of membership and a degree of non-membership of $x \in A$, respectively.

Obviously, each fuzzy set may be represented by the following intuitionistic fuzzy set

$$A = \{< x, \mu_{A'}(x), 1 - \mu_{A'}(x) > | x \in X\} \tag{4}$$

For each intuitionistic fuzzy set in $X$, we will call

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{5}$$

an *intuitionistic fuzzy index* (or a *hesitation margin*) of $x \in A$ and, it expresses a lack of knowledge of whether $x$ belongs to $A$ or not (cf. Atanassov [2]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

In our further considerations we will use the complement set $A^C$

$$A^C = \{< x, \nu_A(x), \mu_A(x) > | x \in X\} \tag{6}$$

The application of intuitionistic fuzzy sets instead of fuzzy sets means the introduction of another degree of freedom into a set description. Such a generalization of fuzzy sets gives us an additional possibility to represent imperfect knowledge what leads to describing many real problems in a more adequate way. We refer the interested reader to Szmidt [16], Szmidt and Kacprzyk [23] where the applications of intiutionistic fuzzy sets to group decision making, negotiations and other situations are presented.

## 3 Geometrical interpretation

Having in mind that for each element $x$ belonging to an intuitionistic fuzzy set $A$, the values of membership, non-membership and the intuitionistic fuzzy index add up to one, i.e.

$$\mu_A(x) + \nu_A(x) + \pi_A(x) = 1$$

and that each: membership, non-membership, and the intuitionistic fuzzy index are from the interval $[0,1]$, we can imagine a unit cube (Figure 1) inside which there is $ABD$ triangle where the above equations are fulfilled. In other words, $ABD$ triangle represents a surface where coordinates of any element belonging

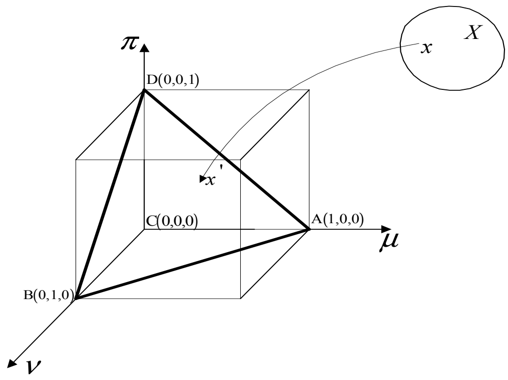


Figure 1: Geometrical representation

to an intuitionistic fuzzy set can be represented. Each point belonging to $ABD$ triangle is described via three coordinates: $(\mu, \nu, \pi)$. Points $A$ and $B$ represent crisp elements. Point $A(1, 0, 0)$ represents elements fully belonging to an intuitionistic fuzzy set as $\mu = 1$. Point $B(0, 1, 0)$ represents elements fully not belonging to an intuitionistic fuzzy set as $\nu = 1$. Point $D(0, 0, 1)$ represents elements about which we are not able to say if they belong or not belong to an intuitionistic fuzzy set (intuitionistic fuzzy index $\pi = 1$). Such an interpretation is intuitively appealing and provides means for the representation of many aspects of imperfect information. Segment $AB$ (where $\pi = 0$) represents elements belonging to classical fuzzy sets ($\mu + \nu = 1$). Any other combination of the values characterizing an intuitionistic fuzzy set can be represented inside the triangle $ABD$. In other words, each element belonging to an intuitionistic fuzzy set can be represented as a point $(\mu, \nu, \pi)$ belonging to the triangle $ABD$ (Figure 2).

It is worth mentioning that the geometrical interpretation is directly related to the definition of an intuitionistic fuzzy set introduced by Atanassov [1], [2], and it does not need any additional assumptions.

By employing the above geometrical representation, we can calculate distances between any two intuitionistic fuzzy sets $A$ and $B$ containing $n$ elements.

## 3.1 Distances between intuitionistic fuzzy sets

In Szmidt [16], Szmidt and Baldwin [17, 18], Szmidt and Kacprzyk [19] it is shown why when calculating distances between intuitionistic fuzzy sets it is expedient to take into account all three parameters describing intuitionistic fuzzy sets. One of the reasons is that when taking into account two parameters only, for elements from classical fuzzy sets (which are a special case of intuitionistic fuzzy sets) we obtain distances from a different interval than for elements belonging to intuitionistic fuzzy sets. It practically makes it impossible to consider by the same formula the two types of sets.

In our further considerations we will use the following distances between fuzzy sets $A, B$ in $X = \{x_{1,}x_2, \ldots, x_n\}$ Szmidt [16], Szmidt and Baldwin [17, 18], Szmidt and Kacprzyk [19]:

- the normalized Hamming distance

$$l_{IFS}(A, B) = \frac{1}{2n} \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|) \qquad (7)$$

- and the normalized Euclidean distance:

$$q_{IFS}(A, B) = (\frac{1}{2n} \sum_{i=1}^{n} (\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 (\pi_A(x_i) - \pi_B(x_i))^2)^{\frac{1}{2}} \qquad (8)$$

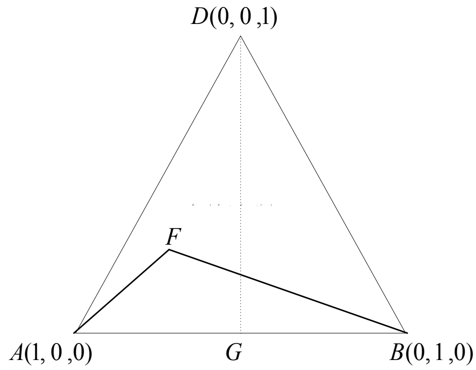


Figure 2: The triangle $ABD$ (cf. Fig. 1) explaining a distance-based measure of fuzziness

For (7), (8) we have:

$$0 \leq l_{IFS}(A, B) \leq 1 \quad (9)$$
$$0 \leq q_{IFS}(A, B) \leq 1 \quad (10)$$

Clearly these distances satisfy the conditions of the metric.

# 4 Entropy

De Luca and Termini [6] first axiomatized a non-probabilistic entropy. The De Luca-Termini axioms formulated for fuzzy sets are intuitive and have been widely employed in the fuzzy literature. They were formulated in the following way. Let $E$ be a set-to-point mapping $E : F(2^x) \to [0, 1]$. Hence $E$ is a fuzzy set defined on fuzzy sets. $E$ is an entropy measure if it satisfies the four De Luca and Termini axioms:

$$E(A) = 0 \quad \text{iff} \quad A \in 2^x \quad (A \text{ non-fuzzy}) \quad (11)$$

$$E(A) = 1 \quad \text{iff} \quad \mu_A(x_i) = 0.5 \quad \text{for all} \quad i \quad (12)$$

$$E(A) \leq E(B) \quad \text{if } A \quad \text{is less fuzzy than } B \quad (13)$$

i.e., if

$$\mu_A(x) \leq \mu_B(x) \quad \text{when} \quad \mu_B(x) \leq 0.5$$

and

$$\mu_A(x) \geq \mu_B(x) \quad \text{when} \quad \mu_B(x) \geq 0.5$$
$$E(A) = E(A^c) \quad (14)$$

Since the De Luca and Termini axioms (11)–(14) were formulated for fuzzy sets (given only by their membership functions, and describing the situation depicted by the segment $AB$ in Figures 1, 2) they may be expressed for the intuitionistic fuzzy sets as follows:

$$E(A) = 0 \quad \text{iff} \quad A \in 2^x \quad (A \text{ non-fuzzy}) \quad (15)$$

$$E(A) = 1 \quad \text{iff} \quad \mu_A(x_i) = \nu_A(x_i) \quad \text{for all} \quad i \quad (16)$$

$$E(A) \leq E(B) \quad \text{if } A \quad \text{is less fuzzy than } B \quad (17)$$

i.e., if

$$\mu_A(x) \leq \mu_B(x) \quad \text{and} \quad \nu_A(x) \geq \nu_B(x) \quad \text{for} \quad \mu_B(x) \leq \nu_B(x)$$

or

$$\mu_A(x) \geq \mu_B(x) \quad \text{and} \quad \nu_A(x) \leq \nu_B(x) \quad \text{for} \quad \mu_B(x) \geq \nu_B(x)$$

$$E(A) = E(A^c) \tag{18}$$

Differences between (11)–(14) and (15)–(18) occur in axioms 2 and 3, as axiom 2 must be fulfilled now not only for point $G$ (Figure 2) but for the whole segment $DG$.

The fuzziness of a fuzzy set, or its entropy, answers the question: how fuzzy is a fuzzy set? The same question may clarly be posed in a case of an intuitionistic fuzzy set. Having in mind our geometric interpretation of intuitionistic fuzzy sets (Figure 1), let us concentrate on the triangle $ABD$ – cf. Figure 2.

A non-fuzzy set (a crisp set) corresponds to the point $A$ (point $A$ represents elements from an intuitionistic fuzzy set fully belonging to it as $(\mu_A, \nu_A, \pi_A) = (1, 0, 0)$) and the point $B$ (point $B$ represents the elements which fully do not belong to the set as $(\mu_B, \nu_B, \pi_B) = (0, 1, 0)$. Points $A$ and $B$ representing a crisp set have the degree of fuzziness equal 0.

A fuzzy set corresponds to the segment $AB$. When we move from point $A$ towards point $B$ along the segment $AB$, we go through points for which the membership function decreases (from 1 at point $A$ to 0 at point $B$), the non-membership function increases (from 0 at point $A$ to 1 at point $B$). For the midpoint $G$ (Figure 2) the values of both the membership and non-membership functions are equal 0.5. So, the midpoint $G$ represents the elements with the degree of fuzziness equal 100% (we do not know if elements represented by point $G$ belong or if they do not belong to our set). On the segment $AG$ the degree of fuzziness grows (from 0% at $A$ to 100% at $G$). The same situation occurs on the segment $BG$. The degree of fuzziness is equal 0% at $B$, grows towards $G$ (here is equal 100%).

An intuitionistic fuzzy set is represented by the triangle $ABD$ and its interior. All points which are above the segment $AB$ represent the elements with an intuitionistic fuzzy index greater than 0. The most undefinded is point $D$. As the intuitionistic fuzzy index for $D$ is equal 1, we can not tell if elements represented by this point belong or do not belong to the set. The distance from $D$ to $A$ (full belonging) is equal to the distance to $B$ (full non-belonging). So, the degree of fuzziness for $D$ is equal 100%. But the same situation occurs for all elements represented by points $x_i$ on the segment $DG$. For $DG$ we have $\mu_{DG}(x_i) = \nu_{DG}(x_i)$, $\pi_{DG}(x_i) \geq 0$ (equality only for point $G$), and certainly $\mu_{DG}(x_i) + \nu_{DG}(x_i) + \pi_{DG}(x_i) = 1$. For every $x_i \in DG$ we have: $distance(A, x_i) = distance(B, x_i)$.

This geometric representation of an intuitionistic fuzzy set motivates a distance-based measure of fuzziness, i.e. an entropy of fuzzy sets. First we will define entropy $E(F)$ for a separate element (represented by point $F$).

**Definition 1**

$$E(F) = 1 - \frac{1}{2}[l_{IFS}(F, F_{far}) - l_{IFS}(F, F_{near})] \tag{19}$$

*where* $l_{IFS}(F, F_{near})$ *is a distance from* $F$ *to the nearer point* $F_{near}$ *among (crisp)* $A$ *and* $B$, $l_{IFS}(F, F_{far})$ *is the distance from* $F$ *to the farer point* $F_{far}$ *among* $A$ *and* $B$.

From (7), considering separately the cases when $\mu_F < \nu_F$, $\mu_F > \nu_F$, $\mu_F = \nu_F$, we can express (19) in the following way

$$E(F) = 1 - \frac{1}{2}|\mu_F - \nu_F| \tag{20}$$

It can be proved that that (19) satisfies axioms (15)–(18).

An interpretation of entropy (19) can be as follow. This entropy measures the whole missing information which may be necessary to have no doubts when classifying (an element represented by) the point $F$ to the area of consideration, i.e. to say say that (an element represented by) $F$ fully belongs (point $A$) or fully does not belong (point $B$) to our set.

Formula (19) describes entropy for a single element belonging to an intuitionistic fuzzy set. For $n$ elements belonging to an intuitionistic fuzzy set we have

$$E = 1 - \frac{1}{2n}\sum_{i=1}^{n}[l_{IFS}(F_i, F_{far}) - l_{IFS}(F_i, F_{near})] = 1 - \frac{1}{2n}\sum_{i=1}^{n}|\mu_{F_i} - \nu_{F_i}| \tag{21}$$

where $F_{far}$ and $F_{near}$ as in (19).

It is worth mentioning several facts as far as the proposed measure of entropy (21) is concerned:

- Although when calculating distances for intuitionistic fuzzy sets it is expedient to take into account all the three functions (memberships, non-memberships, and the intuitionistic fuzzy indices) – see Section 3.1, in the entropy measure (21) the third components (the intuitionistic fuzzy indices) disappear during operation of substraction. It makes the proposed here measure (21) easier to calculate than previously proposed by us measure (cf. Szmidt and Kacprzyk [20]).
- The proposed measure of entropy (21) is a special case of the new measure of similarity we have suggested (Szmidt and Kacprzyk [23]).

Having in mind interrelations between an element from an intuitionistic fuzzy set represented by point $F$, and its complement represented by point $F^C$, i.e., the fact that $F = (\mu_F, \nu_F, \pi_F)$, $F^C = (\nu_F, \mu_F, \pi_F)$, from (7) we have

$$l_{IFS}(F_i, F^C) = |\mu_F - \nu_F| + |\nu_F - \mu_F| + |\pi_F - \pi_F| = 2|\mu_F - \nu_F| \tag{22}$$

what means that (19) can be expressed as:

$$E(F) = 1 - \frac{1}{2} l_{IFS}(F, F^C) \tag{23}$$

So finally, the proposed measure (21) can be expressed as

$$\begin{aligned} E &= 1 - \frac{1}{2n}\sum_{i=1}^{n}[l_{IFS}(F_i, F_{far}) - l_{IFS}(F_i, F_{near})] = 1 - \frac{1}{2n}\sum_{i=1}^{n}|\mu_{F_i} - \nu_{F_i}| \\ &= 1 - \frac{1}{2n}\sum_{i=1}^{n} l_{IFS}(F_i, F_i^C) \end{aligned} \tag{24}$$

The proposed measure of entropy by definition reaches its maximum for elements for which membership function is equal to non-membership function (see the axiom (16) which is a counterpart of the De Luca and Termini axiom (12)). We can treat it as advantage (despite of the value of the intuitionistic fuzzy indices we immediately are able to indicate the missing amount of the knowledge to say if an element belongs or does not belong to an intuitionistic fuzzy set – not paying any attention to the sources of the lack of knowledge). Albeit in some situations it may be useful to be aware as well about the contribution of the intuitionistic fuzzy indices. We would like to stress once more that the sense/meaning of an entropy is expressed properly by (24). Anyway, for some special purposes it could be expedient to have a measure which for each given intuitionistic fuzzy index $\pi$ behaves like entropy, and the same time the measure changes when we consider different intuitionistic fuzzy indices. Let us call such a measure a $\pi$-entropy.

Here we propose a $\pi$-entropy, $E_\pi$, for a single element (represented by point) $F$ belonging to an intuitionistic fuzzy set as:

$$\begin{aligned} E_\pi(F) &= \{1 - \frac{1}{2}[l_{IFS}(F, F_{far}) - l_{IFS}(F, F_{near})]\}/(1 - \pi_F) = [1 - \frac{1}{2} l_{IFS}(F, F^C)]/(1 - \pi_F) = \\ &= [1 - \frac{1}{2}|\mu_F - \nu_F|]/(1 - \pi_F) \end{aligned} \tag{25}$$

Of course, $E_\pi(F)$ does not fulfill the axioms for entropy (e.g. it takes values greater than 1). It is easy to notice that for a given $\pi$, the value of the numerator in (25) takes values from the interval $[0, 1]$ (the maximum for $\mu_F = \nu_F$). For $\pi = 0$, the $\pi$-entropy is equal to the entropy given by (19), (20). When the values of $\pi$ increase, the $\pi$-entropy increases, too. In Figure 3 we can see the values of $\pi$-entropy for the values of the numerator (25) equal to 1 (for $\mu_F = \nu_F$). For $\pi \to 1$ $E_\pi \to \infty$. For other possible values

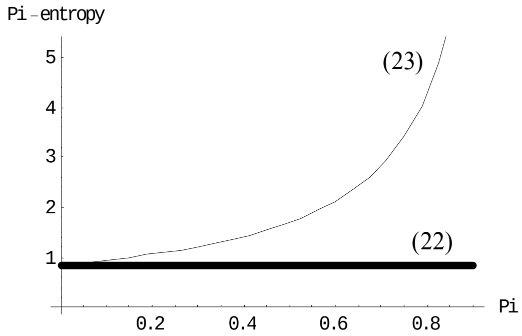


Figure 3: The values of $\pi$-entropy for the values of the numerator (25) equal to 1, and the (constant) values of entropy (24) - the bold line.

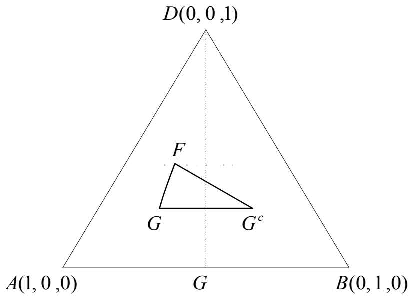


Figure 4: The triangle $ABD$ explaining a ratio-based measure of similarity

of the numerator, the proposed formula (25) behaves in the same way. For many elements an agregation of the values obtained for separate elements (25) is necassary. But as the possible ways of aggregation depend on specific tasks, we do not propose any specific solutions here.

It is worth mentioning that the entropy introduced (24) is a special case of the similarity measure (see Figure 4) we proposed earlier (Szmidt and Kacprzyk [23]). Here we present the similarity measure $Sim(F, G)$ between two elements (represented by points) $F$ and $G$.

**Definition 2**

$$\begin{aligned} Sim(F,G) &= l_{IFS}(F,G^C) - l_{IFS}(F,G) = (\left|\mu(F)-\mu(G^C)\right| + \left|\nu(F)-\nu(G^C)\right| + \\ &+ \left|\pi(F)-\pi(G^C)\right|) - (|\mu(F)-\mu(G)| + |\nu(F)-\nu(G)| + |\pi(F)-\pi(G)|) = \\ &= \left|\mu(F)-\mu(G^C)\right| + \left|\nu(F)-\nu(G^C\right| - |\mu(F)-\mu(G)| - |\nu(F)-\nu(G)|) \end{aligned} \quad (26)$$

*where:* $l_{IFS}(F,G)$ *is a distance from* $F(\mu_F, \nu_F, \pi_F)$ *to* $G(\mu_G, \nu_G, \pi_G)$*,*
$l_{IFS}(F,G^C)$ *is the distance from* $F(\mu_F, \nu_F, \pi_F)$ *to* $G^C(\nu_G, \mu_G, \pi_G)$*,*
$G^C$ *is a complement of* $G$*,*
*the distances* $l_{IFS}(F,G)$ *and* $l_{IFS}(F,G^C)$ *are calculated from (7).*

In a special case when $G = A$ and $G^C = B$ (i.e. when the situation in Figure 4 becomes the situation in Figure 2), the similarity measure (26) becomes closely related to the proposed here entropy measure, namely: $E(F) = 1 - 0.5 * Sim(F, A)$ or $E(F) = 1 - 0.5 * Sim(F, B)$ what depends which one from $A$ and $B$ are nearer to $F$.

Although the Definition 1 of entropy is general in the sense of the applied distances, the detailed formulas were presented for the normalized Hamming distance. Similar formulas can be presented for different distances. For example, for the normalized Euclidean distance (for membesrship function of the considered elements greater than non-membership functions) we have:

$$
\begin{aligned}
E &= 1-\frac{1}{\sqrt{2n}}\sum_{i=1}^{n}[q_{IFS}(F_i,F_{far})-q_{IFS}(F_i,F_{near})]= \\
&= 1-\frac{1}{\sqrt{2n}}\sum_{i=1}^{n}\{\sqrt{(\mu_{F_i}-1)^2+(\nu_{F_i})^2+(\pi_{F_i})^2}-\sqrt{(\mu_{F_i})^2+(\nu_{F_i}-1)^2+(\pi_{F_i})^2}\}= \\
&= 1-\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\{\sqrt{(\nu_{F_i})^2+\pi_{F_i}\nu_{F_i}+(\pi_{F_i})^2}-\sqrt{(\mu_{F_i})^2+\pi_{F_i}\mu_{F_i}+(\pi_{F_i})^2}\} \qquad (27)
\end{aligned}
$$

Unfortunately for the Euclidean distance we can not show that the entropy can be equivalently expressed in the terms of the distances between elements and their complements (like it was for the Hamming distance - (24)).

We could introduce the counterpart of (24) but the new measure is not a direct result of the distances we consider in Definition 1.

$$
\begin{aligned}
E &= 1-\frac{1}{\sqrt{2n}}\sum_{i=1}^{n}q_{IFS}(F_i,F_i^C)=1-\frac{1}{\sqrt{2n}}\sum_{i=1}^{n}\sqrt{(\mu_{F_i}-\nu_{F_i})^2+(\nu_{F_i}-\mu_{F_i})^2}= \\
&= 1-\frac{1}{\sqrt{n}}\sum_{i=1}^{n}|\nu_{F_i}-\mu_{F_i}| \qquad (28)
\end{aligned}
$$

# 5 Concluding remarks

Starting from geometrical interpretation of intuitionistic fuzzy sets, the meaning of the entropy, and the distances for intuitionistic fuzzy sets we have proposed different formulas for calculations. The differences are due to different sort of distances. The simplest formulas are for the Hamming distance (as the hesitation margins disappear in the process of substraction - the formulas are simpler than the ones we have proposed previously (Szmidt and Kacprzyk, 2001)). We proved that for the Hamming distance the calculations can be done in two ways:
a) considering the difference of the distances to the nearer and to the farer crisp elements [cf. (19), (21)], or
b) considering the distances to the complement elements [cf. (24)].

The proposed here measure of entropy is closely related to the similarity measure we proposed in (Szmidt and Kacprzyk [23]).

As the sense of entropy lies in the measuring of the missing information necessary to conclude if elements belong or do not belong to a set, the proposed formulas (21), (24) do not concentrate on the separate, possible reasons of the missing information. On the other hand, in some practical situations it could be expedient to know the reasons why the entropy is high (i.e., because of lack of distinguish between $\mu$ and $\nu$, or/and big values of $\pi$). For such cases we have proposed a function called a $\pi$-entropy (25).

We have also introduced the formulas of entropy for the Euclidean distances. But the simplifications (in the sense of calculations) disappear for the basic definition.

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