Representation of Fuzzy and Intuitionistic Fuzzy Data by Radar Charts

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Abstract

In this paper, we present the idea of visualization of fuzzy and intuitionistic fuzzy data by radar charts, which are useful for representing multivaried data sets, and especially for data from time series. Comparison is given between the run charts and radar charts, as well as between the applications of radar charts in fuzzy and in intuitionistic fuzzy data sets. The graphical presentation of fuzzy data by radar charts seems to be very obvious and natural, while the same done for intuitionistic fuzzy data, as distinguished from the fuzzy data by the addition of second component of set non-membership, seems to bring innovation not only to the field of intuitionistic fuzzy sets, but also to the representational technique of radar charts.

Keywords: Fuzzy sets, Intuitionistic fuzzy sets, Radar charts, Visualization.

1 Introduction

Radar charts (also known as web charts, spider charts, polar charts or Kiviat charts) are a popular method for visual representation and comparison of multivaried data sets, where the factors are represented as radii (vectors, axes, spokes) with a common beginning and equal length, along which data are plotted. Radar charts are essentially analogous to line charts, except that the scale wraps around. Due to their shape, radar charts are also very effective for representation of time series or data of cyclic nature, but this is not obligatory: the chart vectors may be independent.

There are certain common considerations when constructing and interpreting data from radar charts. Points close to the center usually indicate low (or negative) value, while points near the edge indicate a high (or positive) values. However, as [1] comments, "this is an arbitrary decision from a methodological point of view; it is equally possible to define good performance as values approaching zero. The only constraint is that all performance dimensions must be depicted on the same scale".

When interpreting a radar chart, it is important to check each individual axis as well as the chart's overall shape in order to gain a complete understanding of its meaning. One can see to what extent data fluctuates by observing whether the spiral is smooth or has spikes of variability. Radar charts are an effective way to show strengths and weaknesses graphically by enabling the user to observe symmetry or uniformity of data. For instance, if the chart vectors stand for multiple independent criteria, rather than for one criterion changing over time, setting the vectors' order may require some logical rationale. It is somewhat easier to see patterns in the data if the observations are arranged in some non-arbitrary order, and if the variables are assigned to the rays of the star in some meaningful order, [2].

For instance, a pair of criteria, which have been proven to correlate more strongly than another pair of criteria, shall be assigned to closely located endpoints of the chart. The number of criteria and estimated items shall also be kept reasonable, so that the values remain well distinguishable on the chart.

2 Fuzzy data radar charts

Representing fuzzy data seems quite natural in radar charts, the only difference being that all charts vectors are always quantified and standardized to range from 0 in the centre to 1 in each endpoint. For every object represented by a fuzzy data radar chart, the points (one point per vector) represent particular values of the membership function.

The first example shows a line chart (also called $run\ chart$) of one criterion in a time series and its respective radar chart, where the evaluated elements A and B of a fuzzy set exhibit the following values of the membership function, as defined in Table 1.

Table 1 1 2 3 4 5 6 7 8 9 10 11 12 0.4 0,7 8,0 0,8 0.4 0.4 0.475 0,55 0,625 8.0 8,0 8,0 0,6 0,6 0,6 0,6 0,45 0,3 0,37 0,43 0,5 0,5 0,5 0,5 В

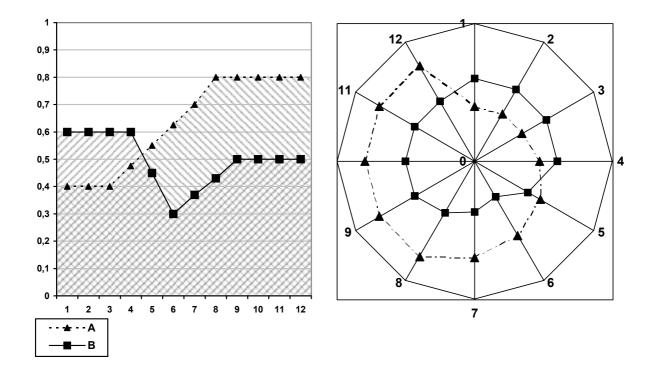


Figure 1.1: Example 1: Line chart.

Figure 1.2: Example 1: Radar chart.

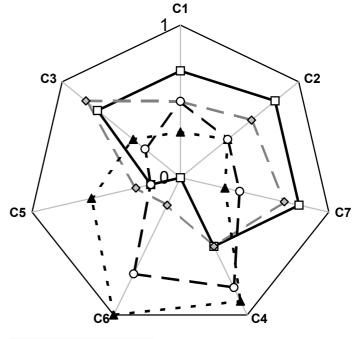
Let us see a second example as well, this time representing two data sets, varying according to different criteria (Table 2): four products evaluated with respect to seven of their leading market characteristics:

Table 2

Products	Product A	Product B	Product C	Product D
Criteria				
C1: Production costs $(0 - low, 1 - high)$	0.7	0.5	0.3	0.5
C2: Customer price (0 – cheap, 1 – expensive)	0.8	0.6	0.4	0.4
C3: Service and product support (0 – unavailable, 1 – well developed)	0.7	0.8	0.4	0.3
C4: Usability and user friendliness (0 – hard to use, 1 – user friendly)	0.5	0.5	0.9	0.8
C5: Energy consumption and waste $(0 - high, 1 - low)$	0.2	0.3	0.6	0.2
C6: Interoperability, standardization $(0 - low, 1 - high)$	0	0.2	1	0.7
C7: Brand popularity (0 – unknown, 1 – top-of-mind brand)	0.8	0.7	0.3	0.4

Let us suppose that the analysis of the seven factors shows largest coefficients of correlation within the pairs of criteria (C1; C2), (C1; C3), (C2; C7), (C2; C4), (C2; C6). On this basis, let us choose to arrange the criteria on the chart vector endpoints in the following order: C1 - C2 - C7 - C4 - C6 - C5 - C3 - C1.

As we can see from Fig. 2, with the chosen technique of radar chart and the proper order of the axes, the products are visualized so that the differences in their fuzzy market presence become instantly visible.

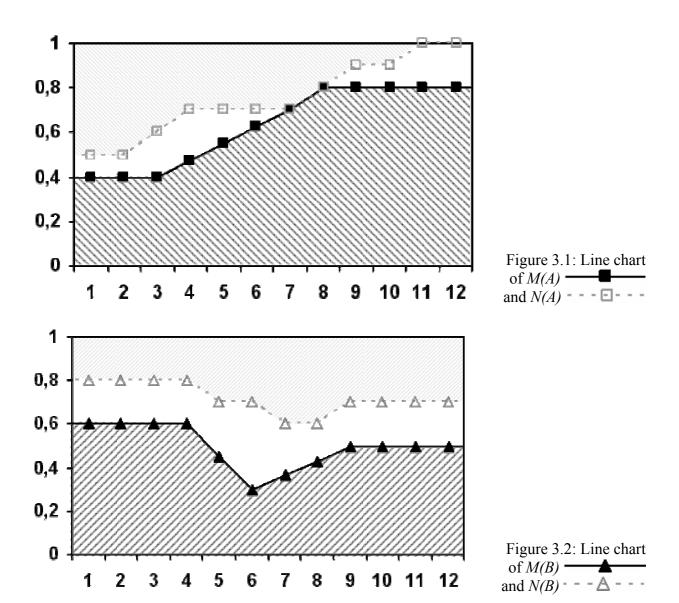


Product A
Product B
Product C
Product C
Product D

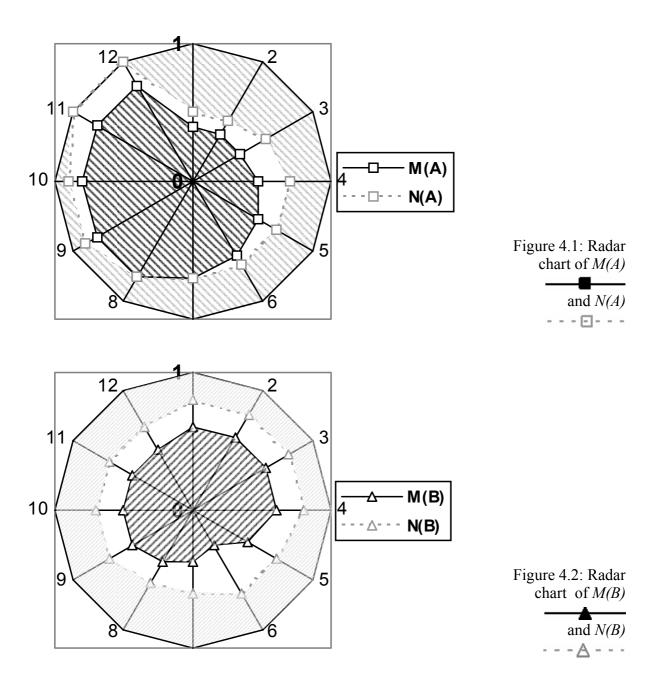
3 Intuitionistic fuzzy data radar charts

An interesting difference occurs when representing intuitionistic fuzzy values: not one, but two points per chart vector are marked, representing respectively the values of the membership and non-membership function, [3].

Let us first remind the classical graphic representation of intuitionistic fuzzy set. For clarity, we will use the line chart representation from the first example, where M(A) and M(B) denote the membership values of A and B, while N(A) and N(B) denote the non-membership functions of both. As we know from the theory of intuitionistic fuzzy sets, $M(A) + N(A) \le 1$ and $M(B) + N(B) \le 1$. There are no other explicit dependencies, for instance between M(A) and M(B) or between N(A) and N(B). Again for clarity both A and B will be represented on separated line charts, respectively on Figure 3.1 and Figure 3.2.



Let us transform these line charts into the respective radar charts, as shown on Figures 4.1 and 4.2. As we can see, the radar chart contains of three regions, the central one corresponding to the area covered by the membership function (dark grey), and the peripheral one corresponding to the non-membership function (light gray). The remaining area in white corresponds to the uncertainty function. Figure 4.1. demonstrates two more special cases: first, when the membership and non-membership values sum up to 1, without value of uncertainty, and second when the non-membership function is equal to 0.



Conclusion

The so proposed visualization technique for fuzzy and intuitionistic fuzzy data is only one of the numerous possible graphic interpretations. It can be especially useful for data in time series and other data with cyclic trait. An interesting issue will be to exchange the places of the membership and non-membership function with respect to the centre of the chart, as well as to explore the sections and unions of the chart lines of two and more elements of the fuzzy or intuitionistic fuzzy set (see Figure 1.2) and other operations over these geometrical objects. Another challenging problem will be the transition from discrete to continuous values of the membership and non-membership functions, i.e. the transition from a polygonal radar chart to a circle with [0;1]-radius.

References

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