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Intuitionistic Fuzzy Generalized Nets as Tools for Modelling and Simulation of Closed Systems

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Introduction

Generalized Nets (GNs, see [1]) as a tool for process modeling give an opportunity to represent processes, which flow in parallel in closed systems. Simulation of these processes helps us to trace the functioning of the system. This can be used for further investigations and optimizations.

The GNs are extensions of the Petri nets and other modifications of them. On the other hand they are objects of next extensions. Two of them are Intuitionistic Fuzzy GNs of type one and of type three (IFGNs1, see [1] and IFGNs3, see [2]). These two extensions are based on the concept of the Intuitionistic Fuzzy Sets (IFSs, see [3]). In a book and series of papers (see [4, 5]) it is shown that functioning and result of work of different types of (abstract) systems in Mesarovich and Takahara sense (see [6]) can be represented by (ordinary) GNs. Here we will discuss the possibility to represent the functioning and result of work of different system types and more especially closed systems illustrating them by an example from medicine.

On the concepts of intuitionistic fuzzy sets and generalized nets

Intuitionistic Fuzzy Sets (***IFSs***, see, e.g. [3]) were defined as a generalization of the ordinary fuzzy sets. Let a (non-fuzzy) set *E* be fixed. An Intuitionistic Fuzzy Set *A* in *E* is defined as an object of the following form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where the functions:

$$\mu_A : E \rightarrow [0,1]$$

and

$$\nu_A: E \rightarrow [0,1]$$

define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Therefore we can define function

$$\pi_A: E \rightarrow [0,1]$$

such that

$$\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x)),$$

that corresponds to the degree of uncertainty of element $x \in E$. Obviously, when

$$\pi_A(x) = 0$$

for each $x \in E$ the set is ordinary fuzzy one.

Generalized Nets (***GNs,*** see [1]) are extensions of the Petri nets and other modifications of them. They are tools intended for detailed modelling of parallel processes.

A GN is a collection of ***transitions***, defined in turn as a set of ***places*** (see Fig.1). For every transition there is an index matrix with elements – predicates. Some GN-places contain ***tokens*** – dynamic elements entering the net with initial characteristics and getting next ones during their movement in the net. Tokens proceed from the input to the output places of the transitions if the predicate corresponding to these places is evaluated as *"true"*. Every token has its own identifier and collects its own history that could influence the development of the whole process modeled by the GNs.

Two time-moments are specified for the GNs: startup and termination of functioning, respectively.

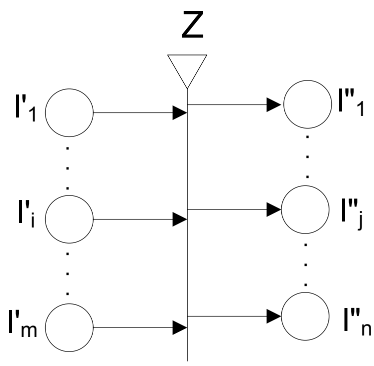

Fig. 1

GNs as well as their extensions can have only a part of their components. In this case they are called reduced GNs and reduced GNs for the respective extended types. Here we shall give the formal definition of a reduced GN without temporal components, place and arc capacities, and token, place and transition priorities. It will be followed by definitions of reduced IFGNs1 and IFGNs3.

Formally, every transition in the used below reduced GN is described by a three-tuple:

$$Z = \langle L', L'', r \rangle,$$

where

(a) L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively); for the transition these are
$L' = \{l'_1, l'_2, \ldots, l'_m\}$ and $L'' = \{l''_1, l''_2, \ldots, l''_n\}$;

(b) r is the transition's *condition* determining which tokens will pass (or *transfer)* from the transition's inputs to its outputs; it has the form of an Index Matrix (IM):

$$r = \begin{array}{c|ccccc} & l''_1 & \ldots & l''_j & \ldots & l''_n \\ \hline l'_1 & & & & & \\ \ldots & & & r_{i,j} & & \\ l'_i & & & (r_{i,j} \; - \; \text{predicate}) & & \\ \ldots & & & (1 \le i \le m, \; 1 \le j \le n) & & \\ l'_m & & & & & \end{array}$$

$r_{i,j}$ is the predicate that corresponds to the i-th input and j-th output place. When its truth value is *"true"*, a token from the i-th input place transfers to the j-th output place; otherwise, this is not possible;

The ordered four-tuple

$$E = \langle A, K, X, \Phi \rangle$$

is called a *Generalized Net* if:

(a) A is a set of transitions;
(b) K is the set of the GN's tokens;
(c) X is the set of all initial characteristics which the tokens can obtain on entering the net;
(d) Φ is the characteristic function that assigns new characteristics to every token when it makes the transfer from an input to an output place of a given transition.

The concept of IFGN1 is introduced by the above way but now the $r_{i,j}$-th component of the r-index matrices are estimated by the intuitionistic fuzzy couple $<r_{i,j}, \mu_A(r_{i,j}), \nu_A(r_{i,j})>$ of the above type.

The IFGN3 is introduced by the IFGN1 with intuitionistic fuzzy estimations of the tokens' characteristics.

A lot of operations (e.g., union, intersection and others), relations (e.g., inclusion, coincidence and others) and operators are defined over the ordinary, reduced and extended GNs. Operators change the GN-forms, the strategies of token transfer and other. They are of six types: global, local, hierarchical, reducing, extending and dynamic operators.

In [4,5] it is shown that there is very strong relationship between the theory of GNs and systems theory. There is proved that the functioning and the results of the work of different types of systems can be represented by some GNs.

In [4] are shown different possibilities for the construction of a GN, which corresponds to a given system. The first option for a description of systems is to represent the whole system with one GN. In this case is drown the possibility to obtain the same results as systems modeled by GNs, after the functioning of these GNs over tokens with initial characteristics the initial parameters of the corresponding systems. So, all information about the system is reflected in the tokens characteristics. This is the simplest way. The second possibility is to build a GN describing the different system components and representing the connections between these components.

Closed systems and their possible representation in the terms of IFGNs are considered in this paper. This is illustrated by examples in medicine.

The closed system is not affected or we are not interested in the possible affects, of the environment. It is influenced only by itself processes represented by tokens and their characteristics. In the closed systems there are no inputs and outputs, respectively the GN does not have input and output places. The closed system status is provoked by the objects, that compose the given system. So we are interested only of the things that could happen inside the system. Everything is caused by the inner objects and their relationships. In the respective GN model this is interpreted by characteristics of some tokens determined previous. When we use IFGNs3 we can give estimations of the degrees of validity and non validity, or of degree of correctness and incorrectness of the tokens characteristics of the net, that models the given closed system. There could be different kinds of tokens in the corresponding GN model. The first type could present the objects of the system. The second type – the relationships between these objects. There could be another type which describes the behaviour of the system as a whole. The status of the system could be traced with another kind of tokens. So, we can have different kinds of tokens for the different types of descriptions related with the corresponding closed system. With IFGNs1 we can estimate the truth value of the predicates of the index matrix *r*. This can help us to evaluate the relations between the objects.

New tokens could not enter the GN-models of these systems and tokens could not go out. This guarantee that the system that is presented by this GN is closed, because the exterior affects are denied. Every relationship between objects of the system can be represented by a characteristic of a suitable token in the GN.

In the paper we consider several possible relations between the objects of a given closed system and their respective GN-interpretations. The correspondence is illustrated with an example of the process of examination of patients.

The GN model

A simple example of GN-model of closed system is shown on Fig.2, where with p_i are marked the patients places, with ph – the general practitioners (GP) and with d_i – the doctors places. α – tokens are moving through places p_i, β – tokens through place ph and γ – tokens through places d_i.

The chosen closed system presents the relations between patients and doctors. Here are given several possible states and events of the system and their presentations in the corresponding GNs. In every one of them there is definite number of general practitioners – m, specialists – n and patients - p. Every patient is presented by α–token and every doctor by β– or γ– token. Initially, these tokens stay in places p_2, ph and d_2 of the GNs. They have the all needed information for the patients and doctors as characteristics.

The given GN model (Fig.2) represents the following situation: closed system, which corresponds to a given health care unit. This unit has definite, invariable number of patients, GPs and doctors. Every one of these participants has its own personal information and is presented by token with corresponding characteristic. Initially, the tokens are in the relevant places and they dispose with the all needed information as an initial characteristic. Through the process of functioning of the system (GN), the tokens obtain next characteristics. This is the way of accumulation of knowledge and history. So, the GN model can be used for information extraction. The characteristics are specific and depend on the participants. We can evaluate their truth values with the use of IFGN3. For this reason degrees of correctness and incorrectness < x_i^α, $\mu_A(x_i^\alpha)$, $\nu_A(x_i^\alpha)$> are defined for *i*-th characteristic of corresponding token α that we will denote by x_i^α, where $\mu_A(x_i^\alpha)$ and $\nu_A(x_i^\alpha)$ represent the degree of validity and degree of non-validity (degree of correctness and incorrectness) of x_i^α. These degrees can be influenced on the degrees of validity and non validity of the truth values of given predicates. These two types of estimations make possible the movement of the tokens from one place to another.

All tokens have general in a part of the characteristics: personal information, kind of participant in the system. But along with this part there are specific characteristics. They are obtained in time of tokens movements through the net, which corresponds to the patients' attendance from the GPs or doctors. The α – tokens characteristics are extended with the names and the skills of the relevant doctors, and with the new diagnoses and treatments. The β– and γ– tokens characteristics are extended with the names, diagnoses and treatments of the relevant patients, and other specific information. This information has intuitionistic fuzzy estimations like degrees of correctness and incorrectness of the personal information, diagnoses and treatments; levels of the skills of the doctors and others. On the other hand, the predicates of the transitions are related with: the personal choices of doctors or patients, the needed treatment, the needed specialists or something else. They can have intuitionistic fuzzy estimations of their degrees of validity and non validity too. But these degrees can be influenced by the estimations of the corresponding characteristics.

The GN model from Fig.2 has three transitions. Transition Z_1 represents the treatment of the patients from the relevant GPs. Z_2 represents the necessity of the treatment of a concrete specialist. The necessity of the treatment of another specialist or GP, or patient's stabilization is represented by transition Z_3.

The transitions can be described in this way:

$$Z_1 = \langle \{ph, p_2, p_6\}, \{ph, p_1, p_2\}, r_1 \rangle,$$

where:

r_1 =

	ph	p_1	p_2
ph	w_{ph_ph}	false	false
p_2	false	w_{p2_p1}	w_{p2_p2}
p_6	false	w_{p6_p1}	w_{p6_p2}

,

where:

w_{ph_ph} = "a patient needs health consultation"

w_{p2_p1} = "the patient needs health consultation from his/her GP",

$w_{p2\_p1} = w_{p6\_p2}$

$w_{p6\_p1} = w_{p2\_p1}$

$w_{p6\_p2} = \neg\, w_{p6\_p1}$.

If one of the patients needs health consultation from his/her GP, one α – token moves from place p_2 to place p_1 (he/ she needs consultation from specialist) or place p_2 (he/ she does not need consultation from specialist). The evaluation of this predicates is based on the intuitionistic fuzzy estimations of the characteristics of the corresponding tokens in case of IFGN3. If one α-token has intuitionistic fuzzy estimations of its characteristics, than we can introduce priorities and thresholds for the predicates for its transitions. Simultaneously, one β – token, which presents the relevant GP, moves from place ph to place ph. The tokens obtain as a characteristic in place p_1: “the complaints and the diagnoses of the given patient; the treatment of the GP; the other needed treatments by specialists”. The β – tokens obtain as characteristic in place ph: “the personal information of the treated patient and other specific information about his/her status” with definite intuitionistc fuzy estimation. The tokens in place p_2 like these in place p_1 also obtain as a characteristic: “the complaints and the diagnoses of the given patient; the treatment of the GP”.

$$Z_2 = <\{p_1, p_5, d_2, d_3\}, \{p_3, d_1, d_2\}, r_2>,$$

where:

$r_2 =$

	p_3	d_1	d_2
p_1	w_{p1_p3}	false	false
p_5	w_{p5_p3}	false	false
d_2	false	w_{d2_d1}	false
d_3	false	false	true

,

where:

w_{p1_p3} = “there is corresponding token in place d_2”;

$w_{p5\_p3} = w_{p1\_p3}$;

w_{d2_d1} = ”there is a patient in place p_1 and he needs health consultation from a doctor”

If the patient needs health consultation from a specialist and the given specialist is available (there is corresponding γ– token in place d_2) the α – token moves from place p_1 to place p_3. In place d_1 the token obtains as a characteristic: “the personal information of the treated patient and other specific information about his/her status”. The tokens in places p_3 and d_2 do not obtain any information as characteristics.

$$Z_3 <\{p_3, p_4, d_1\}, \{p_4, p_5, p_6, d_3\}, r_3>,$$

where:

$r_3 =$

	p_4	p_5	p_6	d_3
p_3	w_{p3_p4}	w_{p3_p5}	w_{p3_p6}	false
p_4	w_{p4_p4}	w_{p4_p5}	w_{p4_p6}	false
d_1	false	false	false	true

,

where:

w_{p3_p4}= ”the patient need more health care”,

w_{p3_p5}= ”the patient needs consultation with another specialist”,

$w_{p3\_p6} = \neg w_{p3\_p4} \wedge \neg w_{p3\_p5} \wedge$ ”after a medical examination by specialist the patient needs consultation from his/her GP or he/she returns to his/her initial state”,

$w_{p4\_p4} = w_{p3\_p4}$,

$w_{p4\_p5} = w_{p3\_p5}$,

$w_{p4\_p6} = w_{p3\_p6}$.

The α – token which moves from place p_3 to place p_4 obtains as a characteristic the information about the relevant specialist, who treats the patient; the diagnoses and the needed next treatment in addition with the whole recent information of the token. The token obtains as a characteristic in place p_5: “personal information of the relevant specialist; the next needed specialist or specialists; the diagnoses; the treatments and the next needed treatments” in addition with the characteristics received up to now. The characteristic of the token in place p_6 is the same as in place p_5 with the exception of the next needed specialist. The tokens in place d_3 do not obtain any characteristics.

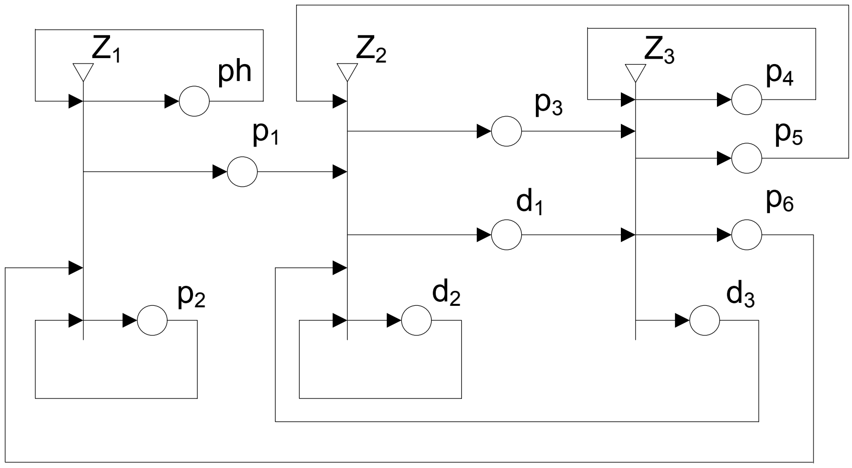

Fig.2. An IFGN model of a closed system: a health care unit as a closed system represented by an IFGN model

There is no problem in case the patients needs of health care consecutively. But at one moment of time two or more patients could need doctors' treatment. This is not problem still, if they needs of different GPs or specialists, who are free at the moment. The difficulties will become when the needed doctor is busy or simultaneously two or more patients try to hold his/her attendance. In the first situation the decision is the patient to wait for his/her release or to refer to another doctor. The second situation is more complicated. A possible decision of it is the use of priorities for the patients and their statuses. In this situation the use of IFGNs of first or third type is necessary. But if there are two patients with equal priorities the solution could be an alternative doctor.

We consider several possible situations that can be generated in the GN-model.

The first situation describes the presence of hospitals with infinity resources as beds, materials, apparatuses, etc. in the terms of the model. In this case the patients have no problems with the time of stay. It is not a critical parameter. So this system will function in the same way for whole model time. There will not have any problems with serving when the number of general practitioners, specialists and patients is infinity. At one moment of time one or more of the doctors could become ill and this could make them patients. So, some of the β– or γ– tokens could belong to the both sides of the participants in the model. The same could be with some of the α–tokens, which could become doctors. And this could be for a short or long period of time.

The second situation presents hospitals with non infinity resources. So after a determinate period of time they will be exhausted and the system, respectively the GN, will stop its functioning. Again, it is not excepted that some of the α– , β– or γ– tokens could belong to the both sides of the participants in the model.

The situation with variant numbers of GPs, specialists and patients is very interesting.
In first case the general practitioners or doctors are less in number from patients. Then it would be suitable queues to be created and there the patient comes with some priority or urgency. These priorities are received from the intuitionistic fuzzy estimations of the tokens characteristics (IFGN3). If the number of patients who need health consultation is many

greater from number of GPs they will be served with some delay as the capacity of practitioners is limited.
The second case is when the number of GPs and specialists is greater than this of patients. It is possible the number of patients who need consultation to be very small and than as a result the great number of doctors have become inactive- there will not have occupation for them. So the system will work inefficiently.

Conclusion

Closed systems are isolated of the affects of the environment. But there are continuous processes, which pass simultaneously inside the system. Its objects are in permanent relationships. The GNs and their extensions IFGNs1 and IFGNs3 are suitable tool for modeling of these relationships and inner processes. The characteristics of the tokens, which represent the relevant participants, can be used for gaining the history of the system through its functioning. And characteristics' intuitionistic fuzzy estimations give additional information and allow extended investigations of the system. The IFGN-model can help us to trace the possible situations of the system, to find the good and bad sides of the system, to optimize the flowing processes.

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