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Index matrix interpretation and intuitionistic fuzzy estimation of the diet problem

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Abstract: The objective of the diet problem is to select a family of foods that will satisfy a set of daily nutritional requirement at minimum cost. In this paper we investigate different aspects and modifications of the diet problem, such as the compatibility/incompatibility of a collection of diets and their merging. We employ the notions of Index Matrix (IM) and Intuitionistic fuzzy sets (IFS), Orderings on IFSs in order to derive an intuitionistic fuzzy estimation of a new, aggregated diet table. We also study the question of compatibility of collection of diets.
Keywords: Index matrix, Intuitionistic fuzzy orderings, Intuitionistic fuzzy estimation, Diet problem.
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1 Preliminaries

The diet problem was one of the first optimization problems studied in the 1930s and 1940s. The objective of the diet problem is to select a family amounts of foods that will satisfy a set of daily nutritional requirement at minimum cost. It has been motivated by the Army's desire to propose a balanced and healthy feeding of the soldiers for minimal cost. The classical problem is expressed as a linear program where the constraints are to satisfy the specified nutritional requirements. In this paper we employ the notions of index matrix (IM) and intuitionistic fuzzy sets (IFS), orderings on IFSs and and intuitionistic fuzzy estimations. Eventually, we propose a matrix with intuitionistic fuzzy values and intuitionistic fuzzy values for the price of the serving for every food. Considering different diets (for example in cases of some illness, loose weight diet, etc) through the operations least upper bound (sup) $\vee_\pi$ and greatest lower bound (inf) $\wedge_\pi$ of π-ordering for $(IFS(X), \preceq_\pi)$ we are going to determine if the conditions (constraints) of these diets meet or they are incompatible.

As mentioned above, the classical diet problem is stated as a linear optimization problem. Suppose that we have a matrix of real numbers $A = \{a_{ij} \colon i = 1, \ldots, m \text{ and } j = 1, \ldots, n\}$, a n-dimensional real vector $c^T = (c_1, \ldots, c_n)$ and a m-dimensional real vector $b^T = (b_1, \ldots, b_m)$ such that. In the standard form of a linear optimization problem we want to minimize the function, called *objective function*, $c^T x$ subject to the constraints $Ax = b$. Of course, x is an n-dimensional vector of unknowns. That is, we want to find out $x_0 \in \{x \mid Ax = b \text{ and } x \geq 0\}$ such that

$$c^T x_0 = \min\{c^T x \mid Ax = b \text{ and } x \geq 0\}. \tag{1}$$

The solution of the above problem may be solved with Matlab, there are many other free libraries for linear optimization in Python as well. In the diet problem come across another form of linear optimization problem, which have different form of the constraints, inequality form, but it can be transformed to the standard case.

In DP case, we have k types of foods, labeled by the index set $K = \{1, \ldots, k\}$, i.e. $F_1, \ldots, F_k$ (apples, carrots, cheese, milk, etc). There are also l types of nutrients, labeled by the index set $L = \{1, \ldots, l\}$, i.e. $N_1, \ldots, N_l$ (Calories, Protein, Fat, Vitamin A, Vitamin C, etc.). Vector $c = (c_1, \ldots, c_k)^T$ represents the price per serving for every food F_i. Matrix $A = \{a_{ij} \colon i \in K, j \in L\}$ now represents the amount of the nutrient N_j in a serving of the food F_i. In the standard DP we are also given two l-dimensional vectors m and M which represent an lower and upper bound respectively of the amount of the corresponding nutrient for the daily portion of the diet. Our DP now can be expressed in the following way: find out $x_0 \in \{x \mid m \leq A^T x \leq M \text{ and } x \geq 0\}$

$$c^T x_0 = \min\{c^T x \mid m \leq A^T x \leq M \text{ and } x \geq 0\}. \tag{2}$$

The above statement can be rewritten in the following way

$$\min\{c^T x \mid [A^T, -A^T]^T x \leq [M, m]^T \text{ and } x \geq 0\},$$

because $m \leq A^T x \leq M$ can be split into $-A^T x \leq -m$ and $A^T x \leq M$. And therefore, we get $\min\{c^T x \mid B' x \leq b'\}$, putting $B' = [A^T, -A^T]^T$ and $b' = [M, m]^T$. In $\min\{c^T x \mid B' x \leq b'\}$ we can put additional positive unknowns $(z_1, \ldots, z_s)$, where s is the dimension of the vector b'. Therefore $B' x \leq b'$ becomes $B' x + z = [B', I][x, z]^T = b'$, where I is the $s \times s$ unity matrix. Now the problem takes the form

$$\min\{[c, 0, \ldots, 0]^T [x, z] \mid [B', I][x, z]^T = b' \text{ and } x \geq 0\},$$

which is the standard form of an optimization problem, that can be solved in Matlab, for instance.

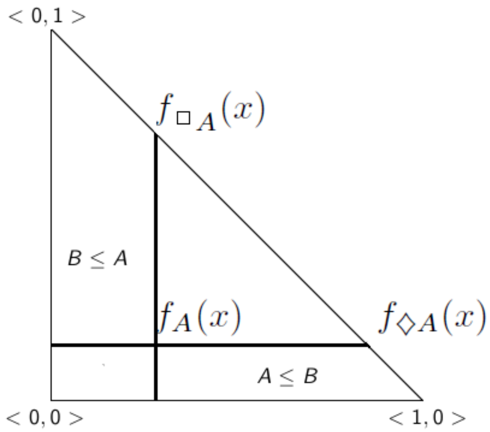

Figure 1: Triangular representation of the the intuitionistic fuzzy sets A and $B \in IFS(X)$ in a particular point $x \in X$, where $f_A(x)$ stands for the point on the plane with coordinates $(\mu_A(x), \nu_A(x))$. $\Box A$ and $\Diamond A$ stand for the two modal operators "necessity" and "possibility" acting on A.

1.1 Introduction to intuitionistic fuzzy sets

A fuzzy set in X (cf. Zadeh [9]) is given by

$$A^{'} = \{\langle x, \mu_{A'}(x)\rangle | x \in X\} \tag{3}$$

where $\mu_{A'}(x) \in [0, 1]$ is the *membership function* of the fuzzy set $A^{'}$. As opposed to the Zadeh's fuzzy set (abbreviated FS), Atanassov (cf. [1], [2]) extended its definition to an intuitionistic fuzzy set (abbreviated IFS) A, given by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{4}$$

where: $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{5}$$

and $\mu_A(x)$, $\nu_A(x) \in [0, 1]$ denote a *degree of membership* and a *degree of non-membership* of $x \in A$, respectively. An additional concept for each IFS in X, that is an obvious result of (4) and (5), is called vspace-2mm

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{6}$$

a *degree of uncertainty* of $x \in A$. It expresses a lack of knowledge of whether x belongs to A or not (cf. Atanassov [1]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$. Uncertainty degree turn out to be relevant for both - applications and the development of theory of IFSs. For instance, distances between IFSs are calculated in the literature in two ways, using two parameters only (cf. Atanassov [1]) or all three parameters (cf. Szmidt and Kacprzyk [8]).

For more detailed information regarding modal operators the reader may refer to [2], Ch. 4.1. "Necessity" and "possibility" operators (denoted $\Box$ and $\Diamond$ respectively) applied on an intuitionistic fuzzy set $A \in IFS(X)$ have been defined as:

$$\begin{aligned} \Box A &= \{\langle x, \quad \mu_A(x), \quad 1 - \mu_A(x) \quad \rangle | x \in X\} \\ \Diamond A &= \{\langle x, \quad 1 - \nu(x), \quad \nu_A(x) \quad \rangle | x \in X\} \end{aligned}$$

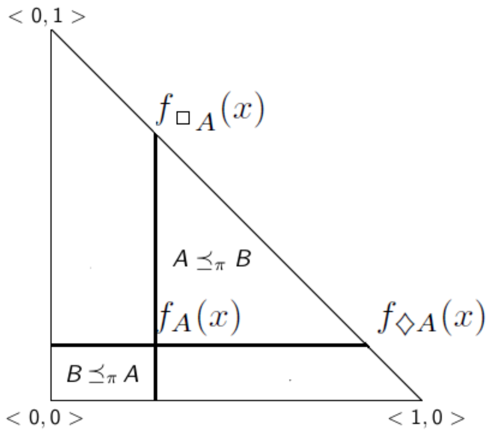

Figure 2: Triangular representation of the the intuitionistic fuzzy sets A and $B \in IFS(X)$ in a particular point $x \in X$, where $f_A(x)$ stands for the point on the plane with coordinates $(\mu_A(x), \nu_A(x))$. $\Box A$ and $\Diamond A$ stand for the two modal operators "necessity" and "possibility" acting on A.

From the above definition it is evident that

$$\star\colon IFS(X) \longrightarrow FS(X) \tag{7}$$

where $\star$ is the prefix operator $\star \in \{\Box, \Diamond\}$, operating on the class of intuitionistic fuzzy sets.

Talking about partial ordering on IFSs, we will by default mean $(IFS(X), \leq)$ where $\leq$ stands for the standard partial ordering in IFS(X). That is, for any two A and $B \in IFS(X)$: $A \leq B$ is satisfied if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for any $x \in X$. On Fig. 1 one may see the triangular representation of the two chosen A and B in a particular point $x \in X$, where $f_A(x)$ stands for the point on the plane with coordinates $(\mu_A(x), \nu_A(x))$. That is, $A \leq B$ means exactly that the point $f(B)$ must lie in the trapezoidal area (or on its border) defined by the points $\langle \mu_A(x), 0\rangle, \langle 1, 0\rangle, f_{\Diamond A}(x), f_A(x)$. On the other hand, $B \leq A$ is satisfied exactly when point $f(B)$ lies in the trapezoidal figure (or on its border) enclosed by the points $f_A(x), f_{\Box A}(x), \langle 0, 1\rangle, \langle 0, \nu_A(x)\rangle$.

The reader is referred to Marinov [6], where a new partial ordering over the class of IFSs has been introduced. Namely, the so called π*-ordering* ($\preceq_\pi$), which turns $(IFS(X), \preceq_\pi)$ into a complete left lattice but not a right one, i.e. it is not a lattice. Left (right) lattice is a partially ordered set in respect with his ordering, for which any two elements have a greatest lower (least upper) bound. A left (right) lattice is called *c*omplete if each subset (not only finite) has a greatest lower (least upper) bound. Complete lattice is called a partially ordered set which is a complete left and complete right lattice simultaneously. The notion of π-ordering has been employed for the introduction of an *index of indeterminacy* measuring how far (close) is an IFS from (to) the family of the usual FSs on the same universe X. A few examples of index of indeterminacy have been introduced based on the structure and properties of the underlying universe. It has to satisfy three corresponding axioms and should not be confused with the degree of uncertainty called also index of indeterminacy by some authors. In contrast to the standard partial ordering $A \leq B$ between two IFSs A and B, the π-ordering $A \preceq_\pi B$ is satisfied iff $\mu_A(x) \leq \mu_B(x)$ and

$\nu_A(x) \leq \nu_B(x)$ for all $x \in X$. The triangular representation on Fig. 2 gives us that $A \preceq_\pi B$ iff for all $x \in X$, $f_B(x)$ lies within (or on the border of) the triangular area defined by the points $f_A(x), f_{\Diamond A}(x)$ and $f_{\Box A}(x)$. On the other hand, $B \preceq_\pi A$ is satisfied iff for all $x \in X$, $f_B(x)$ lies within (or on the border of) the rectangular area defined by the points $\langle 0, 0\rangle, \langle \mu_A(x), 0\rangle, f_A(x)$ and $\langle 0, \nu_A(x)\rangle$. Moreover, the family of maximal elements of $(IFS(X), \preceq_\pi)$ consists exactly of the family of the ordinary fuzzy sets $FS(X)$ and there is a unique minimal element $0_\pi := \langle 0, 0\rangle$, see Fig. 2.

1.2 Introduction to index matrices

Let $\mathcal{I}$ be a fixed set of indices and $\mathcal{R}$ be a partially ordered set, a left (right) lattice, complete lattice or in particular the set of the real numbers. By $IM(\mathcal{R}, \mathcal{I})$ (cf. [3], p. 183) with index sets K and L ($K, L \subset \mathcal{I}$) is denoted the object:

$$A = [K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \dots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \dots & a_{k_2,l_n} \\ \vdots & & & & \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \dots & a_{k_m,l_n} \end{array} \tag{8}$$

(or briefly: $[K, L, \{a_{k_i,l_j}\}]$), where $K = \{k_1, k_2, ..., k_m\}$, $L = \{l_1, l_2, ..., l_n\}$, for $1 \leq i \leq m$, and $1 \leq j \leq n : a_{k_i,l_j} \in \mathcal{R}$. Employing index matrices in analytical expressions the subindices i and j respectively will be omitted.

Taking two IMs $A = [K, L, \{a_{k,l}\}]$ and $B = [P, Q, \{b_{p,q}\}]$ with elements in $\mathcal{R}$, let $\circ$ be some binary operation in $\mathcal{R}$ with range $\mathcal{R}$, i.e. $\circ : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$. And let us suppose that $0_\mathcal{R}$ is the minimal element of $\mathcal{R}$ (or some other appropriate one, to be specified in each concrete case). Then we can define the operation addition of A and B with respect to $(\circ)$, $C = A \oplus_{(\circ)} B = [K \cup P, L \cup Q, \{c_{s,t}\}]$ in the following way (see Fig. 3):

$$c_{s,t} = \begin{cases} a_{k,l} & \text{if } s = k \in K \text{ and } t = l \in L - Q \\ & \text{or } s = k \in K - P \text{ and } t = l \in L; \\ & \\ b_{p,q} & \text{if } s = p \in P \text{ and } t = q \in Q - L \\ & \text{or } s = p \in P - K \text{ and } t = q \in Q; \\ & \\ a_{k,l} \circ b_{p,q} & \text{if } s = k = p \in K \cap P \text{ and } t = l = q \in L \cap Q \\ 0_\mathcal{R} & \text{otherwise} \end{cases} \tag{9}$$

Let us define the $\circ$-row-agregation of the IM $A = [K, L, \{a_{k,l}\}]$ in the following way: $row(A, \circ) = [K_0, L, \{a^{(\circ)}_{k_0,l}\}]$, where $K_0 = \{k_0\}$ such that $k_0 \notin K$ and

$$a^{(\circ)}_{k_0,l} = \circ\{a_{k,l} : k \in K\}, \text{ for all } l \in L$$

In the above introduction of the operation of row-aggregation we suppose that the operation $(\circ)$ is defined over the Cartesian product of any finite degree of the underlying set $\mathcal{R}$ with range $\mathcal{R}$ and moreover, it is commutative. That is,

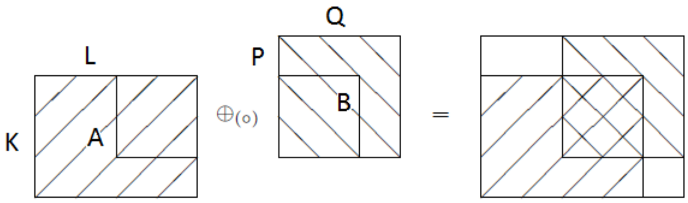

Figure 3: Graphical representation of the operation addition with respect to $(\circ)$ of the IMs $A = [K, L, \{a_{k,l}\}]$ and $B = [P, Q, \{b_{p,q}\}] \in IM(\mathcal{I}, \mathcal{R})$.

$$\begin{array}{rcl} \circ\colon \mathcal{R} \times \cdots \times \mathcal{R} & \longrightarrow & \mathcal{R} \\ (a_1, \ldots, a_n) & \longmapsto & a, \end{array} \tag{10}$$

where $\circ(a_1, \ldots, a_n) = a = \circ(a_{\tau(1)}, \ldots, a_{\tau(n)})$ for any permutation τ of $1, \ldots, n$. It is not obligatory $\circ$ to be associative. For instance, $\circ$ can be taken the $\min$ or $\max$ operation. And an example of non-associative operation is the mean average avg if $\mathcal{R}$ is the field of real numbers. Indeed, let us take three different real numbers $a, b, c \in \mathcal{R}$. Since $avg(a, b, c) = \frac{a+b+c}{3}$ and $avg(avg(a, b), c) = \frac{a+b+2c}{4}$, one can easily remark that $avg(a, b, c) \neq avg(avg(a, b), c)$, unless $c = avg(a, b)$ which is not satisfied in general .

2 IM and IFS interpretations of the diet problem

We are going to show now that in some cases it is more convenient to use index matrices because of the arbitrary character of index sets of the columns. Suppose that we are given an index matrix $A = [K, L, \{a_{k,l}\}] \in IM(\mathcal{I}, \mathcal{R})$, representing the the information about the amounts of the nutritions per serving food and the corresponding lower and upper bounds, $0 \leq m \leq M$ as in (2). Here again K corresponds to the foods, L corresponds to the nutrients and of course we are given a vector $c = \{C_k\colon k \in K\}$ representing the prices of the corresponding foods per serving. Therefore, we can state the following definition:

Definition 1. *A* **diet statement***, in the above introduced denotations, is the triple* (A, m, M)*. A* **diet problem** *on the other hand is the optimization problem, described in the previous section, for the quadruple* (A, m, M, c)*.*

Let in K some of the foods are familiar, for instance we can have different types of apples, types of carrots, ect, which have to be combined in a new diet table. In the matrix corresponding to the new diet table we have to put together the "familiar" foods into equivalence classes, whereas the columns (nutrients) stay unchanged. Therefore, the corresponding element in the new IM, i.e. the amount of the nutrition per serving (unit) becomes the aggregation over all the "familiar" foods in the corresponding subset of K, keeping the rows unchanged. That is, let us split the set K in subsets which don't meet together, i.e. we set an equivalence relation in K - $K_1, \ldots K_s$ such that $K = \cup\{K_i : i = 1, \ldots, s\}$ and $(\forall i, j = 1, \ldots, s)(i \neq j \Rightarrow K_i \cap K_j = \emptyset)$. According to this splitting of K there is a corresponding splitting of the initial IM $A = [K, L, \{a_{k,l}\}]$ into the IMs, $A_i = [K_i, L, \{a_{k,l}\}]$ for $i = 1, \ldots, s$. The matrix corresponding to the new diet table A' will have s rows, built up through the row-aggregation of the index matrices $A_1, \ldots, A_s$. That is, the rows of this matrix will consist of

$$row(A_i, \circ) = [\{k_i\}, L, \{a^{(\circ)}_{k_i,l}\}], \tag{11}$$

where $a^{(\circ)}_{k_i,l} = \circ\{a_{k,l} : k \in K_i\}$, for all $l \in L$ and $(\forall i = 1, \ldots, s)(k_i \notin K \cup L)$. The vector C corresponding to the prices of the foods per serving has also to be aggregated in the same way. Therefore, the new vector corresponding to the prices of the aggregated foods becomes $C' = \{c^{(\circ)}_{k_1}, \ldots, c^{(\circ)}_{k_s}\}^T$, where

$$c^{(\circ)}_{k_i} = \circ\{c_k : k \in K_i\}.$$

The above operation $(\circ)$ has to possess the properties described in (10). As already mentioned such operations are the lub (least upper bound), glb (greatest lower bound) and the mean average. The first two operations coincide with the $\max$ and $\min$ operations for partially ordered sets, which are totally ordered (such as the field of real numbers we are working on).

The mean average operation proves very useful in this situation. Suppose we want to combine all types of apples, each of which are supposed to possess similar amounts of nutrients per serving. The corresponding prices are aggregated through the mean average as well, which looks intuitively suitable. Moreover, while aggregating to the new diet table we may want to be sure that the person adopting this diet will take strictly at least m_l of the nutrition $l \in L$ per daily serving. Therefore, we take $\circ = \min$ in $a^{(\circ)}_{k_i,l} = \circ\{a_{k,l} : k \in K_i\}$. On the other hand, if for some $l \in L$ we want the person to take strictly at most M_l of the nutrition l per daily serving. In this case we take $\circ = \max$ to be sure that eating any arbitrary type of apples (carrots or other types of similar foods) from the resolved modified diet problem, the person won't take more than the maximum amount of the nutrition l per serving (which wouldn't be the case if we have taken $\circ = avg$).

Other interesting case of row aggregation of the initial DP matrix gives us intuitionistic fuzzy estimations as values for the matrix representing the new diet table, i.e. the diet table of the modified DP. Let us consider the operation

$$\circledast : \mathcal{R} \times \cdots \times \mathcal{R} \longrightarrow \mathcal{R} \times \mathcal{R}, \tag{12}$$

such that for each $(a_1, \ldots, a_n) \in \mathcal{R}^n$ we have that

$$\circledast(a_1, \ldots, a_n) = (\min\{a_j : j = 1, \ldots, n\}, \max\{a_j : j = 1, \ldots, n\}).$$

Supposing that $\tilde{a} = \max\{a_{ij} : i \in K, j \in L\}$ we can take the interval $[0, \tilde{a}]$ as a range interval for IFS or equivalently interval valued sets (IVS). That is, we would have: $\mu : X \to [0, \tilde{a}]$ and $\nu : X \to [0, \tilde{a}]$ such that

$$0 \leq \mu(x) + \nu(x) \leq \tilde{a}. \tag{13}$$

By dividing $\mu(x)$ and $\nu(x)$ by $\tilde{a}$ we normalize the range interval and get IFSs in the standard way, as described earlier. Interval valued sets are practically almost the same as intuitionistic fuzzy sets, but in place of the second argument (degree of uncertainty ν), the value $1 - \nu$ is applied (or $\tilde{a} - \nu$) in our case. Applying the new aggregation operation in (11), i.e. $\circ = \circledast$, we obtain an interval valued estimation. Which gives us an appropriate information about the bounds of the original data (in contrast to the mean average operation, for instance). The operation

$$\circledast_{ifs}(a_1, \ldots, a_n) = (\min\{a_j : j = 1, \ldots, n\}, \tilde{a} - \max\{a_j : j = 1, \ldots, n\}), \quad (14)$$

provides us with an intuitionistic fuzzy estimation. Since $-\max\{a_j : j = 1, \ldots, n\} = \min\{-a_j : j = 1, \ldots, n\}$, the expression (14) for the operation $\circledast_{ifs}$ can be rewritten as

$$\circledast_{ifs}(a_1, \ldots, a_n) = (\min\{a_j : j = 1, \ldots, n\}, \min\{\tilde{a} - a_j : j = 1, \ldots, n\}). \quad (15)$$

3 Compatibility of a set of diets

Let us now suppose that a person has to adopt a few diets simultaneously. This situation could occur, for instance, if the person is suffering from few diseases simultaneously. We are going to model the question of *compatibility/incompatibility* of a collection of different diets and match them in a *strong* and *weak* way, respectively.

Let us fix a diet table, corresponding to a particular index matrix $A = [K, L, \{a_{k,l}\}] \in IM(\mathcal{I}, \mathcal{R})$. For the chosen matrix A there are many corresponding constraints (m, M), forming particular *diets* (A, m, M) (as introduced in Definition 1).

Definition 2. *Let us say that two diets $D_1 = (A, m_1, M_1)$ and $D_2 = (A, m_2, M_2)$, corresponding to one diet table A are* **compatible** *iff for each nutrition $l \in L$ the intersection of the intervals $[m_{1,l}, M_{1,l}]$ and $[m_{2,l}, M_{2,l}]$ is not empty.*

The two diets D_1 and D_2 are compatible by definition, if $[m_{1,l}, M_{1,l}] \cap [m_{2,l}, M_{2,l}] \neq \emptyset$ for each nutrition $l \in L$. But in fact, we have that

$$[m_{1,l}, M_{1,l}] \cap [m_{2,l}, M_{2,l}] = [\max(m_{1,l}, m_{2,l}), \min(M_{1,l}, M_{2,l})].$$

Remark 1. *Since we are working on a fixed collection of nutritions L, the notions $\min(m_1, m_2)$ and $\max(m_1, m_2)$ will stand for the finite series $\{\min(m_{1,l}, m_{2,l}) : l \in L\}$ and $\{\max(m_{1,l}, m_{2,l}) : l \in L\}$, respectively.*

Therefore, for the constraints of the two diets (m_1, M_1) and (m_2, M_2), we can just write $(m_1, M_1) \wedge (m_2, M_2) = [\max(m_1, m_2), \min(m_1, m_2)]$. By analogy, we can introduce the operation $(m_1, M_1) \vee (m_2, M_2) = [\min(m_1, m_2), \max(m_1, m_2)]$, which is always defined, even if the diets D_1 and D_2 are not compatible.

Definition 3. *Suppose that the diets D_1 and D_2 are compatible, then their* **strong matching** *is defined as*

$$D_1 \Cap D_2 := (A, \max(m_1, m_2), \min(M_1, M_2)).$$

Even if D_1 and D_2 are not compatible, let us define their **weak matching** *as*

$$D_1 \Cup D_2 := (A, \min(m_1, m_2), \max(M_1, M_2)).$$

One may easily remark that the weak matching can always be defined, since for each nutrition $l \in L$ we have that $\min(m_{1,l}, m_{2,l}) \leq \max(M_{1,l}, M_{2,l})$, while $\max(m_{1,l}, m_{2,l}) \leq \min(M_{1,l}, M_{2,l})$ is not satisfied in general. If one wants to adopt a few diets simultaneously, his final diet has to meet the constraints of all the composing diets. That is exactly the above defined strong matching $D_1 \Cap D_2$ of the two diets. On the other hand, the meaning of the weak matching $D_1 \Cup D_2$ is that its constraints are actually the weakest of the two composing diets. That is, the constraints of the weak matching, for each nutrition the person has to meet just one of the constraints of the composing diets. The operations of diets matching can be extended to the case of any finite collection of diets. As one may easily check, the operations $\Cup$ and $\Cap$ are commutative and associative. Therefore, for a collection of diets $\mathcal{D}$ corresponding to a fixed diet table $A \in IM(\mathcal{I}, \mathcal{R})$, one may build up their strong (if it exists) and weak matching and write down $\Cap\mathcal{D}$ and $\Cup\mathcal{D}$, respectively.

So far we have been working with ordinary intervals in this section. But all this can be translated in the intuitionistic fuzzy language, as done in the previous section. In this way, we can represent the constraints (m, M) of the diet $D = (A, m, M)$ as elements of $IFS(L)$ with a range interval $[0, \tilde{M}]$, where $\tilde{M}$ is a large enough real value. The switch to the IF-language is worth, since in [6] have already been developed interesting formulas for the so called *index of indeterminacy*, measuring how close/far is an IFS to/from an ordinary FS. And these formulas can be adequately applied in our diet investigation.

As already mentioned in the introductory section, $\preceq_\pi$ is a complete left lattice but not a right one. For any usual fuzzy set $F \in FS(X)$, i.e. $F \in IFS(X)$ such that

$$\pi_F = 1 - \mu_F - \nu_F = 0$$

on X, let us define

$$\Theta(F) := \{A \mid A \in IFS(X) \,\&\, A \preceq_\pi F\}.$$

It has been shown (see [6], Sect. 2) that

$$(\Theta(F), \preceq_\pi) \subsetneq (IFS(X), \preceq_\pi)$$

is a complete lattice, i.e.

$$(\forall \mathcal{A} \subseteq \Theta(F))(\exists \inf \mathcal{A} \in \Theta(F) \,\&\, \exists \sup \mathcal{A} \in \Theta(F)).$$

In the above expression, inf and sup stand for the greatest lower bound and least upper bound related to the π-ordering, to be denoted also $\wedge_\pi$ and $\vee_\pi$. One can easily check the following statement:

Proposition 1. *The operation strong matching $\Cup$ corresponds to the operation $\vee_\pi$ and the operation weak matching $\Cap$ corresponds to the operation $\wedge_\pi$.*

It turns out that the constraints (in the IF-iterpretation) of the diets D_1 and D_2 - (m_1, M_1) and $(m_2, M_2) \in IFS(L)$ are compatible iff for them there is a π-supremum or equivalently (see [6], Remark 3) iff there is an ordinary fuzzy set $F \in FS(L)$, such that $(m_1, M_1), (m_2, M_2) \in \Theta(F)$.

The index of indeterminacy of the constraints of a diet $D = (A, m, M)$, i.e. $ind_\pi(m, M) \in [0, 1]$ measures now the degree of strictness of the diet. That is, the smaller the value of the index, the stricter the diet is supposed to be.

4 Conclusion

In this paper we have investigated the diet problem from different points of view, employing the framework of the index matrices. A few interesting operations have been defined, such as putting together some of the foods from a diet table and an appropriate IF-aggregation of the values for the obtained resulting diet table. Different aspects of the question of compatibility/incompatibility of a collection of diets have been observed, by introducing the notions of strict and weak matching, respectively. The π-ordering and index of indeterminacy for IFSs, measuring the degree of strictness of the diet, have also been applied.

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