

ON AN INTUITIONISTIC FUZZY NORM

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The intuitionistic fuzzy sets are defined in [1] and two norms are introduced in [2] for them. The intuitionistic fuzzy propositional calculus is introduced in [3-5] (see also [6]) and the corresponding two norms are introduced in [7] in the forms:

$$\sigma_1(A) = \mu(A) + \gamma(A) \quad \text{and} \quad \sigma_2(A) = (\mu(A) + \gamma(A))^{\frac{2}{2}},$$

where A is a propositional norm. The form of the third norm is defined as:

$$\sigma_3(A) = \frac{\mu(A) - \gamma(A) + 1}{2}.$$

Easily it can be seen that σ_3 is not comparable with σ_1 and σ_2 .

Let us below write $\sigma(A)$ instead of $\sigma_3(A)$.

This norm is a modification of an expression from [8].

THEOREM: For every two propositional forms A and B and for every $\alpha, \beta \in [0, 1]$:

- (a) $\sigma(\neg A) = 1 - \sigma(A);$
- (b) $\sigma(A \wedge B) \leq \min(\sigma(A), \sigma(B));$
- (c) $\sigma(A \vee B) \geq \max(\sigma(A), \sigma(B));$
- (d) $\sigma(A \supset B) \geq \max(\sigma(\neg A), \sigma(B));$
- (e) $\sigma(\exists x A(x)) \geq \max_{x \in S} \sigma(A(x));$
- (f) $\sigma(\forall x A(x)) \leq \min_{x \in S} \sigma(A(x));$
- (g) $\sigma(\Box A) = \mu(A);$
- (h) $\sigma(\Diamond A) = 1 - \gamma(A);$
- (i) $\sigma(D_\alpha(A)) = \mu(A) + \alpha(1 - \sigma_1(A));$

$$(j) \sigma(F_{\alpha, \beta}(A)) = \sigma(A) + \frac{\alpha - \beta}{2} \cdot (1 - \sigma_1(A))$$

for every $\alpha, \beta \in [0, 1]$ such that $\alpha + \beta \leq 1$;

$$(k) \sigma(G_{\alpha, \beta}(A)) \geq \sigma(A) \cdot \max(\alpha, \beta);$$

$$(l) \sigma(H_{\alpha, \beta}(A)) \in [\max(0, \alpha \cdot \sigma(A) - \frac{\beta}{2} \cdot (1 - \sigma_1(A))), \sigma(A) - \frac{\beta}{2} \cdot (1 - \sigma_1(A))];$$

$$(m) \sigma(H_{\alpha, \beta}^*(A)) \in [\alpha \cdot \sigma(A) + \beta \cdot (1 - \sigma_1(A)), (1 + \beta) \cdot \sigma(A)];$$

- (n) $\sigma(J_{\alpha, \beta}(A)) \geq \sigma(A) + \frac{\alpha}{2} \cdot (1 - \sigma_1(A));$
- (o) $\sigma(J_{\alpha, \beta}^*(A)) \geq \sigma(A) + \frac{\alpha}{2} \cdot (1 - \sigma_1(A));$
- (p) $\sigma(\neg A) \geq \max(1/2, \sigma_1(A));$
- (q) $\sigma(\exists A) \leq \min(1/2, \sigma_1(A));$
- (r) $\sigma(P_{\alpha, \beta}(A)) \leq \min(\frac{1 + \alpha - \beta}{2}, \sigma_1(A)), \text{ for } \alpha + \beta \leq 1;$
- (s) $\sigma(Q_{\alpha, \beta}(A)) \leq \max(\frac{1 + \alpha - \beta}{2}, \sigma_1(A)), \text{ for } \alpha + \beta \leq 1,$

where S is a set of propositional forms ($A, B \in S$).

All the above norms can be transformed for the case of IFSs.

Finally, we shall note that

$$\sigma(R(A)) = \sigma(A),$$

where the operator R is defined in [9] by

$$V(R(A)) = \langle \mu(A) \cdot (1 - \tau(A)), \tau(A) \cdot (1 - \mu(A)) \rangle.$$

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