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A STUDY ON SOME INTUITIONISTIC FUZZY IMPLICATIONS

V. Tasseva, D. Peneva

Centre for Biomedical Engineering "Prof. Ivan Daskalov"
Bulgarian Academy of Sciences,
Acad. G. Bonchev Str., Bl. 105, Sofia-1113, BULGARIA
e-mails: {violeta, desi}@clbme.bas.bg

INTRODUCTION

Intuitionistic Fuzzy Sets (IFSs, see [1]) are an extension of fuzzy sets allowing levels of membership and non-membership to be independently defined for each element of the set. On the base of fuzzy logic was introduced the Intuitionistic Fuzzy Logic (IFL), which undergo many changes and innovations during the last year. In a number of papers was shown the generation of many new implications and the examination of their properties. In [2] are introduced the all 174 implications that have been obtained previously.

Let E be an arbitrary universe. IFS A in E is defined as an object of the following form

$$A = \{<x, \mu_A(x), \nu_A(x)>|x \in E\},$$

where the functions:

$$\mu_A : E \to [0,1]$$

and

$$\nu_A: E \to [0,1]$$

define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Therefore, we can define a function

$$\pi_A: E \to [0,1]$$

such that

$$\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x)),$$

which corresponds to the degree of uncertainty about the membership of an element $x \in E$.

In this paper we will study properties of some of the new intuitionistic fuzzy implications shown in [2] and will exam whether or not they satisfy the given below eight axioms from the classical logic.

Property number	*Form of property*
I.	**A → (B v C) R_1 (A → B) v (A → C)**
II.	**A → (B v C) R_2 (A → B) ^ (A → C)**
III.	**A → (B ^ C) R_3 (A → B) ^ (A → C)**
IV.	**A → (B ^ C) R_4 (A → B) v (A → C)**
V.	**(A v B) → C R_5 (A → C) v (B → C)**
VI.	**(A v B) → C R_6 (A → C) ^ (B → C)**
VII.	**(A ^ B) → C R_7 (A → C) ^ (B → C)**
VIII.	**(A ^ B) → C R_8 (A → C) v (B → C)**

Table 1: List of properties

We will consider the following new implications introduced in [2].

Implication number	*Form of implicaion*
$X_{4,2}$	$\{ < x, 1-sg(\nu_B(x)-\nu_A(x)), \mu_A(x).sg(\nu_B(x)-\nu_A(x)) > \mid x \in E \}$
$X_{4,3}$	$\{ < x, 1-(1-\nu_A(x)).sg(\nu_B(x)-\nu_A(x)), \mu_A(x).sg(\nu_B(x)-\nu_A(x)) > \mid x \in E \}$
$X_{4,4}$	$\{ < x, \max(\mu_B(x), \nu_A(x)), \min(\nu_B(x), \mu_A(x)) > \mid x \in E \}$
$X_{4,5}$	$\{ < x, \min(1, \mu_B(x)+\nu_A(x)), \max(0, \nu_B(x)+\mu_A(x)-1) > \mid x \in E \}$

Table 2: List of examined intuitionistic fuzzy implications

In this table, sg and $\overline{sg}$ are two functions defined as:

$$sg(x) = \begin{cases} 1, x > 0 \\ 0, x \le 0 \end{cases} \quad \text{and} \quad \overline{sg}(x) = \begin{cases} 0, x > 0 \\ 1, x \le 0 \end{cases}$$

EXAMINAITON OF THE IMPLICATIONS' PROPERTIES

$X_{4,2} = \{ < x, 1-sg(\nu_B(x)-\nu_A(x)), \mu_A(x).sg(\nu_B(x)-\nu_A(x)) > \mid x \in E \}$,

I. A → (B v C) R_1 (A → B) v (A → C)

B v C = $\max(\mu_B(x), \mu_C(x)), \min(\nu_B(x), \nu_C(x))$
A → (B v C) = $1 - sg(\min(\nu_B(x), \nu_C(x)) - \nu_A(x)), \mu_A(x).sg(\min(\nu_B(x), \nu_C(x)) - \nu_A(x))$

(A → B) v (A → C) = $\max(1 - sg(\nu_B(x) - \nu_A(x)), 1 - sg(\nu_C(x) - \nu_A(x))), \min(\mu_A(x).sg(\nu_B(x) - \nu_A(x)), \mu_A(x).sg(\nu_C(x) - \nu_A(x))$

$1 - sg(\min(\nu_B(x), \nu_C(x)) - \nu_A(x)) - \max(1 - sg(\nu_B(x) - \nu_A(x)), 1 - sg(\nu_C(x) - \nu_A(x))) = 1 - sg(\min(\nu_B(x), \nu_C(x)) - \nu_A(x)) - 1 + \min(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) = \min(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) - \min(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) = 0$

$\mu_A(x).sg(\min(\nu_B(x), \nu_C(x)) - \nu_A(x)) - \min(\mu_A(x).sg(\nu_B(x) - \nu_A(x)), \mu_A(x).sg(\nu_C(x) - \nu_A(x)) = \mu_A(x).\min(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) - \mu_A(x).\min(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) = 0.$

So, the form of the relation R_1 is =

II. $A \rightarrow (B \vee C)\ \ R_2\ \ (A \rightarrow B) \wedge (A \rightarrow C)$

$(A \rightarrow B) \wedge (A \rightarrow C) = \min(1 - sg(\nu_B(x) - \nu_A(x)), 1 - sg(\nu_C(x) - \nu_A(x))), \max(\mu_A(x).sg(\nu_B(x) - \nu_A(x)), \mu_A(x).sg(\nu_C(x) - \nu_A(x))$

$1 - sg(\min(\nu_B(x), \nu_C(x)) - \nu_A(x)) - \min(1 - sg(\nu_B(x) - \nu_A(x)), 1 - sg(\nu_C(x) - \nu_A(x))) = 1 - sg(\min(\nu_B(x), \nu_C(x)) - \nu_A(x)) - 1 + \max(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) = \max(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) - \min(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) \geq 0$

$\mu_A(x).sg(\min(\nu_B(x), \nu_C(x)) - \nu_A(x)) - \max(\mu_A(x).sg(\nu_B(x) - \nu_A(x)), \mu_A(x).sg(\nu_C(x) - \nu_A(x)) = \mu_A(x).\min(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) - \mu_A(x).\max(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) \leq 0$

So, the form of the relation R_2 is $\geq$

III. $A \rightarrow (B \wedge C)\ \ R_3\ \ (A \rightarrow B) \wedge (A \rightarrow C)$

$A \rightarrow (B \wedge C) = 1 - sg(\max(\nu_B(x), \nu_C(x)) - \nu_A(x)), \mu_A(x).sg(\max(\nu_B(x), \nu_C(x)) - \nu_A(x))$

$1 - sg(\max(\nu_B(x), \nu_C(x)) - \nu_A(x)) - \min(1 - sg(\nu_B(x) - \nu_A(x)), 1 - sg(\nu_C(x) - \nu_A(x))) = 1 - \max(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) - 1 + \max(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) = 0$

$\mu_A(x).sg(\max(\nu_B(x), \nu_C(x)) - \nu_A(x)) - \max(\mu_A(x).sg(\nu_B(x) - \nu_A(x)), \mu_A(x).sg(\nu_C(x) - \nu_A(x)) = \mu_A(x).\max(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) - \mu_A(x).\max(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) = 0$

So, the form of the relation R_3 is =

IV. $A \rightarrow (B \wedge C)\ \ R_4\ \ (A \rightarrow B) \vee (A \rightarrow C)$

$1 - sg(\max(\nu_B(x), \nu_C(x)) - \nu_A(x)) - \max(1 - sg(\nu_B(x) - \nu_A(x)), 1 - sg(\nu_C(x) - \nu_A(x))) = 1 - \max(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) - 1 + \min(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) \leq 0$

$\mu_A(x).sg(\max(\nu_B(x), \nu_C(x)) - \nu_A(x)) - \min(\mu_A(x).sg(\nu_B(x) - \nu_A(x)), \mu_A(x).sg(\nu_C(x) - \nu_A(x)) = \mu_A(x).\max(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) - \mu_A(x).\min(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) \geq 0$

So, the form of the relation R_4 is $\leq$

V. $(A \vee B) \rightarrow C\ \ R_5\ \ (A \rightarrow C) \vee (B \rightarrow C)$

$(A \vee B) \rightarrow C = 1 - sg(\nu_C(x) - \min(\nu_A(x), \nu_B(x))), \max(\mu_A(x), \mu_B(x)).sg(\nu_C(x) - \min(\nu_A(x), \nu_B(x)))$

$(A \rightarrow C) \vee (B \rightarrow C) = \max(1 - sg(\nu_C(x) - \nu_A(x)), 1 - sg(\nu_C(x) - \nu_B(x))), \min(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x)))$

$1 - sg(\nu_C(x) - \min(\nu_A(x), \nu_B(x))) - \max(1 - sg(\nu_C(x) - \nu_A(x)), 1 - sg(\nu_C(x) - \nu_B(x))) = 1 - \max(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) - 1 + \min(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) \leq 0$

$\max(\mu_A(x), \mu_B(x)).sg(\nu_C(x) - \min(\nu_A(x), \nu_B(x))) - \min(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) = \max(\mu_A(x), \mu_B(x)).\max(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) - \min(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) \geq 0.$

So, the form of the relation R_5 is $\leq$

VI. $(A \vee B) \rightarrow C$ R_6 $(A \rightarrow C) \wedge (B \rightarrow C)$

$(A \rightarrow C) \wedge (B \rightarrow C) = \min(1 - sg(\nu_C(x) - \nu_A(x)), 1 - sg(\nu_C(x) - \nu_B(x))), \max(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x)))$

$1 - sg(\nu_C(x) - \min(\nu_A(x), \nu_B(x))) - \min(1 - sg(\nu_C(x) - \nu_A(x)), 1 - sg(\nu_C(x) - \nu_B(x))) = 1 - \max(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) - 1 + \max(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) = 0$

$\max(\mu_A(x), \mu_B(x)).sg(\nu_C(x) - \min(\nu_A(x), \nu_B(x))) - \max(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) = \max(\mu_A(x), \mu_B(x)).\max(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) - \max(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) \geq 0$

So, the form of the relation R_6 is $\leq$

VII. $(A \wedge B) \rightarrow C$ R_7 $(A \rightarrow C) \wedge (B \rightarrow C)$

$(A \wedge B) \rightarrow C = 1 - sg(\nu_C(x) - \max(\nu_A(x), \nu_B(x))), \min(\mu_A(x), \mu_B(x)).sg(\nu_C(x) - \max(\nu_A(x), \nu_B(x)))$

$1 - sg(\nu_C(x) - \max(\nu_A(x), \nu_B(x))) - \min(1 - sg(\nu_C(x) - \nu_A(x)), 1 - sg(\nu_C(x) - \nu_B(x))) = 1 - \min(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) - 1 + \max(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) \geq 0$

$\min(\mu_A(x), \mu_B(x)).sg(\nu_C(x) - \max(\nu_A(x), \nu_B(x))) - \max(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) = \min(\mu_A(x), \mu_B(x)).\min(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) - \max(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) \leq 0$

So, the form of the relation R_7 is $\geq$

VIII. $(A \wedge B) \rightarrow C$ R_8 $(A \rightarrow C) \vee (B \rightarrow C)$

$1 - sg(\nu_C(x) - \max(\nu_A(x), \nu_B(x))) - \max(1 - sg(\nu_C(x) - \nu_A(x)), 1 - sg(\nu_C(x) - \nu_B(x))) = 1 - \min(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) - 1 + \min(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) = 0$

$\min(\mu_A(x), \mu_B(x)).sg(\nu_C(x) - \max(\nu_A(x), \nu_B(x))) - \min(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) = \min(\mu_A(x), \mu_B(x)).sg(\min((\nu_C(x) - \nu_A(x)), (\nu_C(x) - \nu_B(x))) - \min(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x)))$

1) $\mu_A(x) \leq \mu_B(x) \Rightarrow \mu_A(x).sg(\min((\nu_C(x) - \nu_A(x)), (\nu_C(x) - \nu_B(x))) - \min(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x)))$,
   a) $(\nu_C(x) - \nu_A(x)) \leq 0 \Rightarrow \mu_A(x).sg(\nu_C(x) - \nu_A(x)) - \min(\mu_A(x).0, \mu_B(x).sg(\nu_C(x) - \nu_B(x))) = \mu_A(x).0 - \mu_A(x).0 = 0 - 0 = 0$,
   b) $(\nu_C(x) - \nu_B(x)) \leq 0 \Rightarrow \mu_A(x).sg(\nu_C(x) - \nu_B(x)) - \min(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).0) = \mu_A(x).0 - \mu_B(x).0 = 0 - 0 = 0$,
   c) $(\nu_C(x) - \nu_A(x)) = 1 \Rightarrow \mu_A(x).sg(\nu_C(x) - \nu_B(x)) - \min(\mu_A(x).1, \mu_B(x).sg(\nu_C(x) - \nu_B(x)))$,
      - $(\nu_C(x) - \nu_B(x)) \leq 0 \Rightarrow \mu_A(x).0 - \mu_B(x).0) = \mu_A(x).0 - \mu_B(x).0 = 0 - 0 = 0$,
      - $(\nu_C(x) - \nu_B(x)) = 1 \Rightarrow \mu_A(x).1 - \min(\mu_A(x).1, \mu_B(x).1) = \mu_A(x) - \mu_A(x) = 0$,
   d) $(\nu_C(x) - \nu_B(x)) = 1$ – analoguously to case c),
2) $\mu_A(x) > \mu_B(x) \Rightarrow \mu_B(x).sg(\min((\nu_C(x) - \nu_A(x)), (\nu_C(x) - \nu_B(x))) - \min(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x)))$ – the deduction is analoguous to case 1).

So, the form of the relation R_8 is =

$X_{4,3}$ = { < x,1–(1–ν_A(x)).sg(ν_B(x)–ν_A(x)), μ_A(x).sg(ν_B(x)–ν_A(x)) > | $x\in$ E },

I. A → (B v C) R_1 (A → B) v (A → C)

B v C = max(μ_B(x), μ_C(x)), min(ν_B(x), ν_C(x))
A → (B v C) = 1 – (1 – ν_A(x)).sg(min(ν_B(x), ν_C(x)) – ν_A(x)), μ_A(x).sg(min(ν_B(x), ν_C(x)) – ν_A(x))

(A → B) v (A → C) =max(1 – (1 – ν_A(x)).sg(ν_B(x)–ν_A(x)),1 – (1 – ν_A(x)).sg(ν_C(x)–ν_A(x))), min(μ_A(x).sg(ν_B(x) – ν_A(x)), μ_A(x).sg(ν_C(x) – ν_A(x))

1 – (1 – ν_A(x)).sg(min(ν_B(x), ν_C(x)) – ν_A(x)) – max(1 – (1 – ν_A(x)).sg(ν_B(x) – ν_A(x)),1 – (1 – ν_A(x)).sg(ν_C(x) – ν_A(x))) = 1 – (1 – ν_A(x)).min(sg(ν_B(x) – ν_A(x)), sg(ν_B(x) – ν_C(x))) – 1 + min((1 – ν_A(x)).sg(ν_B(x) – ν_A(x)), (1 – ν_A(x)).sg(ν_C(x) – ν_A(x))) = 0

μ_A(x).sg(min(ν_B(x), ν_C(x)) – ν_A(x)) – min(μ_A(x).sg(ν_B(x) – ν_A(x)), μ_A(x).sg(ν_C(x) – ν_A(x)) = 0

R_1 is =

II. A → (B v C) R_2 (A → B) ^ (A → C)

(A → B) ^ (A → C) = min(1 – (1 – ν_A(x)).sg(ν_B(x) – ν_A(x)), 1 – (1 – ν_A(x)).sg(ν_C(x)– ν_A(x))), max(μ_A(x).sg(ν_B(x) – ν_A(x)), μ_A(x).sg(ν_C(x) – ν_A(x))

1 – (1 – ν_A(x)).sg(min(ν_B(x), ν_C(x)) – ν_A(x)) – min(1 – (1 – ν_A(x)).sg(ν_B(x) – ν_A(x)), 1 – (1 – ν_A(x)).sg(ν_C(x) – ν_A(x))) = 1 – (1 – ν_A(x)).min(sg(ν_B(x) – ν_A(x)), sg(ν_B(x) – ν_C(x))) – 1 + max((1 – ν_A(x)).sg(ν_B(x) – ν_A(x)), (1 – ν_A(x)).sg(ν_C(x) – ν_A(x))) = 0 ≥ 0

μ_A(x).sg(min(ν_B(x), ν_C(x)) – ν_A(x)) – max(μ_A(x).sg(ν_B(x) – ν_A(x)), μ_A(x).sg(ν_C(x) – ν_A(x)) = μ_A(x).min(sg(ν_B(x) – ν_A(x)), sg(ν_C(x) – ν_A(x))) – μ_A(x).max(sg(ν_B(x) – ν_A(x)), sg(ν_C(x) – ν_A(x))) ≤ 0

So, the form of the relation R_2 is ≥

III. A → (B ^ C) R_3 (A → B) ^ (A → C)

A → (B ^ C) = 1 – (1 – ν_A(x)).sg(max(ν_B(x), ν_C(x)) – ν_A(x)), μ_A(x).sg(max(ν_B(x), ν_C(x)) – ν_A(x))

1 – (1 – ν_A(x)).sg(max(ν_B(x), ν_C(x)) – ν_A(x)) – min(1 – (1 – ν_A(x)).sg(ν_B(x) – ν_A(x)), 1 – (1 – ν_A(x)).sg(ν_C(x) – ν_A(x))) = 1 – (1 – ν_A(x)).max(sg(ν_B(x) – ν_A(x)), sg(ν_C(x) – ν_A(x))) – 1 + max((1 – ν_A(x)).sg(ν_B(x) – ν_A(x)), (1 – ν_A(x)).sg(ν_C(x) – ν_A(x))) = 0

μ_A(x).sg(max(ν_B(x), ν_C(x)) – ν_A(x)) – max(μ_A(x).sg(ν_B(x) – ν_A(x)), μ_A(x).sg(ν_C(x) – ν_A(x)) = 0

So, the form of the relation R_3 is =

IV. A → (B ^ C) R_4 (A → B) v (A → C)

$1 - (1 - \nu_A(x)).sg(\max(\nu_B(x), \nu_C(x)) - \nu_A(x)) - \max(1 - (1 - \nu_A(x)).sg(\nu_B(x) - \nu_A(x)), 1 - (1 - \nu_A(x)).sg(\nu_C(x) - \nu_A(x))) = 1 - (1 - \nu_A(x)).\max(sg(\nu_B(x) - \nu_A(x)), sg(\nu_C(x) - \nu_A(x))) - 1 + \min((1 - \nu_A(x)).sg(\nu_B(x) - \nu_A(x)), (1 - \nu_A(x)).sg(\nu_C(x) - \nu_A(x))) \leq 0$

$\mu_A(x).sg(\max(\nu_B(x), \nu_C(x)) - \nu_A(x)) - \min(\mu_A(x).sg(\nu_B(x) - \nu_A(x)), \mu_A(x).sg(\nu_C(x) - \nu_A(x)) \geq 0$

So, the form of the relation R_4 is ≤

V. (A v B) → C R_5 (A → C) v (B → C)

$(A \vee B) \rightarrow C = 1 - (1 - \min(\nu_A(x), \nu_B(x))).sg(\nu_C(x) - \min(\nu_A(x), \nu_B(x))), \max(\mu_A(x), \mu_B(x)).sg(\nu_C(x) - \min(\nu_A(x), \nu_B(x)))$

$(A \rightarrow C) \vee (B \rightarrow C) = \max(1 - (1 - \nu_A(x)).sg(\nu_C(x) - \nu_A(x)), 1 - (1 - \nu_B(x)).sg(\nu_C(x) - \nu_B(x))), \min(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x)))$

$1 - (1 - \min(\nu_A(x), \nu_B(x))).sg(\nu_C(x) - \min(\nu_A(x), \nu_B(x))) - \max(1 - (1 - \nu_A(x)).sg(\nu_C(x) - \nu_A(x)), 1 - (1 - \nu_B(x)).sg(\nu_C(x) - \nu_B(x))) = 1 - (1 - \min(\nu_A(x), \nu_B(x))).\max(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) - 1 + \min((1 - \nu_A(x)).sg(\nu_C(x) - \nu_A(x)), (1 - \nu_B(x)).sg(\nu_C(x) - \nu_B(x))) \leq 0$

$\max(\mu_A(x), \mu_B(x)).sg(\nu_C(x) - \min(\nu_A(x), \nu_B(x))) - \min(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) = \max(\mu_A(x), \mu_B(x)).\max(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) - \min(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) \geq 0$

So, the form of the relation R_5 is ≤

VI. (A v B) → C R_6 (A → C) ^ (B → C)

$(A \rightarrow C) \wedge (B \rightarrow C) = \min(1 - (1 - \nu_A(x)).sg(\nu_C(x) - \nu_A(x)), 1 - (1 - \nu_B(x)).sg(\nu_C(x) - \nu_B(x))), \max(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x)))$

$1 - (1 - \min(\nu_A(x), \nu_B(x))).sg(\nu_C(x) - \min(\nu_A(x), \nu_B(x))) - \min(1 - (1 - \nu_A(x)).sg(\nu_C(x) - \nu_A(x)), 1 - (1 - \nu_B(x)).sg(\nu_C(x) - \nu_B(x))) = 1 - (1 - \min(\nu_A(x), \nu_B(x))).\max(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) - 1 + \max((1 - \nu_A(x)).sg(\nu_C(x) - \nu_A(x)), (1 - \nu_B(x)).sg(\nu_C(x) - \nu_B(x))) \leq 0$

$\max(\mu_A(x), \mu_B(x)).sg(\nu_C(x) - \min(\nu_A(x), \nu_B(x))) - \max(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) = \max(\mu_A(x), \mu_B(x)).\max(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) - \max(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) \geq 0$

So, the form of the relation R_6 is ≤

VII. (A ^ B) → C R_7 (A → C) ^ (B → C)

$(A \wedge B) \rightarrow C = 1 - (1 - \max(\nu_A(x), \nu_B(x))).sg(\nu_C(x) - \max(\nu_A(x), \nu_B(x))), \min(\mu_A(x), \mu_B(x)).sg(\nu_C(x) - \max(\nu_A(x), \nu_B(x)))$

$1 - (1 - \max(\nu_A(x), \nu_B(x))).sg(\nu_C(x) - \max(\nu_A(x), \nu_B(x))) - \min(1 - (1 - \nu_A(x)).sg(\nu_C(x) - \nu_A(x)), 1 - (1 - \nu_B(x)).sg(\nu_C(x) - \nu_B(x))) = 1 - (1 - \max(\nu_A(x), \nu_B(x))).\min(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) - 1 + \max((1 - \nu_A(x)).sg(\nu_C(x) - \nu_A(x)), (1 - \nu_B(x)).sg(\nu_C(x) - \nu_B(x))) \geq 0$

$\min(\mu_A(x), \mu_B(x)).sg(\nu_C(x) - \max(\nu_A(x), \nu_B(x))) - \max(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) = \min(\mu_A(x), \mu_B(x)).\min(sg(\nu_C(x) - \nu_A(x)), sg(\nu_C(x) - \nu_B(x))) - \max(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) \leq 0$

So, the form of the relation R_7 is $\geq$

VIII. $(A \wedge B) \to C$ R_8 $(A \to C) \vee (B \to C)$

$1 - (1 - \max(\nu_A(x), \nu_B(x))).sg(\nu_C(x) - \max(\nu_A(x), \nu_B(x))) - \max(1 - (1 - \nu_A(x)).sg(\nu_C(x) - \nu_A(x)), 1 - (1 - \nu_B(x)).sg(\nu_C(x) - \nu_B(x))) = \min((1 - \nu_A(x)).sg(\nu_C(x) - \nu_A(x)), (1 - \nu_B(x)).sg(\nu_C(x) - \nu_B(x))) - \min((1 - \nu_A(x)), (1 - \nu_B(x))).sg(\min((\nu_C(x) - \nu_A(x)), (\nu_C(x) - \nu_B(x)))$

1) $\nu_C(x) \leq \nu_A(x)$ => $sg(\nu_C(x) - \nu_A(x)) = 0$, $sg(\min((\nu_C(x) - \nu_A(x)), (\nu_C(x) - \nu_B(x))) = 0$ => $0 = 0$;
2) $\nu_C(x) > \nu_A(x)$
   a) $\nu_C(x) \leq \nu_B(x)$ => $sg(\nu_C(x) - \nu_B(x)) = 0$,
      $sg(\min((\nu_C(x) - \nu_A(x)), (\nu_C(x) - \nu_B(x))) = 0$ => $0 = 0$;
   b) $\nu_C(x) > \nu_B(x)$ => $sg(\nu_C(x) - \nu_A(x)) = sg(\nu_C(x) - \nu_B(x)) =$
      $= sg(\min((\nu_C(x) - \nu_A(x)), (\nu_C(x) - \nu_B(x))) = 1$ =>
      $\min((1 - \nu_A(x)), (1 - \nu_B(x))) - \min((1 - \nu_A(x)), (1 - \nu_B(x))) = 0$;

$\min(\mu_A(x), \mu_B(x)).sg(\nu_C(x) - \max(\nu_A(x), \nu_B(x))) - \min(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x))) = \min(\mu_A(x), \mu_B(x)).sg(\min((\nu_C(x) - \nu_A(x)), (\nu_C(x) - \nu_B(x)))) - \min(\mu_A(x).sg(\nu_C(x) - \nu_A(x)), \mu_B(x).sg(\nu_C(x) - \nu_B(x)))$

1) $\nu_C(x) \leq \nu_A(x)$ => $sg(\nu_C(x) - \nu_A(x)) = 0$,
   $sg(\min((\nu_C(x) - \nu_A(x)), (\nu_C(x) - \nu_B(x)))) = 0$, => $0 = 0$;
2) $\nu_C(x) > \nu_A(x)$
   a) $\nu_C(x) \leq \nu_B(x)$ => $sg(\nu_C(x) - \nu_B(x)) = 0$,
      $sg(\min((\nu_C(x) - \nu_A(x)), (\nu_C(x) - \nu_B(x))) = 0$ => $0 = 0$;
   b) $\nu_C(x) > \nu_B(x)$ => $sg(\nu_C(x) - \nu_A(x)) = sg(\nu_C(x) - \nu_B(x)) =$
      $= sg(\min((\nu_C(x) - \nu_A(x)), (\nu_C(x) - \nu_B(x))) = 1$ =>
      $\min(\mu_A(x), \mu_B(x)) - \min(\mu_A(x), \mu_B(x)) = 0$;

So, the form of the relation R_8 is =

$X_{4,4} = \{ < x, \max(\mu_B(x), \nu_A(x)), \min(\nu_B(x), \mu_A(x)) > | x \in E \}$,

I. $A \to (B \vee C)$ R_1 $(A \to B) \vee (A \to C)$

$B \vee C = \max(\mu_B(x), \mu_C(x)), \min(\nu_B(x), \nu_C(x))$
$A \to (B \vee C) = \max(\max(\mu_B(x), \mu_C(x)), \nu_A(x)), \min(\min(\nu_B(x), \nu_C(x)), \mu_A(x))$

$(A \to B) \vee (A \to C) = \max(\max(\mu_B(x), \nu_A(x)), \max(\mu_C(x), \nu_A(x))), \min(\min(\nu_B(x), \mu_A(x)), \min(\nu_C(x), \mu_A(x)))$

$\max(\max(\mu_B(x), \mu_C(x)), \nu_A(x)) - \max(\max(\mu_B(x), \nu_A(x)), \max(\mu_C(x), \nu_A(x))) = \max(\mu_B(x), \mu_C(x), \nu_A(x)) - \max(\max(\mu_B(x), \mu_C(x)), \nu_A(x)) = 0$

$\min(\min(\nu_B(x), \nu_C(x)), \mu_A(x)) - \min(\min(\nu_B(x), \mu_A(x)), \min(\nu_C(x), \mu_A(x))) = \min(\nu_B(x), \nu_C(x), \mu_A(x)) - \min(\min(\nu_B(x), \nu_C(x)), \mu_A(x)) = 0$

R_1 is =

II. A → (B v C) R_2 (A → B) ^ (A → C)

(A → B) ^ (A → C) = $\min(\max(\mu_B(x), \nu_A(x)), \max(\mu_C(x), \nu_A(x))), \max(\min(\nu_B(x), \mu_A(x)), \min(\nu_C(x), \mu_A(x)))$

$\max(\max(\mu_B(x), \mu_C(x)), \nu_A(x)) - \min(\max(\mu_B(x), \nu_A(x)), \max(\mu_C(x), \nu_A(x))) = \max(\mu_B(x), \mu_C(x), \nu_A(x)) - \max(\min(\mu_B(x), \mu_C(x)), \nu_A(x)) \geq 0$

$\min(\min(\nu_B(x), \nu_C(x)), \mu_A(x)) - \max(\min(\nu_B(x), \mu_A(x)), \min(\nu_C(x), \mu_A(x))) = \min(\nu_B(x), \nu_C(x), \mu_A(x)) - \min(\max(\nu_B(x), \nu_C(x)), \mu_A(x)) \leq 0$

So, the form of the relation R_2 is ≥

III. A → (B ^ C) R_3 (A → B) ^ (A → C)

A → (B ^ C) = $\max(\min(\mu_B(x), \mu_C(x)), \nu_A(x)), \min(\max(\nu_B(x), \nu_C(x)), \mu_A(x))$

$\max(\min(\mu_B(x), \mu_C(x)), \nu_A(x)) - \min(\max(\mu_B, \nu_A(x)), \max(\mu_C, \nu_A(x))) = \max(\min(\mu_B(x), \mu_C(x)), \nu_A(x)) - \max(\min(\mu_B(x), \mu_C(x)), \nu_A(x)) = 0$

$\min(\max(\nu_B(x), \nu_C(x)), \mu_A(x)) - \max(\min(\nu_B(x), \mu_A(x)), \min(\nu_C(x), \mu_A(x))) = \min(\max(\nu_B(x), \nu_C(x)), \mu_A(x)) - \min(\max(\nu_B(x), \nu_C(x)), \mu_A(x)) = 0$

So, the form of the relation R_3 is =

IV. A → (B ^ C) R_4 (A → B) v (A → C)

$\max(\min(\mu_B(x), \mu_C(x)), \nu_A(x)) - \max(\max(\mu_B(x), \nu_A(x)), \max(\mu_C(x), \nu_A(x))) = \max(\min(\mu_B(x), \mu_C(x)), \nu_A(x)) - \max(\max(\mu_B(x), \mu_C(x)), \nu_A(x)) \leq 0$

$\min(\max(\nu_B(x), \nu_C(x)), \mu_A(x)) - \min(\min(\nu_B(x), \mu_A(x)), \min(\nu_C(x), \mu_A(x))) = \min(\max(\nu_B(x), \nu_C(x)), \mu_A(x)) - \min(\min(\nu_B(x), \nu_C(x)), \mu_A(x)) \geq 0$

So, the form of the relation R_4 is ≤

V. (A v B) → C R_5 (A → C) v (B → C)

(A v B) → C = $\max(\mu_C(x), \min(\nu_A(x), \nu_B(x))), \min(\max(\mu_A(x), \mu_B(x)), \nu_C(x))$

$(A \to C) \vee (B \to C) = \max(\max(\mu_C(x), \nu_A(x)), \max(\mu_C(x), \nu_B(x))), \min(\min(\nu_C(x), \mu_A(x)), \min(\nu_C(x), \mu_B(x)))$

$\max(\mu_C(x), \min(\nu_A(x), \nu_B(x))) - \max(\max(\mu_C(x), \nu_A(x)), \max(\mu_C(x), \nu_B(x))) = \max(\mu_C(x), \min(\nu_A(x), \nu_B(x))) - \max(\max(\nu_B(x), \nu_A(x)), \mu_C(x)) \leq 0$

$\min(\max(\mu_A(x), \mu_B(x)), \nu_C(x)) - \min(\min(\nu_C(x), \mu_A(x)), \min(\nu_C(x), \mu_B(x))) = \min(\max(\mu_A(x), \mu_B(x)), \nu_C(x)) - \min(\min(\mu_B(x), \mu_A(x)), \nu_C(x)) \geq 0$

So, the form of the relation R_5 is ≤

VI. (A v B) → C R_6 (A → C) ^ (B → C)

$(A \to C) \wedge (B \to C) = \min(\max(\mu_C(x), \nu_A(x)), \max(\mu_C(x), \nu_B(x))), \max(\min(\nu_C(x), \mu_A(x)), \min(\nu_C(x), \mu_B(x)))$

$\max(\mu_C(x), \min(\nu_A(x), \nu_B(x))) - \min(\max(\mu_C(x), \nu_A(x)), \max(\mu_C(x), \nu_B(x))) = \max(\mu_C(x), \min(\nu_A(x), \nu_B(x))) - \max(\min(\nu_B(x), \nu_A(x)), \mu_C(x)) = 0$

$\min(\max(\mu_A(x), \mu_B(x)), \nu_C(x)) - \max(\min(\nu_C(x), \mu_A(x)), \min(\nu_C(x), \mu_B(x))) = \min(\max(\mu_A(x), \mu_B(x)), \nu_C(x)) - \min(\max(\mu_B(x), \mu_A(x)), \nu_C(x)) = 0$

So, the form of the relation R_6 is =

VII. (A ^ B) → C R_7 (A → C) ^ (B → C)

$(A \wedge B) \to C = \max(\mu_C(x), \max(\nu_A(x), \nu_B(x))), \min(\min(\mu_A(x), \mu_B(x)), \nu_C(x))$

$\max(\mu_C(x), \max(\nu_A(x), \nu_B(x))) - \min(\max(\mu_C(x), \nu_A(x)), \max(\mu_C(x), \nu_B(x))) = \max(\mu_C(x), \nu_A(x), \nu_B(x)) - \max(\min(\nu_B(x), \nu_A(x)), \mu_C(x)) \geq 0$

$\min(\min(\mu_A(x), \mu_B(x)), \nu_C(x)) - \max(\min(\nu_C(x), \mu_A(x)), \min(\nu_C(x), \mu_B(x))) = \min(\mu_A(x), \mu_B(x), \nu_C(x)) - \min(\max(\mu_B(x), \mu_A(x)), \nu_C(x)) \leq 0$

So, the form of the relation R_7 is ≥

VIII. (A ^ B) → C R_8 (A → C) v (B → C)

$\max(\mu_C(x), \max(\nu_A(x), \nu_B(x))) - \max(\max(\mu_C(x), \nu_A(x)), \max(\mu_C(x), \nu_B(x))) = \max(\mu_C(x), \nu_A(x), \nu_B(x)) - \max(\nu_B(x), \nu_A(x), \mu_C(x)) = 0$

$\min(\min(\mu_A(x), \mu_B(x)), \nu_C(x)) - \min(\min(\nu_C(x), \mu_A(x)), \min(\nu_C(x), \mu_B(x))) = \min(\mu_A(x), \mu_B(x), \nu_C(x)) - \min(\mu_B(x), \mu_A(x), \nu_C(x)) = 0$

So, the form of the relation R_8 is =

$X_{4,5} = \{ < x, \min(1, \mu_B(x)+\nu_A(x)), \max(0, \nu_B(x)+ \mu_A(x)-1) > | x \in E \},$

I. $A \rightarrow (B \vee C)$ R_1 $(A \rightarrow B) \vee (A \rightarrow C)$

$A \rightarrow (B \vee C) = \min(1, \max(\mu_B(x), \mu_C(x)) + \nu_A(x)), \max(0, \min(\nu_B(x), \nu_C(x)) + \mu_A(x) - 1)$

$(A \rightarrow B) \vee (A \rightarrow C) = \max(\min(1,\mu_B(x) + \nu_A(x)), \min(1, \mu_C(x) + \nu_A(x))), \min(\max(0, \nu_B(x) + \mu_A(x) - 1), \max(0, \nu_C(x) + \mu_A(x) - 1))$

$\min(1, \max(\mu_B(x), \mu_C(x)) + \nu_A(x)) - \max(\min(1,\mu_B(x) + \nu_A(x)), \min(1, \mu_C(x) + \nu_A(x))) = \min(1, \max(\mu_B(x), \mu_C(x)) + \nu_A(x)) - \min(1, \max(\mu_B(x) + \nu_A(x), \mu_C(x) + \nu_A(x)) = \min(1, \max(\mu_B(x), \mu_C(x)) + \nu_A(x)) - \min(1, \max(\mu_B(x), \mu_C(x)) + \nu_A(x)) = 0$

$\max(0, \min(\nu_B(x), \nu_C(x)) + \mu_A(x) - 1) - \min(\max(0, \nu_B(x) + \mu_A(x) - 1), \max(0, \nu_C(x) + \mu_A(x) - 1)) = \max(0, \min(\nu_B(x), \nu_C(x)) + \mu_A(x) - 1) - \max(0, \min(\nu_B(x) + \mu_A(x) - 1, \nu_C(x) + \mu_A(x) - 1)) = 0$

R_1 is =

II. $A \rightarrow (B \vee C)$ R_2 $(A \rightarrow B) \wedge (A \rightarrow C)$

$(A \rightarrow B) \wedge (A \rightarrow C) = \min(\min(1, \mu_B(x) + \nu_A(x)), \min(1, \mu_C(x) + \nu_A(x))), \max(\max(0, \nu_B(x) + \mu_A(x) - 1), \max(0, \nu_C(x) + \mu_A(x) - 1))$

$\min(1, \max(\mu_B(x), \mu_C(x)) + \nu_A(x)) - \min(\min(1, \mu_B(x) + \nu_A(x)), \min(1, \mu_C(x) + \nu_A(x))) = \min(1, \max(\mu_B(x), \mu_C(x)) + \nu_A(x)) - \min(1, \min(\mu_B(x) + \nu_A(x), \mu_C(x) + \nu_A(x)) = \min(1, \max(\mu_B(x), \mu_C(x)) + \nu_A(x)) - \min(1, \min(\mu_B(x), \mu_C(x)) + \nu_A(x)) \geq 0$

$\max(0, \min(\nu_B(x), \nu_C(x)) + \mu_A(x) - 1) - \max(\max(0, \nu_B(x) + \mu_A(x) - 1), \max(0, \nu_C(x) + \mu_A(x) - 1)) = \max(0, \min(\nu_B(x), \nu_C(x)) + \mu_A(x) - 1) - \max(0, \max(\nu_B(x), \nu_C(x)) + \mu_A(x) - 1) \leq 0$

So, the form of the relation R_2 is $\geq$

III. $A \rightarrow (B \wedge C)$ R_3 $(A \rightarrow B) \wedge (A \rightarrow C)$

$A \rightarrow (B \wedge C) = \min(1, \min(\mu_B(x), \mu_C(x)) + \nu_A(x)), \max(0, \max(\nu_B(x), \nu_C(x)) + \mu_A(x) - 1)$

$\min(1, \min(\mu_B(x), \mu_C(x)) + \nu_A(x)) - \min(\min(1, \mu_B(x) + \nu_A(x)), \min(1, \mu_C(x) + \nu_A(x))) = 0$

$\max(0, \max(\nu_B(x), \nu_C(x)) + \mu_A(x) - 1) - \max(\max(0, \nu_B(x) + \mu_A(x) - 1), \max(0, \nu_C(x) + \mu_A(x) - 1)) = 0$

So, the form of the relation R_3 is =

IV. $A \rightarrow (B \wedge C)$ R_4 $(A \rightarrow B) \vee (A \rightarrow C)$

$\min(1, \min(\mu_B(x), \mu_C(x)) + \nu_A(x)) - \max(\min(1,\mu_B(x) + \nu_A(x)), \min(1, \mu_C(x) + \nu_A(x))) = \min(1, \min(\mu_B(x), \mu_C(x)) + \nu_A(x)) - \min(1, \max(\mu_B(x) + \nu_A(x), \mu_C(x) + \nu_A(x)) = \min(1, \min(\mu_B(x), \mu_C(x)) + \nu_A(x)) - \min(1, \max(\mu_B(x), \mu_C(x)) + \nu_A(x)) \leq 0$

$\max(0, \max(\nu_B(x), \nu_C(x)) + \mu_A(x) - 1) - \min(\max(0, \nu_B(x) + \mu_A(x) - 1), \max(0, \nu_C(x) + \mu_A(x) - 1)) = \max(0, \max(\nu_B(x), \nu_C(x)) + \mu_A(x) - 1) - \max(0, \min(\nu_B(x) + \mu_A(x) - 1, \nu_C(x) + \mu_A(x) - 1) = \max(0, \max(\nu_B(x), \nu_C(x)) + \mu_A(x) - 1) - \max(0, \min(\nu_B(x), \nu_C(x)) + \mu_A(x) - 1)\geq 0$

So, the form of the relation R_4 is ≤

V. (A v B) → C R_5 (A → C) v (B → C)

(A v B) → C = min(1, $\mu_C(x)$ + min($\nu_A(x)$, $\nu_B(x)$)), max(0, $\nu_C(x)$ + max($\mu_A(x)$, $\mu_B(x)$) – 1)

(A → C) v (B → C) =max(min(1, $\mu_C(x) + \nu_A(x)$), min(1, $\mu_C(x) + \nu_B(x)$)), min(max(0, $\nu_C(x) + \mu_A(x)$ – 1), max(0, $\nu_C(x) + \mu_B(x)$ – 1))

min(1, $\mu_C(x)$ + min($\nu_A(x)$, $\nu_B(x)$)) – max(min(1, $\mu_C(x) + \nu_A(x)$), min(1, $\mu_C(x) + \nu_B(x)$)) = min(1, $\mu_C(x)$ + min($\nu_A(x)$, $\nu_B(x)$)) – min(1, $\mu_C(x)$ + max($\nu_A(x)$, $\nu_B(x)$)) ≤ 0

max(0, $\nu_C(x)$ + max($\mu_A(x)$, $\mu_B(x)$) – 1) – min(max(0, $\nu_C(x) + \mu_A(x)$ – 1), max(0, $\nu_C(x) + \mu_B(x)$ – 1)) = max(0, $\nu_C(x)$ + max($\mu_A(x)$, $\mu_B(x)$) – 1) – max(0, min($\nu_C(x) + \mu_A(x)$ – 1, $\nu_C(x) + \mu_B(x)$ – 1)) ≥ 0

So, the form of the relation R_5 is ≤

VI. (A v B) → C R_6 (A → C) ^ (B → C)

(A → C) ^ (B → C) =min(min(1, $\mu_C(x) + \nu_A(x)$), min(1, $\mu_C(x) + \nu_B(x)$)), max(max(0, $\nu_C(x) + \mu_A(x)$ – 1), max(0, $\nu_C(x) + \mu_B(x)$ – 1))

min(1, $\mu_C(x)$ + min($\nu_A(x)$, $\nu_B(x)$)) – min(min(1, $\mu_C(x) + \nu_A(x)$), min(1, $\mu_C(x) + \nu_B(x)$)) = min(1, $\mu_C(x)$ + min($\nu_A(x)$, $\nu_B(x)$)) – min(1, min($\mu_C(x) + \nu_A(x)$, $\mu_C(x) + \nu_B(x)$)) = min(1, $\mu_C(x)$ + min($\nu_A(x)$, $\nu_B(x)$)) – min(1, $\mu_C(x)$ + min($\nu_A(x)$, $\nu_B(x)$)) = 0

max(0, $\nu_C(x)$ + max($\mu_A(x)$, $\mu_B(x)$) – 1) – max(max(0, $\nu_C(x) + \mu_A(x)$ – 1), max(0, $\nu_C(x) + \mu_B(x)$ – 1)) = 0

So, the form of the relation R_6 is =

VII. (A ^ B) → C R_7 (A → C) ^ (B → C)

(A ^ B) → C = min(1, $\mu_C(x)$ + max($\nu_A(x)$, $\nu_B(x)$)), max(0, $\nu_C(x)$ + min($\mu_A(x)$, $\mu_B(x)$) – 1)

min(1, $\mu_C(x)$ + max($\nu_A(x)$, $\nu_B(x)$)) – min(min(1, $\mu_C(x) + \nu_A(x)$), min(1, $\mu_C(x) + \nu_B(x)$)) = min(1, $\mu_C(x)$ + max($\nu_A(x)$, $\nu_B(x)$)) – min(1, $\mu_C(x)$ + min($\nu_A(x)$, $\nu_B(x)$)) ≥ 0

max(0, $\nu_C(x)$ + min($\mu_A(x)$, $\mu_B(x)$) – 1) – max(max(0, $\nu_C(x) + \mu_A(x)$ – 1), max(0, $\nu_C(x) + \mu_B(x)$ – 1)) ≤ 0

So, the form of the relation R_7 is ≥

VIII. (A ^ B) → C R_8 (A → C) v (B → C)

min(1, $\mu_C(x)$ + max($\nu_A(x)$, $\nu_B(x)$)) – max(min(1, $\mu_C(x) + \nu_A(x)$), min(1, $\mu_C(x) + \nu_B(x)$)) = min(1, $\mu_C(x)$ + max($\nu_A(x)$, $\nu_B(x)$)) – min(1, $\mu_C(x)$ + max($\nu_A(x)$, $\nu_B(x)$)) = 0

$\max(0, \nu_C(x) + \min(\mu_A(x), \mu_B(x)) - 1) - \min(\max(0, \nu_C(x) + \mu_A(x) - 1), \max(0, \nu_C(x) + \mu_B(x) - 1)) =$
$\max(0, \nu_C(x) + \min(\mu_A(x), \mu_B(x)) - 1) - \max(0, \nu_C(x) + \min(\mu_A(x), \mu_B(x)) - 1) = 0$

So, the form of the relation R_8 is =

MAIN RESULTS

ImplicationNo	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
4.2	=	≥	=	≤	≤	≤	≥	=
4.3	=	≥	=	≤	≤	≤	≥	=
4.4	=	≥	=	≤	≤	=	≥	=
4.5	=	≥	=	≤	≤	=	≥	=

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