

First weight-center operator

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Abstract: In the papers [3, 4, 5] there were introduced modifications of the weight-center operator and some basic properties were shown. Now we prove some more of relations on first weight-center operator.

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1 Introduction

Intuitionistic fuzzy sets (IFSs) A, B in E by [1] are objects of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in E\}$$

$$B = \{\langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in E\}$$

where $\mu_A : E \rightarrow \langle 0, 1 \rangle$, $\nu_A : E \rightarrow \langle 0, 1 \rangle$ and for every $x \in E$: $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

For every two IFSs A and B a lot of operations, relations and operators are defined in [1], but we need only a few of these:

$$A \subseteq B \Leftrightarrow (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x))$$

$$\square(A) = \{\langle x, \mu_A(x), 1 - \mu_A(x) \rangle \mid x \in E\}$$

$$\diamond(A) = \{\langle x, 1 - \nu_A(x), \nu_A(x) \rangle \mid x \in E\}$$

In [2] was introduced the following operator, defined for IFSs over a finite universe E :

$$W(A) = \left\{ \left\langle x, \frac{\sum_{y \in E} \mu_A(y)}{card(E)}, \frac{\sum_{y \in E} \nu_A(y)}{card(E)} \right\rangle \mid x \in E \right\},$$

where $card(E)$ is the number of the elements of a finite universe E .

2 First weight-center operator

Let $B \neq U^*$ where $U^* = \{\langle x, 0, 0 \rangle \mid x \in E\}$. The first modification of weight-center operator over IFSs A and B over the finite universe E is introduce by:

$$W_B^1(A) = \left\{ \left\langle x, \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)}, \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)} \right\rangle \mid x \in E \right\}$$

It is inspired by paper of Vania Peneva and Ivan Popchev [6].

We see that $W_B^1(A)$ is an IFS because, for every $x \in E$:

$$\begin{aligned} 0 &\leq \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)} + \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)} \leq \\ &\leq \frac{\sum_{y \in E} \mu_A(y)}{\text{card}(E)} + \frac{\sum_{y \in E} \nu_A(y)}{\text{card}(E)} = \frac{\sum_{y \in E} \mu_A(y) + \nu_A(y)}{\text{card}(E)} \leq 1 \end{aligned}$$

Theorem 1. For every two IFSs A and $B \neq U^*$ over the finite universe E :

$$(a) \quad \square W_B^1(A) \supseteq W_B^1(\square A),$$

$$(b) \quad \diamond W_B^1(A) \subseteq W_B^1(\diamond A).$$

Proof. (a)

$$\begin{aligned} \square W_B^1(A) &= \square \left(\left\{ \left\langle x, \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)}, \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)} \right\rangle \mid x \in E \right\} \right) = \\ &= \left\{ \left\langle x, \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)}, 1 - \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)} \right\rangle \mid x \in E \right\} \supseteq \\ &\supseteq \left\{ \left\langle x, \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)}, \frac{\mu_B(x)}{\sum_{y \in E} \mu_B(y)} - \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)} \right\rangle \mid x \in E \right\} = \\ &= \left\{ \left\langle x, \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)}, \frac{\mu_B(x) \cdot \text{card}(E)}{\text{card}(E) \sum_{y \in E} \mu_B(y)} - \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)} \right\rangle \mid x \in E \right\} = \\ &= \left\{ \left\langle x, \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)}, \frac{\mu_B(x) \cdot \sum_{y \in E} 1}{\text{card}(E) \sum_{y \in E} \mu_B(y)} - \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)} \right\rangle \mid x \in E \right\} = \end{aligned}$$

$$\begin{aligned}
&= \left\{ \left\langle x, \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)}, \frac{\sum_{y \in E} \mu_B(x)}{\text{card}(E) \sum_{y \in E} \mu_B(y)} - \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)} \right\rangle \mid x \in E \right\} = \\
&= \left\{ \left\langle x, \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)}, \frac{\sum_{y \in E} (1 - \mu_A(y)) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)} \right\rangle \mid x \in E \right\} = \\
&= W_B^1(\{\langle x, \mu_A(x) \rangle \mid x \in E\}) = W_B^1(\square A)
\end{aligned}$$

(b)

$$\begin{aligned}
\diamond W_B^1(A) &= \diamond \left(\left\{ \left\langle x, \frac{\sum_{y \in E} \mu_A(y) \cdot \mu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \mu_B(y)}, \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)} \right\rangle \mid x \in E \right\} \right) = \\
&= \left\{ \left\langle x, 1 - \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)}, \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)} \right\rangle \mid x \in E \right\} \subseteq \\
&\subseteq \left\{ \left\langle x, \frac{\nu_B(x)}{\sum_{y \in E} \nu_B(y)} - \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)}, \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)} \right\rangle \mid x \in E \right\} = \\
&= \left\{ \left\langle x, \frac{\nu_B(x) \cdot \text{card}(E)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)} - \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)}, \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)} \right\rangle \mid x \in E \right\} = \\
&= \left\{ \left\langle x, \frac{\nu_B(x) \cdot \sum_{y \in E} 1}{\text{card}(E) \sum_{y \in E} \nu_B(y)} - \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)}, \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)} \right\rangle \mid x \in E \right\} = \\
&= \left\{ \left\langle x, \frac{\sum_{y \in E} \nu_B(x)}{\text{card}(E) \sum_{y \in E} \nu_B(y)} - \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)}, \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)} \right\rangle \mid x \in E \right\} = \\
&= \left\{ \left\langle x, \frac{\sum_{y \in E} (1 - \nu_A(y)) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)}, \frac{\sum_{y \in E} \nu_A(y) \cdot \nu_B(x)}{\text{card}(E) \cdot \sum_{y \in E} \nu_B(y)} \right\rangle \mid x \in E \right\} = \\
&= W_B^1(\{\langle x, 1 - \nu_A(x), \nu_A(x) \rangle \mid x \in E\}) = W_B^1(\diamond A)
\end{aligned}$$

This completes the proof. \square

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