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Modifications of the weight-center operator, defined over intuitionistic fuzzy sets. Part 2

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Abstract: In [2], K. Atanassov and A. Ban introduced the operator $W(A)$, defined for IFSs over a finite universe E. In [5] the first modifications of the weight-center operator was proposed and studied. In the present paper, the second modification is considered over a countable universe.
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1 Introduction

In the present part of the research, a second modifications of the weight-center operator W, defined over intuitionistic fuzzy sets in [2], will be introduced.

Initially, we give some basic definitions, related to the Intuitionistic Fuzzy Sets (IFSs), following [1].

Let a set E be fixed. An IFS A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where the functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

For every two IFSs A and B a lot of operations, relations and operators are defined (see, e.g. [1]), the most important of which, related to the present research, are:

$$\begin{array}{lcl}
A \subseteq B & iff & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)), \\
A \subset_{\Box} B & iff & (\forall x \in E)(\mu_A(x) \leq \mu_B(x)), \\
A \subset_{\Diamond} B & iff & (\forall x \in E)(\nu_A(x) \geq \nu_B(x)), \\
A = B & iff & (\forall x \in E)(\mu_A(x) = \mu_B(x) \; \& \; \nu_A(x) = \nu_B(x)), \\
\overline{A} & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}, \\
C(A) & = & \{\langle x, K, L\rangle | x \in E\}, \\
I(A) & = & \{\langle x, k, l\rangle | x \in E\}, \\
C_\nu(A) & = & \{\langle x, \mu_A(x), L\rangle | x \in E\}, \\
I_\mu(A) & = & \{\langle x, k, \nu_A(x)\rangle | x \in E\},
\end{array}$$

where

$$K = \sup_{y \in E} \mu_A(y),$$

$$L = \inf_{y \in E} \nu_A(y),$$

$$k = \inf_{y \in E} \mu_A(y),$$

$$l = \sup_{y \in E} \nu_A(y).$$

In [2] we introduced the following operator, defined for IFSs over a finite universe E:

$$W(A) = \{\langle x, \frac{\sum\limits_{y \in E} \mu_A(y)}{card(E)}, \frac{\sum\limits_{y \in E} \nu_A(y)}{card(E)}\rangle | x \in E\}, \tag{1}$$

where $card(E)$ is the number of the elements of a finite universe E.

In [1] the operators $H_{\alpha,\beta}$ and $J_{\alpha,\beta}$ are defined by

$$H_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle | x \in E\},$$

$$J_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \beta.\nu_A(x)\rangle | x \in E\}.$$

Obviously, for every IFS X over universe E,

$$H_{0,0}(X) = \{\langle x, 0, \nu_X(x)\rangle | x \in E\},$$

$$J_{0,0}(X) = \{\langle x, \mu_X(x), 0\rangle | x \in E\}.$$

Let the IFS $B \neq H_{0,0}(B)$ and $B \neq J_{0,0}(B)$. Therefore,

$$\sum_{y \in E} \mu_B(y) > 0$$

and

$$\sum_{y \in E} \nu_B(y) > 0.$$

In [5] we modified the operator W to the form

$$W_B^1(A) = \{\langle x, \frac{(\sum\limits_{y \in E} \mu_A(y)).\mu_B(x)}{card(E) \sum\limits_{y \in E} \mu_B(y)}, \frac{(\sum\limits_{y \in E} \nu_A(y)).\nu_B(x)}{card(E) \sum\limits_{y \in E} \nu_B(y)}\rangle | x \in E\}, \tag{2}$$

where $B \neq H_{0,0}(B)$ and $B \neq J_{0,0}(B)$.

It is inspired by papers of Ricardo Alberto Marques Pereira and Rita Almeida Ribeiro [3], and Vania Peneva and Ivan Popchev [4].

2 Second modification of the weight-center operator

Let A and B be two IFSs over the finite universe E.

Now, we introduce *"the second modified weight-center operator"* over IFSs A and B over the finite universe E.

Let everywhere below, the IFS $B \neq H_{0,0}(B)$ and $B \neq J_{0,0}(B)$, and let $||A|| \leq ||B||$. Therefore,

$$\sum_{y \in E} (\mu_B(y) + \nu_B(y)) > 0$$

and

$$\sum_{y \in E} (\mu_A(y) + \nu_A(y)) \leq \sum_{y \in E} (\mu_B(y) + \nu_B(y)).$$

The new operator has the form

$$W_B^2(A) = \{\langle x, \frac{(\sum\limits_{y \in E} \mu_A(y)).\mu_B(x)}{2\max(\sum\limits_{y \in E} \mu_B(y), \sum\limits_{y \in E} \nu_B(y))}, \frac{(\sum\limits_{y \in E} \nu_A(y)).\nu_B(x)}{2\max(\sum\limits_{y \in E} \mu_B(y), \sum\limits_{y \in E} \nu_B(y))}\rangle | x \in E\}, \tag{2}$$

First, we see that the definition is correct, i.e., $W_B^2(A)$ is an IFS. Really, for every $x \in E$,

$$0 \leq \frac{(\sum\limits_{y \in E} \mu_A(y)).\mu_B(x)}{2\max(\sum\limits_{y \in E} \mu_B(y), \sum\limits_{y \in E} \nu_B(y))} \leq \frac{\sum\limits_{y \in E} \mu_A(y) + \nu_A(y)}{2\max(\sum\limits_{y \in E} \mu_B(y), \sum\limits_{y \in E} \nu_B(y))} < 1,$$

$$0 \leq \frac{(\sum\limits_{y \in E} \nu_A(y)).\nu_B(x)}{2\max(\sum\limits_{y \in E} \mu_B(y), \sum\limits_{y \in E} \nu_B(y))} \leq \frac{\sum\limits_{y \in E} \mu_A(y) + \nu_A(y)}{2\max(\sum\limits_{y \in E} \mu_B(y), \sum\limits_{y \in E} \nu_B(y))} < 1,$$

and

$$\frac{(\sum\limits_{y\in E}\mu_A(y)).\mu_B(x)}{2\max(\sum\limits_{y\in E}\mu_B(y),\sum\limits_{y\in E}\nu_B(y))}+\frac{(\sum\limits_{y\in E}\nu_A(y)).\nu_B(x)}{2\max(\sum\limits_{y\in E}\mu_B(y),\sum\limits_{y\in E}\nu_B(y))}$$

$$=\frac{(\sum\limits_{y\in E}\mu_A(y)).\mu_B(x)+(\sum\limits_{y\in E}\nu_A(y)).\nu_B(x)}{2\max(\sum\limits_{y\in E}\mu_B(y),\sum\limits_{y\in E}\nu_B(y))}$$

$$\leq\frac{\sum\limits_{y\in E}\mu_A(y)+\sum\limits_{y\in E}\nu_A(y)}{2\max(\sum\limits_{y\in E}\mu_B(y),\sum\limits_{y\in E}\nu_B(y))}\leq\frac{\sum\limits_{y\in E}\mu_B(y)+\sum\limits_{y\in E}\nu_B(y)}{2\max(\sum\limits_{y\in E}\mu_B(y),\sum\limits_{y\in E}\nu_B(y))}\leq 1.$$

Theorem 1. For every two IFSs A and $B \neq U^*$ over the finite universe E:

(a) $\overline{W_B^2(\overline{A})} = W_{\overline{B}}^2(A)$,

(b) $I(W_B^2(A)) = W_B^2(I(A))$,

(c) $C(W_B^2(A)) = W_B^2(C(A))$,

(d) $I_\mu(W_B^2(A)) = W_B^2(I_\mu(A))$,

(c) $C_\nu(W_B^2(A)) = W_B^2(C_\nu(A))$.

Proof: Let us check the validity of (a) for given IFSs A and $B \neq U^*$ over universe E.

$$\overline{W_B^2(\overline{A})} = \overline{W_B^2(\{\langle x, \nu_A(x), \mu_A(x)\rangle | x\in E\})}$$

$$=\overline{\{\langle x,\frac{(\sum\limits_{y\in E}\nu_A(y)).\mu_B(x)}{2\max(\sum\limits_{y\in E}\mu_B(y),\sum\limits_{y\in E}\nu_B(y))},\frac{(\sum\limits_{y\in E}\mu_A(y)).\nu_B(x)}{2\max(\sum\limits_{y\in E}\mu_B(y),\sum\limits_{y\in E}\nu_B(y))}\rangle|x\in E\}}$$

$$=\{\langle x,\frac{(\sum\limits_{y\in E}\mu_A(y)).\nu_B(x)}{2\max(\sum\limits_{y\in E}\nu_B(y),\sum\limits_{y\in E}\mu_B(y))},\frac{(\sum\limits_{y\in E}\nu_A(y)).\mu_B(x)}{2\max(\sum\limits_{y\in E}\nu_B(y),\sum\limits_{y\in E}\mu_B(y))}\rangle|x\in E\}=W_{\overline{B}}^2(A).$$

(b) – (e) are proved analogously. $\square$

Theorem 2. For every two IFSs A and $B \neq U^*$ over the finite universe E:

$$W_B^1(A)A \subset_{\square} W_B^2(A),$$

$$W_B^1(A)A \subset_{\diamond} W_B^2(A).$$

3 Conclusion

In the next authors' research, we will introduce another new modification of operator W_B and will study some of its properties.

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