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An (α, β)-tautology and a problem, related to it

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Abstract: The concept of an (α, β)-intuitionistic fuzzy tautology is introduced and it is shown that it generalizes the currently existing tautologies in intuitionistic fuzzy logic. An open problem is formulated about implications' capacity to satisfy the Modus Ponens in the form of (α, β)-tautology.
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In intuitionistic fuzzy propositional calculus (see, e.g., [1, 2, 4]), if x is a variable or a formula, then its truth-value is represented by the ordered pair

$$V(x) = \langle a, b \rangle,$$

so that $a, b, a + b \in [0, 1]$, where a and b are degrees of validity and of non-validity of x, respectively. There, the notion of Intuitionistic Fuzzy Tautology (IFT) is introduced by:

$$x \text{ is an IFT if and only if for } V(x) = \langle a, b \rangle \text{ holds: } a \geq b,$$

while x is a tautology if and only if $a = 1$ and $b = 0$. Therefore, as in the case of ordinary logics, x is a tautology, if $V(x) = \langle 1, 0 \rangle$. In [1, 2], the formula x is called "sure" if and only if

$a \geq 0.5 \geq b$. In [5] the same concept is called “truth Intuitionistic Fuzzy tautology” (truth-IFT), that from linguistic point of view is better and by this reason, it will be used from now on.

Let $\alpha, \beta \in [0, 1]$. Here, we introduce an (α, β)-intuitionistic fuzzy tautology (shortly: (α, β)-IFT) by:

x is an (α, β)-IFT if and only if for $V(x) = \langle a, b \rangle$ holds: $a \geq \alpha$ and $b \leq \beta$.

We should mention that:

- If $\alpha = 1$ and $\beta = 0$, then we obtain the (standard) tautology;
- If $\alpha = \beta = 0.5$, then we obtain that x is truth-IFT;
- If $\alpha = \beta$, then x is an IFT.

Let x be an (α, β)-IFT and let for the variable y for which $V(y) = \langle c, d \rangle$, $x \to y$ be an (α, β)-IFT, where $\to$ is some intuitionistic fuzzy implication, described in [3]. For example, if we like to use the intuitionistic fuzzy form of Kleene-Dienes's implication, mentioned in [3] by $\to_4$, we will see that $\langle \max(b, c), \min(a, d) \rangle$ had to be an (α, β)-IFT, i.e.,

$$\max(b, c) \geq \alpha,$$

$$\min(a, d) \leq \beta$$

and

$$a \geq \alpha \quad \text{and} \quad b \leq \beta.$$

Now, let us discuss the conditions, that α and β must satisfy.

We see, that if $\alpha < \beta$, then it is possible that $d \geq \beta > \alpha \geq c$ and therefore y will be no (α, β)-IFT.

If $\alpha \geq \beta$, then from $\min(a, d) \leq \beta$ it follows that $d \leq \beta$ and from $\max(b, c) \geq \alpha$ it follows that $c \geq \alpha$, i.e., y is an (α, β)-IFT.

Therefore, the above discussion shows why in the case of the IFTs, the Modus Ponens is not always valid, as it is shown in [4].

An **Open Problem** is to study for which i, implication $\to_i$ satisfies the Modus Ponens in the form of an (α, β)-IFT.

The above definition and problem, related to it, were stimulated by Marcin Detyniecki, Marie-Jeanne Lesot and Paul Moncuquet's paper [5].

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