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Intuitionistic fuzzy estimation of the doctoral comprehensive examination

Evdokia Sotirova[1], Anthony Shannon[2], Sotir Sotirov[1] and Maciej Krawczak[3]

[1] Prof. Asen Zlatarov University, Burgas-8000, Bulgaria
e-mails: {esotirova, ssotirov}@btu.bg

[2] Faculty of Engineering & IT, University of Technology, Sydney, NSW 2007, Australia
e-mails: tshannon38@gmail.com, Anthony.Shannon@uts.edu.au

[3] Systems Research Institute – Polish Academy of Sciences
Wyzsza Szkola Informatyki Stosowanej i Zarzadzania, Warsaw, Poland
e-mail: krawczak@ibspan.waw.pl

Abstract: This paper describes a method for determining an intuitionistic fuzzy estimation of PhD candidature using the theories of Index Matrices and Intuitionistic Fuzzy Sets. The estimated degrees of satisfaction of the criteria by candidates are clustered by a neural network. In this way the procedure for assessment of the candidates for the dissertation stage can be measured and analyzed.
Keywords: Intuitionistic Fuzzy Sets, Index Matrices, Self-organizing Maps.
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1 Introduction

The preparation of PhD candidates is a complex process. The basic steps of that process are submission of documents for PhD enrollment, the process of evaluation of progress in the preparation of thesis or dissertation, preparation for the PhD examination, checking the deadline for taking the PhD examination, submission of the PhD thesis, selection of thesis examiners/reviewers, and thesis defense [13, 14, 15]. A thorough process can improve the preparation of these candidates.

The process of PhD candidature is structured. It is a function of the various entry criteria; for example: the evaluation from successful passing of the PhD comprehensive examinations, research experience, academic referees, academic qualifications, another professional experience, and so on. We shall focus on the evaluation of the PhD comprehensive examination stage of doctoral candidature, and we shall use the theories of Index Matrices

(Ims, see [1, 2, 3]) and Intuitionistic Fuzzy Set (IFS, see [4, 5]) to do so. Then the next step is the clustering of the calculated intuitionistic fuzzy estimations for which we use a Self-organizing Map (SOM), [7].

A SOM is a neural network which uses unsupervised learning. It produces high dimensional data into the low dimensional space (called a *map*), [5, 6, 8, 9, 10]. The SOM neural network is the appropriate way to represent and cluster the data. It discretizes representation of the input space of the training samples in the sense that they use a neighborhood function to preserve the topological properties of the input space.

With an SOM, clustering is performed by having several units compete for the current object. Once the data have been entered into the system, the network of artificial neurons is trained by providing information about inputs.

This paper, with its focus on only one part of the process, is a continuation of previous research of the authors, [11, 12, 13]. It is realized that the nature and scope, and indeed the existence, of comprehensive doctoral examinations vary from country to country, and sometimes even among jurisdictions within the one country.

2 Implementation

A. Determination of the intuitionistic fuzzy evaluations

Suppose we have i PhD candidates ($i = 1, 2,\ldots, n$) and j criteria ($j = 1, 2,\ldots, m$). The criterion c_j has weight t_j. Each candidate has to be evaluated individually for each criterion.
So we use the following sets:

- Set of candidates $\{k_1, k_2,\ldots, k_n\}$,
- Set of criteria $\{c_1, c_2,\ldots, c_m\}$,
- Set of weights of the criteria $\{t_1, t_2,\ldots, t_m\}$.

Step A1. Construction of the index matrix T with weight coefficients corresponding to each criterion:

$$T = \begin{array}{c|ccccc} & c_1 & \ldots & c_j & \ldots & c_m \\ \hline t & t_1 & \ldots & t_j & \ldots & t_m \end{array}. \tag{1}$$

Step A2. Construction of the index matrix K with the points $a_{i,j}$ of each candidate according to each criterion.

$$K = \begin{array}{c|ccccc} & k_1 & \ldots & k_i & \ldots & k_n \\ \hline c_1 & a_{1,1} & \ldots & a_{1,i} & \ldots & a_{1,n} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ c_j & a_{j,1} & \ldots & a_{j,i} & \vdots & a_{j,n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_m & a_{m,1} & \ldots & a_{m,i} & \ldots & a_{m,n} \end{array}. \tag{2}$$

Step A3. Construction of the index matrix K_c with the points $b_{i,j}$ of candidates according to criteria using the weight coefficients.

$$K_c = \begin{array}{c|ccccc} & k_1 & \dots & k_i & \dots & k_n \\ \hline c_1 & b_{1,1} & \dots & b_{1,i} & \dots & b_{1,n} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ c_j & b_{j,1} & \dots & b_{j,i} & \vdots & b_{j,n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_m & b_{m,1} & \dots & b_{m,i} & \dots & b_{m,n} \end{array}, \tag{3}$$

where

$$b_{j,i} = t_j.\ a_{j,i},$$

for $i = 1,\dots, n, j = 1,\dots, m$.

Step A4. Construction of the index matrix F by applying operation sum-row-aggregation [3] over the index matrix K_c.

$$F = \begin{array}{c|ccccc} & k_1 & \dots & k_i & \dots & k_n \\ \hline f & f_1 & \dots & f_i & \dots & f_n \end{array}. \tag{4}$$

More precisely

$$F = \rho_{sum}(K_c, f) = \begin{array}{c|ccccc} & k_1 & \dots & k_i & \dots & k_n \\ \hline f & \sum_{j=1}^{m} b_{j,1} & \dots & \sum_{j=1}^{m} b_{j,i} & \dots & \sum_{j=1}^{m} b_{j,n} \end{array}. \tag{5}$$

Step A5. Determination of the intuitionistic fuzzy evaluations of the PhD candidates.

The evaluations for each of the PhD candidates using the elements of the index matrix F are represented by intuitionistic fuzzy estimations [4, 5]. They have the form $\langle \mu_i, \nu_i \rangle$, where μ_i and ν_i determine the degrees of satisfaction and not-satisfaction of the criteria for the i-th candidate.

The i-th candidate, $i = 1, 2,\dots, n$ has f_i points. Let f^{min} represent the points for the minimal required evaluation from having successfully passed the PhD comprehensive examination. Then f^{max} represents the maximal points obtained from the candidates, $f_{max} = max(f_1, \dots, f_i, \dots, f_n)$.

In this case the evaluations of the i-th PhD candidate $\langle \mu_i, \nu_i \rangle$, $\mu_i, \nu_i \in [0, 1]$, $\mu_i + \nu_i \le 1$ is:

$$\mu_i = \frac{f_i}{f_{max}}, \tag{6}$$

$$\nu_i = \begin{cases} \dfrac{f_{min} - f_i}{f_{max}}, & \text{if } f_{min} \ge f_i \\ 0, & \text{if } f_{min} < f_i \end{cases}, \tag{7}$$

The degree of uncertainty π_i is

$$\pi_i = \begin{cases} \dfrac{f_{max} - f_i - (f_{min} - f_i)}{f_{max}}, & \text{if } f_{min} \ge f_i \\ \dfrac{f_{max} - f_i}{f_{max}}, & \text{if } f_{min} < f_i \end{cases}. \tag{8}$$

B. Clustering of the evaluations

The SOM used for the clustering of the intuitionistic fuzzy assessments has 6 neurons in one layer. The vector for the SOM learning SOM has 62 different points which describe the working area of the neural network. The number of neurons determines the number of clusters, which hit corresponding points. In the two dimensional SOM 3×2 neurons are used; that is, 6 neurons.

We use Matlab program for creating, learning and testing the SOM (Fig. 1).

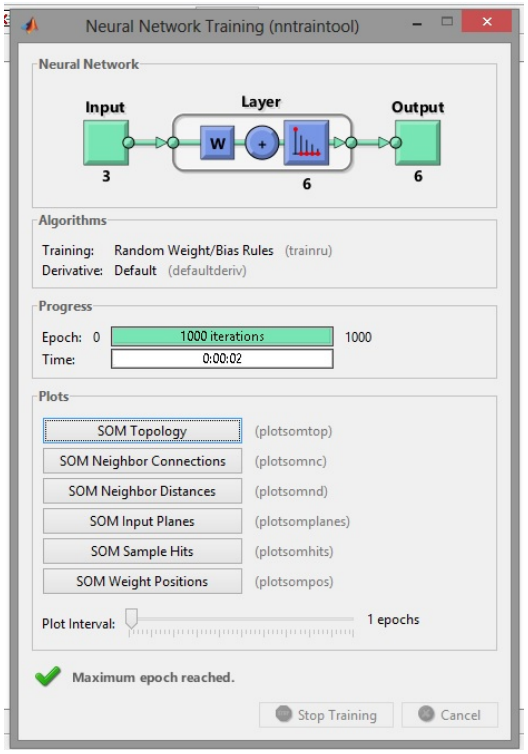

Figure 1. The structure and parameters of the SOM

Graphical representation of the clusters is shown in Fig. 2.

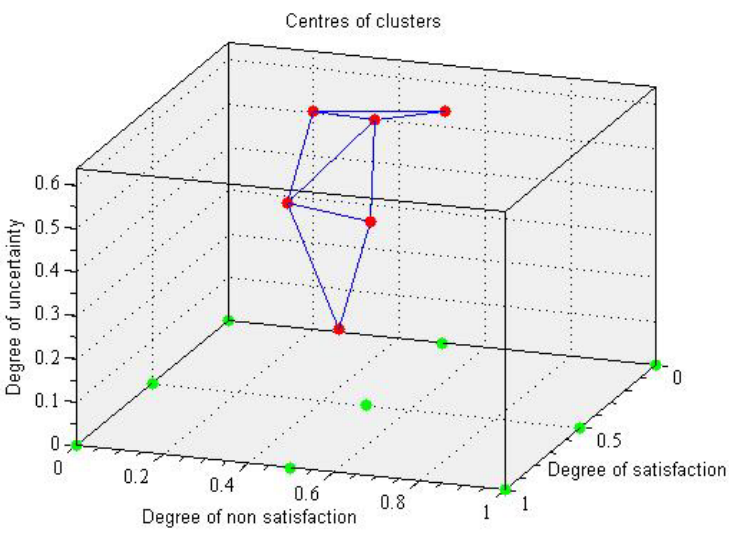

Figure 2. Graphical representation of the clusters

The test vectors enter the inputs of the SOM. Every test vector hits the cluster that represents evaluations which correspond to the points obtained from the candidates according to (6), (7) and (8). For example, in cluster number 6 there are the PhD candidates who exhibit the highest level for satisfying the criteria.

The example test vectors are shown in Table 1.

	μ	π	ν	Number of cluster
Vector 3	0.02	0.67	0.31	1
Vector 22	0.97	0.03	0	6
Vector 29	0.2	0.67	0.13	1
Vector 35	0.62	0.38	0.00	4
Vector 35	0.30	0.67	0,03	2
Vector 47	0.71	0.29	0	5
Vector 51	0,52	0,48	0	3
Vector 58	0.83	0.17	0	6

Table 1. Test vectors

- The test vectors with values for $\mu \in [0.80, 1.00]$, $\pi \in [0.00, 0.20]$ and $\nu = 0.00$ are classified in cluster 6. This reflects the cases where the candidates have successfully passed the PhD comprehensive examination (the degree of 'not-satisfaction', ν, is zero), and have very high assessment marks on the other criteria.
- The test vectors with values for $\mu \in [0.70, 0.77]$, $\pi \in [0.23, 0.30]$ and $\nu = 0.00$ are classified in cluster 5. This reflects the cases when the candidates passed successfully the PhD comprehensive examination, and have high assessment for the other criteria.
- The test vectors with values for $\mu \in [0.57, 0.67]$, $\pi \in [0.33, 0.43]$ and $\nu = 0.00$ are classified in cluster 4. This reflects the cases when the candidates passed successfully the PhD comprehensive examination, and have medium assessment for the other criteria.
- The test vectors with values $\mu \in [0.43, 0.53]$, $\pi \in [0.47, 0.57]$ and $\nu = 0.00$ are classified in cluster 3. This reflects the cases when the candidates passed successfully the PhD comprehensive examination, and have low assessment for the other criteria.
- The test vectors with values $\mu \in [0.27, 0.40]$, $\pi \in [0.60, 0.67]$ and $\nu \in [0.00, 0.07]$ are classified in cluster 2. This reflects the cases when the PhD candidates have a low or minimal assessment for the PhD comprehensive examination, but above minimal required (the degree of the satisfaction μ), very low assessment for the other criteria (the high degree of the uncertainty π) and minimal or low level for the degree of not satisfaction (ν).
- The test vectors with values $\mu \in [0.00, 0.23]$, $\pi = 0.67$ and $\nu \in [0.10, 0.33]$ are classified in cluster 1. This reflects the cases when the candidates did not pass successfully a PhD comprehensive examination.

The average values of the μ, ν and π in the clusters are represented in Fig. 3.

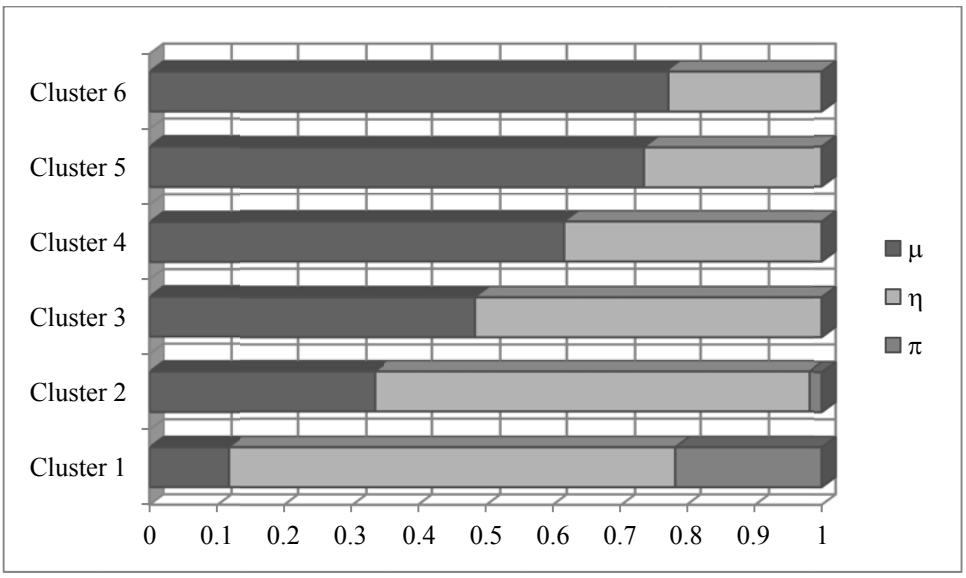

Figure 3. Graphical representation of the average values of the μ, ν and π in the clusters

3 Conclusion

In the present paper we have introduced a method for calculating an intuitionistic fuzzy estimation of the progress of PhD candidates. We used IM for the determination of the points obtained from each candidate according to the various criteria. Then the intuitionistic fuzzy evaluations of the PhD candidates were calculated by the use of elements of the index matrix *F*. The evaluated degrees of satisfactory progress by the candidates, or lack of satisfactory progress, in relation to the criteria are clustered by SOM into 6 clusters. The proposed procedure can measure selection and/or progress of the doctoral candidates.

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