**Notes on Intuitionistic Fuzzy Sets**
**Print ISSN 1310–4926, Online ISSN 2367–8283**
**Vol. 23, 2017, No. 5, 112–124**

# Properties of an intuitionistic fuzzy kernel and an intuitionistic fuzzy subsemiautomaton

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**Received:** 26 September 2017 **Accepted:** 15 November 2017

**Abstract:** In this paper, intuitionistic fuzzy kernel and intuitionistic fuzzy subsemiautomaton are defined over an intuitionistic fuzzy semiautomaton (IFSA) $S = (Q, \Sigma, A)$. Proved the existence of intuitionistic fuzzy homomorphism and strong intuitionistic fuzzy homomorphism on both intuitionistic fuzzy kernel and intuitionistic fuzzy subsemiautomaton over $S$. It is proved that product of two intuitionistic fuzzy kernel is an intuitionistic fuzzy kernel on $S$; product of intuitionistic fuzzy kernel and intuitionistic fuzzy subsemiautomaton is an intuitionistic fuzzy subsemiautomaton over $S$.
**Keywords:** Intuitionistic fuzzy semiautomaton, Intuitionistic fuzzy kernel and intuitionistic fuzzy subsemiautomaton.
**AMS Classification:** 03F55, 03F45.

## 1 Introduction

Zadeh [12] was the first to introduce fuzzy set. In view of a fuzzy finite state automaton there may be more than one fuzzy state transition from a state on an input symbol with a given membership value given by Santos, Wee and Fu [10, 11]. This development was followed by the postulation called deterministic fuzzy finite state automaton as in Malik and Mordeson [8], in which there can be at most one fuzzy transition on a input symbol, which can be constructed equivalently from a fuzzy finite state automaton. Atanassov [2, 3] initiated the concept of an intuitionistic fuzzy set

(IFS). An Atanassov intuitionistic fuzzy set is considered as a generalization of fuzzy set and has been found to be useful to deal with vagueness. In the sense of Atanassov an IFS is characterized by a pair of functions valued in $[0, 1]$, the membership function and the non-membership function. Using the notions of intuitionistic fuzzy sets, Jun [7] introduced the concept of intuitionistic fuzzy finite state machines. Rosenfeld [9] defined fuzzy subgroup. Anthony and Sherwood [1] replaced "min" in Rosenfeld axiom by t-norm and introduced T-Fuzzy subgroup. This group semiautomaton was fuzzified by Das [5] and he introduced fuzzy semiautomaton, fuzzy kernel and fuzzy subsemiautomaton over a finite group. Bhakat [4] introduced $(\epsilon, \varepsilon)$-fuzzy normal, quasinormal and maximal subgroup. Also they introduced fuzzy kernel and fuzzy subsemiautomaton of a fuzzy semiautomaton over a finite group using the notions of a fuzzy normal subgroup and a fuzzy subgroup of a group. Intuitionistic fuzzy homomorphism and strong intuitionistic fuzzy homomorphism are defined in [6].

In this paper we derive some of the properties of an intuitionistic fuzzy kernel, intuitionistic fuzzy subsemiautomaton and discuss some results concerning them.

# 2 Intuitionistic fuzzy kernel

**Definition 1.** *An intuitionistic fuzzy subset $\lambda = (\lambda_\mu, \lambda_\nu)$ of $Q$ is called an intuitionistic fuzzy subgroup of $Q$ if the following properties hold:*

*(i)* $\lambda_\mu(p * q) \geq \lambda_\mu(p) \wedge \lambda_\mu(q)$ *and* $\lambda_\nu(p * q) \leq \lambda_\nu(p) \vee \lambda_\nu(q)$

*(ii)* $\lambda_\mu(p) = \lambda_\mu(p^{-1})$ *and* $\lambda_\nu(p) = \lambda_\nu(p^{-1})$ $\forall p, q \in Q$

**Definition 2.** *An intuitionistic fuzzy subgroup $\lambda = (\lambda_\mu, \lambda_\nu)$ of $Q$ is called an intuitionistic fuzzy normal subgroup of $Q$ if $\lambda_\mu(p * q * p^{-1}) \geq \lambda_\mu(q)$ and $\lambda_\nu(p * q * p^{-1}) \leq \lambda_\nu(q)$ $\forall p, q \in Q$*

**Definition 3.** *Let $\lambda = (\lambda_\mu, \lambda_\nu)$ and $\rho = (\rho_\mu, \rho_\nu)$ be an intuitionistic fuzzy subsets of $Q$. The product $\lambda \times \rho = ((\lambda \times \rho)_\mu, (\lambda \times \rho)_\nu)$ of $\lambda$ and $\rho$ is defined by*
$(\lambda \times \rho)_\mu(p) = \vee\{\lambda_\mu(q) \wedge \rho_\mu(r) \mid q, r \in Q \text{ such that } p = q * r\}$
*and* $(\lambda \times \rho)_\nu(p) = \wedge\{\lambda_\nu(q) \vee \rho_\nu(r) \mid q, r \in Q \text{ such that } p = q * r\}$ $\forall p \in Q$

**Definition 4.** *Let $\lambda$ and $\rho$ be an intuitionistic fuzzy subgroups of $Q$ such that $\lambda \subseteq \rho$. Then $\lambda$ is called an intuitionistic fuzzy normal subgroup of $\rho$ if*
$\lambda_\mu(p * q * p^{-1}) \geq \lambda_\mu(q) \wedge \rho_\mu(p)$ *and* $\lambda_\nu(p * q * p^{-1}) \leq \lambda_\nu(q) \vee \rho_\nu(p)$ $\forall p, q \in Q$

**Definition 5.** *An intuitionistic fuzzy semiautomaton IFSA over a finite group $(Q, *)$ is a 3-tuple $(Q, \Sigma, A)$ where $\Sigma$ is a finite set and $A = (\mu_A, \nu_A)$ is a fuzzy subset of $Q \times \Sigma \times Q$.*

**Definition 6.** *Let $\lambda$ and $\rho$ be an intuitionistic fuzzy subsets of the groups G and H, respectively. Let $f : G \to H$ be a group intuitionistic fuzzy homomorphism. An intuitionistic fuzzy subsets $f(\lambda) = (f(\lambda)_\mu, f(\lambda)_\nu)$ of H and $f^{-1}(\rho) = (f^{-1}(\rho)_\mu, f^{-1}(\rho)_\nu)$ of G are defined as follows:*

$$f(\lambda)_\mu(h) = \begin{cases} \vee\{\lambda_\mu(g) \mid g \in G \text{ such that } f(g) = h\} & \text{if } f^{-1}(H) \neq \phi \\ 0 & \text{if } f^{-1}(H) = \phi \end{cases}$$

*for all $h \in H$, and $f^{-1}(\rho)_\mu(g) = \rho_\mu(f(g))$ for all $g \in G$ and*

$$f(\lambda)_\nu(h) = \begin{cases} \wedge\{\lambda_\nu(g) \mid g \in G \text{ such that } f(g) = h\} \text{ if } f^{-1}(H) \neq \phi \\ 1 \quad \text{if } f^{-1}(H) = \phi \end{cases}$$

*for all $h \in H$, and $f^{-1}(\rho)_\nu(g) = \rho_\nu(f(g))$ for all $g \in G$*

**Definition 7.** *Let $S = (Q, \Sigma, A)$ and $T = (Q_1, \Sigma_1, A_1)$ be IFSA's over a finite group. A pair of functions $(f, g)$ where $f : Q \to Q_1, g : \Sigma \to \Sigma_1$, is called an intuitionistic fuzzy homomorphism from $S$ into $T$, written $(f, g) : S \to T$ if the following conditions hold:*

*(i) $f$ is a group intuitionistic fuzzy homomorphism*

*(ii) $\mu_A(p, x, q) \leq \mu_{A_1}(f(p), g(x), f(q))$ and $\nu_A(p, x, q) \geq \nu_{A_1}(f(p), g(x), f(q))$ $\forall p, q \in Q, x \in \Sigma$.*

*The pair $(f, g)$ is called a strong intuitionistic fuzzy homomorphism from $S$ into $T$ if it satisfies (i) of above definition and the added condition*
*$\mu_{A_1}(f(p), g(x), f(q)) = \vee\{\mu_A(p, x, r) \mid r \in Q, f(r) = f(q)\}$ and*
*$\nu_{A_1}(f(p), g(x), f(q)) = \wedge\{\nu_A(p, x, r) \mid r \in Q, f(r) = f(q)\}$ $\forall p, q \in Q, x \in \Sigma$.*

**Definition 8.** *Let $S = (Q, \Sigma, A)$ be an IFSA over a finite group. A fuzzy subset $\lambda$ of $Q$ is called an intuitionistic fuzzy kernel of $S$ if the following conditions hold:*

*(i) $\lambda$ is an intuitionistic fuzzy normal subgroup of $Q$.*

*(ii) $\lambda_\mu(p * r^{-1}) \geq \mu_A(q * k, x, p) \wedge \mu_A(q, x, r) \wedge \lambda(k)$ and*
*$\lambda_\nu(p * r^{-1}) \leq \nu_A(q * k, x, p) \vee \nu_A(q, x, r) \vee \lambda(k)$ $\forall p, q, k, r \in Q, x \in \Sigma$*

**Theorem 1.** *An intuitionistic fuzzy normal subgroup $\lambda$ of $Q$ is an intuitionistic fuzzy kernel of $S$ if and only if*
*$\lambda_\mu(p * r^{-1}) \geq \mu^*_A(q * k, x, p) \wedge \mu^*_A(q, x, r) \wedge \lambda_\mu(k)$ and*
*$\lambda_\nu(p * r^{-1}) \leq \nu^*_A(q * k, x, p) \vee \nu^*_A(q, x, r) \vee \lambda_\nu(k)$ $\forall p, q, k, r \in Q, x \in \Sigma^*$*

*Proof.* Let $\lambda$ be an intuitionistic fuzzy kernel of $S$. we prove the theorem by induction on $|x| = n$. Let $n = 0$. Then $x = \epsilon$. Let $p, q, k, r \in Q$. If $p = q * k, r = q$, then

$$\mu^*_A(q * k, \epsilon, p) \wedge \mu^*_A(q, \epsilon, r) \wedge \lambda_\mu(k) \leq \lambda_\mu(q * k * q^{-1})$$

and

$$\nu^*_A(q * k, \epsilon, p) \vee \nu^*_A(q, \epsilon, r) \vee \lambda_\nu(k) \geq \lambda_\nu(q * k * q^{-1})$$

Since $\lambda$ is a fuzzy normal subgroup. If $p \neq q * k$ or $r \neq q$, then

$$\mu^*_A(q * k, \epsilon, p) \wedge \mu^*_A(q, \epsilon, r) \wedge \lambda_\mu(k) \leq \lambda_\mu(p * r^{-1})$$

and

$$\nu^*_A(q * k, \epsilon, p) \vee \nu^*_A(q, \epsilon, r) \vee \lambda_\nu(k) \geq \lambda_\nu(p * r^{-1})$$

Thus the result holds for $n = 0$. Suppose that the result holds for all $y \in \Sigma^*$, where $|y| \leq n-1, n > 0$. Let $x \in \Sigma^*$ be such that $x = ya, y \in \Sigma^*, a \in \Sigma, |y| = n-1, n > 0$. Then

$$\begin{aligned}
\mu_A^*(q*k,x,p) \wedge \mu_A^*(q,x,r) \wedge \lambda_\mu(k) &= (\vee\{\mu_A^*(q*k,y,u) \wedge \mu_A(u,a,p) \mid u \in Q\}) \\
&\quad \wedge (\wedge\{\mu_A^*(q,y,v) \wedge \mu_A(v,a,r) \mid v \in Q\}) \wedge \lambda_\mu(k) \\
&= \vee\{\vee\{\mu_A^*(q*k,y,u) \wedge \mu_A(u,a,p) \wedge \mu_A^*(q,y,v) \\
&\quad \wedge \mu_A(v,a,r) \wedge \lambda_\mu(k) \mid u \in Q\} \mid v \in Q\} \\
&\leq \vee\{\vee\{\lambda_\mu(u*v^{-1}) \wedge \mu_A(u,a,p) \wedge \mu_A(v,a,r) \mid u \in Q\} \\
&\quad \mid v \in Q\} \\
&\leq \vee\{\vee\{\lambda_\mu(v^{-1}*u) \wedge \mu_A(v*v^{-1}*u,a,p) \\
&\quad \wedge \mu_A(v,a,r) \mid u \in Q\} \mid v \in Q\} \\
&\quad (since\ \lambda_\mu\ is\ a\ normal\ fuzzy\ subgroup) \\
&\leq \lambda_\mu(p*r^{-1})
\end{aligned}$$

and

$$\begin{aligned}
\nu_A^*(q*k,x,p) \vee \nu_A^*(q,x,r) \vee \lambda_\nu(k) &= (\wedge\{\nu_A^*(q*k,y,u) \vee \nu_A(u,a,p) \mid u \in Q\}) \\
&\quad \vee (\vee\{\nu_A^*(q,y,v) \vee \nu_A(v,a,r) \mid v \in Q\}) \vee \lambda_\nu(k) \\
&= \wedge\{\wedge\{\nu_A^*(q*k,y,u) \vee \nu_A(u,a,p) \vee \nu_A^*(q,y,v) \\
&\quad \vee \nu_A(v,a,r) \vee \lambda_\nu(k) \mid u \in Q\} \mid v \in Q\} \\
&\geq \wedge\{\wedge\{\lambda_\nu(u*v^{-1}) \vee \nu_A(u,a,p) \vee \nu_A(v,a,r) \mid u \in Q\} \\
&\quad \mid v \in Q\} \\
&\geq \wedge\{\wedge\{\lambda_\nu(v^{-1}*u) \vee \nu_A(v*v^{-1}*u,a,p) \\
&\quad \vee \nu_A(v,a,r) \mid u \in Q\} \mid v \in Q\} \\
&\quad (since\ \lambda_\nu\ is\ a\ normal\ fuzzy\ subgroup) \\
&\geq \lambda_\nu(p*r^{-1})
\end{aligned}$$

Hence the desired conditions holds. The converse follows easily. □

**Theorem 2.** *Let $T = (Q_1, \Sigma, A_1)$ be an IFSA over a finite group and let $f$ be a homomorphism from $S$ into $T$. If $\lambda$ is an intuitionistic fuzzy kernel of $T$, then $f^{-1}(\lambda_\mu, \lambda_\nu)$ is an intuitionistic fuzzy kernel of $S$.*

*Proof.* Let $\lambda$ be an intuitionistic fuzzy kernel of $T$. Then $\lambda$ is an intuitionistic fuzzy subgroup of $Q$. Since $f^{-1}$ is a function from $Q_1$ into $Q$, $f^{-1}(\lambda_\mu, \lambda_\nu)$ is an intuitionistic fuzzy subgroup of $Q$. Let $p, q \in Q, x \in \Sigma$. Then

$$\begin{aligned}
&\mu_A(q*k,x,p) \wedge \mu_A(q,x,r) \wedge f^{-1}(\lambda)_\mu(k) \\
&\qquad = \mu_A(q*k,x,p) \wedge \mu_A(q,x,r) \wedge (\vee\{\lambda_\mu(k_1) \\
&\qquad \mid k_1 \in Q_1, f^{-1}(k_1) = k\}) \\
&\qquad = \vee\{\mu_A(q*k,x,p) \wedge \mu_A(q,x,r) \wedge \lambda_\mu(k_1) \\
&\qquad \mid k_1 \in Q_1, f^{-1}(k_1) = k\}
\end{aligned}$$

Now let $p, q, r, k \in Q$ be such that $f^{-1}(p_1) = p, f^{-1}(q_1) = q, f^{-1}(r_1) = r$ and $f^{-1}(k_1) = k$. Then

$$\begin{aligned}
&\mu_A(q*k, x, p) \wedge \mu_A(q, x, r) \wedge \lambda_\mu(k) \\
&\qquad = \mu_A(f^{-1}(q_1 * k_1), x, f^{-1}(p_1)) \wedge \mu_A(f^{-1}(q), x, f^{-1}(r)) \wedge \lambda_\mu(k) \\
&\qquad = (\vee\{\mu_{A_1}(q_1 * k_1, x, a_1) \mid a_1 \in Q_1, f^{-1}(a_1) = f^{-1}(p_1)\}) \\
&\qquad\quad \wedge (\vee\{\mu_{A_1}(q_1, x, b_1) \mid b_1 \in Q_1, f^{-1}(b_1) = f^{-1}(r_1)\}) \wedge \lambda_\mu(k) \\
&\qquad = \vee\{\vee\{\mu_{A_1}(q_1 * k_1, x, a_1) \wedge \mu_{A_1}(q_1, x, b_1) \wedge \lambda_\mu(k) \\
&\qquad\quad \mid a_1 \in Q_1, f^{-1}(a_1) = f^{-1}(p_1)\} \mid b_1 \in Q_1, f^{-1}(b_1) = f^{-1}(r_1)\} \\
&\qquad \leq \vee\{\vee\{\lambda_\mu(a_1 * b_1^{-1}) \mid a_1 \in Q_1, f^{-1}(a_1) = f^{-1}(p_1)\} \\
&\qquad\quad \mid b_1 \in Q_1, f^{-1}(b_1) = f^{-1}(r_1)\} \\
&\qquad \leq f^{-1}(\lambda)_\mu(p * r^{-1})
\end{aligned}$$

Hence

$$\begin{aligned}
f^{-1}(\lambda)_\mu(p * r^{-1}) &\geq \vee\{\mu_A(q*k, x, p) \wedge \mu_A(q, x, r) \wedge \lambda_\mu(k_1) \mid k_1 \in Q_1, f^{-1}(k_1) = k\} \\
&= \mu_A(q*k, x, p) \wedge \mu_A(q, x, r) \wedge f^{-1}(\lambda)_\mu(k)
\end{aligned}$$

and

$$\begin{aligned}
&\nu_A(q*k, x, p) \vee \nu_A(q, x, r) \vee f^{-1}(\lambda)_\nu(k) \\
&\qquad = \nu_A(q*k, x, p) \vee \nu_A(q, x, r) \vee (\wedge\{\lambda_\nu(k_1) \\
&\qquad\quad \mid k_1 \in Q_1, f^{-1}(k_1) = k\}) \\
&\qquad = \wedge\{\nu_A(q*k, x, p) \vee \nu_A(q, x, r) \vee \lambda_\nu(k_1) \\
&\qquad\quad \mid k_1 \in Q_1, f^{-1}(k_1) = k\}
\end{aligned}$$

Now let $p, q, r, k \in Q$ be such that $f^{-1}(p_1) = p, f^{-1}(q_1) = q, f^{-1}(r_1) = r$ and $f^{-1}(k_1) = k$. Then

$$\begin{aligned}
&\nu_A(q*k, x, p) \vee \nu_A(q, x, r) \vee \lambda_\nu(k) \\
&\qquad = \nu_A(f^{-1}(q_1 * k_1), x, f^{-1}(p_1)) \vee \nu_A(f^{-1}(q_1), x, f^{-1}(r_1)) \vee \lambda_\nu(k) \\
&\qquad = (\wedge\{\nu_{A_1}(q_1 * k_1, x, a_1) \mid a_1 \in Q_1, f^{-1}(a_1) = f^{-1}(p_1)\}) \\
&\qquad\quad \vee (\wedge\{\nu_{A_1}(q_1, x, b_1) \mid b_1 \in Q_1, f^{-1}(b_1) = f^{-1}(p_1)\}) \vee \lambda_\nu(k) \\
&\qquad = \wedge\{\wedge\{\nu_{A_1}(q_1 * k_1, x, a_1) \vee \nu_{A_1}(q_1, x, b_1) \vee \lambda_\nu(k) \mid a_1 \in Q_1, \\
&\qquad\qquad f^{-1}(a_1) = f^{-1}(p_1)\} \mid b_1 \in Q_1, f^{-1}(b_1) = f^{-1}(r_1)\} \\
&\qquad \geq \wedge\{\wedge\{\lambda_\nu(a_1 * b_1^{-1}) \mid a_1 \in Q_1, f^{-1}(a_1) = f^{-1}(p_1)\} \mid b_1 \in Q_1, \\
&\qquad\qquad f^{-1}(b_1) = f^{-1}(r_1)\} \\
&\qquad \geq f^{-1}(\lambda)_\nu(p * r^{-1})
\end{aligned}$$

Hence

$$\begin{aligned}
f^{-1}(\lambda)_\nu(p * r^{-1}) &\leq \wedge\{\nu_A(q*k, x, p) \vee \nu_A(q, x, r) \vee \lambda_\nu(k_1) \mid k_1 \in Q_1, f^{-1}(k_1) = k\} \\
&= \nu_A(q*k, x, p) \vee \nu_A(q, x, r) \vee f^{-1}(\lambda)_\nu(k)
\end{aligned}$$

Thus $f^{-1}(\lambda)$ is an intuitionistic fuzzy kernel of $S$. $\square$

**Theorem 3.** *Let $T = (Q_1, \Sigma, A_1)$ be an IFSA over a finite group and let $f$ be a strong homomorphism from $S$ onto $T$. If $\lambda$ is an intuitionistic fuzzy kernel of $S$, then $f(\lambda_\mu, \lambda_\nu)$ is an intuitionistic fuzzy kernel of $T$.*

*Proof.* Let $\lambda$ be an intuitionistic fuzzy kernel of $S$. Then $\lambda$ is an intuitionistic normal subgroup of $Q$. Since $f$ is an epimorphism from $Q$ onto $Q_1$, $f(\lambda_\mu, \lambda_\nu)$ is an intuitionistic fuzzy normal subgroup of $Q_1$. Let $p_1, q_1, r_1, k_1 \in Q_1, x \in \Sigma$. Then

$$\begin{aligned}&\mu_{A_1}(q_1 * k_1, x, p_1) \wedge \mu_{A_1}(q_1, x, r_1) \wedge f(\lambda)_\mu(k_1)\\ &= \mu_{A_1}(q_1 * k_1, x, p_1) \wedge \mu_{A_1}(q_1, x, r_1) \wedge (\vee\{\lambda_\mu(k) \mid k \in Q, f(k) = k_1\})\\ &= \vee\{\mu_{A_1}(q_1 * k_1, x, p_1) \wedge \mu_{A_1}(q_1, x, r_1) \wedge \lambda_\mu(k) \mid k \in Q, f(k) = k_1\}\end{aligned}$$

Now let $p, q, r, k \in Q$ be such that $f(p) = p_1, f(q) = q_1, f(r) = r_1$ and $f(k) = k_1$. Then

$$\begin{aligned}&\mu_{A_1}(q_1 * k_1, x, p_1) \wedge \mu_{A_1}(q_1, x, r_1) \wedge \lambda_\mu(k)\\ &= \mu_{A_1}(f(q * k), x, f(p)) \wedge \mu_{A_1}(f(q), x, f(r)) \wedge \lambda_\mu(k)\\ &= (\vee\{\mu_A(q * k, x, a) \mid a \in Q, f(a) = f(p)\})\\ &\quad \wedge (\vee\{\mu_A(q, x, b) \mid b \in Q, f(b) = f(r)\}) \wedge \lambda_\mu(k)\\ &= \{\vee\{\mu_A(q * k, x, a) \wedge \mu_A(q, x, b) \wedge \lambda_\mu(k) \mid a \in Q,\\ &\qquad f(a) = f(p)\} \mid b \in Q, f(b) = f(r)\}\\ &\leq \vee\{\vee\{\lambda_\mu(a * b^{-1}) \mid a \in Q, f(a) = f(p)\} \mid b \in Q,\\ &\qquad f(b) = f(r)\}\\ &\leq f(\lambda)_\mu(p_1 * r_1^{-1})\end{aligned}$$

Hence

$$\begin{aligned}f(\lambda)_\mu(p_1 * r_1^{-1}) &\geq \vee\{\mu^*_{A_1}((q_1 * k_1), x, p_1) \wedge \mu_{A_1}(q_1, x, r_1) \wedge \lambda_\mu(k) \mid k \in Q, f(k) = k_1\}\\ &= \mu_{A_1}(q_1 * k_1, x, p_1) \wedge \mu_{A_1}(q_1, x, r_1) \wedge f(\lambda)_\mu(k_1)\end{aligned}$$

and

$$\begin{aligned}&\nu_{A_1}(q_1 * k_1, x, p_1) \vee \nu_{A_1}(q_1, x, r_1) \vee f(\lambda)_\nu(k_1)\\ &= \nu_{A_1}(q_1 * k_1, x, p_1) \vee \nu_{A_1}(q_1, x, r_1) \vee (\wedge\{\lambda_\nu(k) \mid k \in Q, f(k) = k_1\})\\ &= \wedge\{\nu_{A_1}(q_1 * k_1, x, p_1) \vee \nu_{A_1}(q_1, x, r_1) \vee \lambda_\nu(k) \mid k \in Q, f(k) = k_1\}\end{aligned}$$

Now let $p, q, r, k \in Q$ be such that $f(p) = p_1, f(q) = q_1, f(r) = r_1$ and $f(k) = k_1$. Then

$$\begin{aligned}
&\nu_{A_1}(q_1 * k_1, x, p_1) \vee \nu_{A_1}(q_1, x, r_1) \vee \lambda_\nu(k) \\
&\qquad = \nu_{A_1}(f(q * k), x, f(p)) \vee \nu_{A_1}(f(q), x, f(r)) \vee \lambda_\nu(k) \\
&\qquad = (\wedge\{\nu_A(q * k, x, a) \mid a \in Q, f(a) = f(p)\}) \\
&\qquad\quad \vee (\wedge\{\nu_A(q, x, b) \mid b \in Q, f(b) = f(p)\}) \vee \lambda_\nu(k) \\
&\qquad = \{\wedge\{\nu_A(q * k, x, a) \vee \nu_A(q, x, b) \vee \lambda_\nu(k) \mid a \in Q, \\
&\qquad\qquad f(a) = f(p)\} \mid b \in Q, f(b) = f(r)\} \\
&\qquad \geq \wedge\{\wedge\{\lambda_\nu(a * b^{-1}) \mid a \in Q, f(a) = f(p)\} \mid b \in Q, \\
&\qquad\qquad f(b) = f(r)\} \\
&\qquad \geq f(\lambda)_\nu(p_1 * r_1^{-1})
\end{aligned}$$

Hence

$$\begin{aligned}
f(\lambda)_\nu(p_1 * r_1^{-1}) &\leq \wedge\{\nu^*_{A_1}((q_1 * k_1), x, p_1) \vee \nu_{A_1}(q_1, x, r_1) \vee \lambda_\nu(k) \mid k \in Q, f(k) = k_1\} \\
&= \nu_{A_1}(q_1 * k_1, x, p_1) \vee \nu_{A_1}(q_1, x, r_1) \vee f(\lambda)_\mu(k_1)
\end{aligned}$$

Thus $f(\lambda)$ is an intuitionistic fuzzy kernel of $T$. □

**Theorem 4.** *If $\lambda$ and $\sigma$ are intuitionistic fuzzy kernels of $S$, then $\lambda \times \sigma$ is an intuitionistic fuzzy kernel of $S$.*

*Proof.* Since $\lambda$ and $\sigma$ are intuitionistic fuzzy normal subgroups of $Q$, we prove that $\lambda \times \sigma$ is an intuitionistic fuzzy normal subgroup of $Q$ and $\lambda \times \sigma = \sigma \times \lambda$. Now

$$\begin{aligned}
(\lambda \times \sigma)_\mu(p * q * p^{-1}) &= \vee\{\lambda_\mu(r) \wedge \sigma_\mu(s) \mid r, s \in Q \textit{ such that } p * q * p^{-1} = r * s\} \\
&\geq \lambda_\mu(r) \wedge \sigma_\mu(s) \\
&\geq (\lambda \times \sigma)_\mu(q)
\end{aligned}$$

and

$$\begin{aligned}
(\lambda \times \sigma)_\nu(p * q * p^{-1}) &= \vee\{\lambda_\nu(r) \wedge \sigma_\nu(s) \mid r, s \in Q \textit{ such that } p * q * p^{-1} = r * s\} \\
&\leq \lambda_\nu(r) \wedge \sigma_\nu(s) \\
&\leq (\lambda \times \sigma)_\nu(q)
\end{aligned}$$

$$\begin{aligned}
(\lambda \times \sigma)_\mu(p * r^{-1}) &\geq \lambda_\mu(p * q^{-1}) \wedge \sigma_\mu(q * r^{-1}) \\
&\geq (\mu_A(a * b * c, x, p) \wedge \mu_A(a * b, x, q) \wedge \lambda_\mu(c)) \\
&\quad \wedge (\mu_A(a * b, x, p) \wedge \mu_A(a, x, r) \wedge \sigma_\mu(b)) \\
&= \mu_A(a * b * c, x, p) \wedge \mu_A(a, x, r) \wedge \lambda_\mu(c) \wedge \sigma_\mu(b) \\
&\qquad (\textit{since } \mu_A(a * b * c, x, p) \leq \mu_A(a * b, x, p)) \forall a, b, c, p, r \in Q, x \in \Sigma.
\end{aligned}$$

and

$$\begin{aligned}(\lambda\times\sigma)_\nu(p*r^{-1}) &\le \lambda_\nu(p*q^{-1})\vee\sigma_\nu(q*r^{-1})\\ &\le (\nu_A(a*b*c,x,p)\vee\nu_A(a*b,x,q)\vee\lambda_\nu(c))\\ &\quad\vee(\nu_A(a*b,x,p)\vee\nu_A(a,x,r)\vee\sigma_\nu(b))\\ &=\nu_A(a*b*c,x,p)\vee\nu_A(a,x,r)\vee\lambda_\nu(c)\vee\sigma_\nu(b)\\ &\quad(since\ \nu_A(a*b*c,x,p)\ge\nu_A(a*b,x,p))\forall a,b,c,p,r\in Q,x\in\Sigma.\end{aligned}$$

Thus for all $p,q,r,k\in Q,x\in\Sigma$

$$\begin{aligned}(\lambda\times\sigma)_\mu(p*r^{-1}) &\ge \vee\{\mu_A(q*b*c,x,p)\wedge\mu_A(q,x,r)\wedge\lambda_\mu(c)\wedge\sigma_\mu(b)\mid b,c\in Q,b*c=k\}\\ &=(\mu_A(q*k,x,p)\wedge\mu_A(q,x,r))\wedge(\vee\{\lambda_\mu(c)\wedge\sigma_\mu(b)\mid b,c\in Q,b*c=k\})\\ &=\mu_A(q*k,x,p)\wedge\mu_A(q,x,r)\wedge(\lambda\times\sigma)_\mu(k)\end{aligned}$$

and

$$\begin{aligned}(\lambda\times\sigma)_\nu(p*r^{-1}) &\le \wedge\{\nu_A(q*b*c,x,p)\vee\nu_A(q,x,r)\vee\lambda_\nu(c)\vee\sigma_\nu(b)\mid b,c\in Q,b*c=k\}\\ &=(\nu_A(q*k,x,p)\vee\nu_A(q,x,r))\vee(\wedge\{\lambda_\nu(c)\vee\sigma_\nu(b)\mid b,c\in Q,b*c=k\})\\ &=\nu_A(q*k,x,p)\vee\nu_A(q,x,r)\vee(\lambda\times\sigma)_\nu(k)\end{aligned}$$

Thus $\lambda\times\sigma$ is an intuitionistic fuzzy kernel of $S$. □

# 3 Intuitionistic fuzzy subsemiautomaton

**Definition 9.** *An intuitionistic fuzzy subset $\lambda$ of $Q$ is called an intuitionistic fuzzy subsemiautomaton of $S$ if the following conditions hold*

*(i)* *$\lambda$ is an intuitionistic fuzzy subgroup of $Q$*

*(ii)* $\lambda_\mu(p)\ge\mu_A(q,x,p)\wedge\lambda_\mu(q)$ *and* $\lambda_\nu(p)\le\nu_A(q,x,p)\vee\lambda_\nu(q)\ \forall p,q\in Q,x\in\Sigma$

**Theorem 5.** *An intuitionistic fuzzy subgroup $\lambda$ of $Q$ is an intuitionistic fuzzy subsemiautomaton of $S$ if and only if*
$\lambda_\mu(p)\ge\mu_A^*(q,x,p)\wedge\lambda_\mu(q)$ *and*
$\lambda_\nu(p)\le\nu_A^*(q,x,p)\vee\lambda_\nu(k)\ \forall p,q\in Q,x\in\Sigma^*$

*Proof.* Let $\lambda$ be an intuitionistic fuzzy subsemiautomaton of $S$. We prove the theorem by induction on $|x|=n$. Let $n=0$. Then $x=\epsilon$. Let $p,q\in Q$. If $p=q$, then

$$\mu_A^*(q,\epsilon,p)\wedge\lambda_\mu(q)\le\lambda_\mu(p*q*p^{-1})$$

and

$$\nu_A^*(q,\epsilon,p)\vee\lambda_\nu(q)\ge\lambda_\nu(p*q*p^{-1})$$

Since $\lambda$ is an intuitionistic fuzzy subgroup. If $p \neq q$, then

$$\mu_A^*(q, \epsilon, p) \wedge \lambda_\mu(q) \leq \lambda_\mu(p)$$

and

$$\nu_A^*(q, \epsilon, p) \vee \lambda_\nu(q) \geq \lambda_\nu(p)$$

Thus the result holds for $n = 0$. Suppose that the result holds for all $y \in \Sigma^*$, where $|y| \leq n - 1, n > 0$. Let $x \in \Sigma^*$ be such that $x = ya, y \in \Sigma^*$,
$a \in \Sigma, |y| = n - 1, n > 0$. Then

$$\begin{aligned}
&\mu_A^*(q, x, p) \wedge \lambda_\mu(q) \\
&\qquad = (\vee\{\mu_A^*(q, y, u) \wedge \mu_A(u, a, p) \mid u \in Q\}) \wedge \lambda_\mu(q) \\
&\qquad = \vee\{\vee\{\mu_A^*(q, y, u) \wedge \mu_A(u, a, p) \wedge \lambda_\mu(q) \mid u \in Q\} \\
&\qquad \leq \{\vee\{\lambda_\mu(u) \wedge \mu_A(u, a, p) \mid u \in Q\} \\
&\qquad \leq \lambda_\mu(p)
\end{aligned}$$

and

$$\begin{aligned}
&\nu_A^*(q, x, p) \vee \lambda_\nu(q) \\
&\qquad = (\wedge\{\nu_A^*(q, y, u) \vee \nu_A(u, a, p) \mid u \in Q\})\lambda\nu(q) \\
&\qquad = \wedge\{\wedge\{\nu_A^*(q, y, u) \vee \nu_A(u, a, p) \vee \lambda\nu(q) \mid u \in Q\} \\
&\qquad \geq \{\wedge\{\lambda_\nu(u) \vee \nu_A(u, a, p) \mid u \in Q\} \\
&\qquad \geq \lambda_\nu(p)
\end{aligned}$$

Hence the desired conditions holds. The converse follows easily. □

**Theorem 6.** *Let $T = (Q_1, \Sigma, A_1)$ be an IFSA over a finite group and let $f$ be a homomorphism from $S$ into $T$. If $\lambda$ is an intuitionistic fuzzy subsemiautomaton of $T$, then $f^{-1}(\lambda_\mu, \lambda_\nu)$ is an intuitionistic fuzzy subsemiautomaton of $S$.*

*Proof.* Let $\lambda$ be an intuitionistic fuzzy subsemiautomaton of $T$. Then $\lambda$ is an intuitionistic fuzzy subgroup of $Q$. Since $f^{-1}$ is a function from $Q_1$ into $Q, f^{-1}(\lambda_\mu, \lambda_\nu)$ is an intuitionistic fuzzy subgroup of $Q$. Let $p, q \in Q, x \in \Sigma$. Then

$$\begin{aligned}
&\mu_A(q, x, p) \wedge f^{-1}(\lambda)_\mu(q) \\
&\qquad = \mu_A(q, x, p) \wedge (\vee\{\lambda_\mu(q_1) \mid q_1 \in Q_1, f^{-1}(q_1) = q\}) \\
&\qquad = \vee\{\mu_A(q, x, p) \wedge \lambda_\mu(q_1) \mid q_1 \in Q_1, f^{-1}(q_1) = q\}
\end{aligned}$$

Now let $p, q \in Q$ be such that $f^{-1}(p_1) = p, f^{-1}(q_1) = q$. Then

$$\begin{aligned}
\mu_A(q, x, p) \wedge \lambda_\mu(q) &= \mu_A(f^{-1}(q_1), x, f^{-1}(p_1)) \wedge \lambda_\mu(q) \\
&= (\vee\{\mu_{A_1}(q_1, x, a_1) \mid a_1 \in Q_1, f^{-1}(a_1) = f^{-1}(p_1)\}) \wedge \lambda_\mu(q) \\
&= \vee\{\vee\{\mu_{A_1}(q_1, x, a_1) \wedge \lambda_\mu(q) \mid a_1 \in Q_1, f^{-1}(a_1) = f^{-1}(p_1)\} \\
&\leq \vee\{\vee\{\lambda_\mu(a_1) \mid a_1 \in Q_1, f^{-1}(a_1) = f^{-1}(p_1)\} \\
&\leq f^{-1}(\lambda)_\mu(p)
\end{aligned}$$

Hence

$$f^{-1}(\lambda)_{\mu}(p) \geq \vee\{\mu_A(q,x,p) \wedge \lambda_{\mu}(q_1) \mid q_1 \in Q_1, f^{-1}(q_1) = q\}$$
$$= \mu_A(q,x,p) \wedge f^{-1}(\lambda)_{\mu}(q)$$

and

$$\nu_A(q,x,p) \vee f^{-1}(\lambda)_{\nu}(q)$$
$$= \nu_A(q,x,p) \vee (\wedge\{\lambda_{\nu}(q_1) \mid q_1 \in Q_1, f^{-1}(q_1) = q\})$$
$$= \wedge\{\nu_A(q,x,p) \vee \lambda_{\nu}(q_1) \mid q_1 \in Q_1, f^{-1}(q_1) = q\}$$

Now let $p, q \in Q$ be such that $f^{-1}(p_1) = p, f^{-1}(q_1) = q$. Then

$$\nu_A(q,x,p) \vee \lambda_{\nu}(q) = \nu_A(f^{-1}(q_1), x, f^{-1}(p_1)) \vee \lambda_{\nu}(q)$$
$$= (\wedge\{\nu_{A_1}(q_1,x,a_1) \mid a_1 \in Q_1, f^{-1}(a_1) = f^{-1}(p_1)\}) \vee \lambda_{\nu}(q)$$
$$= \wedge\{\wedge\{\nu_{A_1}(q_1,x,a_1) \vee \lambda_{\nu}(q) \mid a_1 \in Q_1, f^{-1}(a_1) = f^{-1}(p_1)\}$$
$$\geq \wedge\{\wedge\{\lambda_{\nu}(a_1) \mid a_1 \in Q_1, f^{-1}(a_1) = f^{-1}(p_1)\}$$
$$\geq f^{-1}(\lambda)_{\nu}(p)$$

Hence

$$f^{-1}(\lambda)_{\nu}(p) \leq \wedge\{\nu_A(q,x,p) \vee \lambda_{\nu}(q_1) \mid q_1 \in Q_1, f^{-1}(q_1) = q\}$$
$$= \nu_A(q,x,p) \vee f^{-1}(\lambda)_{\nu}(q)$$

Thus $f^{-1}(\lambda)$ is an intuitionistic fuzzy subsemiautomaton of $S$. □

**Theorem 7.** *Let $T = (Q_1, \Sigma, A_1)$ be an IFSA over a finite group and let $f$ be a strong homomorphism from $S$ onto $T$. If $\lambda$ is an intuitionistic fuzzy subsemiautomaton of $T$, then $f(\lambda_{\mu}, \lambda_{\nu})$ is an intuitionistic fuzzy subsemiautomaton of $S$.*

*Proof.* Let $\lambda$ be an intuitionistic fuzzy subsemiautomaton of $S$. Then $\lambda$ is an intuitionistic fuzzy subgroup of $Q$. Since $f$ is an epimorphism from $Q$ onto $Q_1$, $f(\lambda_{\mu}, \lambda_{\nu})$ is an intuitionistic fuzzy subgroup of $Q_1$. Let $p_1, q_1 \in Q_1, x \in \Sigma$. Then

$$\mu_{A_1}(q_1,x,p_1) \wedge f(\lambda)_{\mu}(q_1) = \mu_{A_1}(q_1,x,p_1) \wedge (\vee\{\lambda_{\mu}(q) \mid q \in Q, f(q) = q_1\})$$
$$= \vee\{\mu_{A_1}(q_1,x,p_1) \wedge \lambda_{\mu}(q) \mid q \in Q, f(q) = q_1\}$$

Now let $p, q \in Q$ be such that $f(p) = p_1, f(q) = q_1$. Then

$$\mu_{A_1}(q_1,x,p_1) \wedge \lambda_{\mu}(q) = \mu_{A_1}(f(q), x, f(p)) \wedge \lambda_{\mu}(q)$$
$$= (\vee\{\mu_A(q,x,a) \mid a \in Q, f(a) = f(p)\}) \wedge \lambda_{\mu}(q)$$
$$= \{\vee\{\mu_A(q,x,a) \wedge \lambda_{\mu}(q) \mid a \in Q, f(a) = f(p)\}$$
$$\leq \vee\{\vee\{\lambda_{\mu}(a) \mid a \in Q, f(a) = f(p)\}$$
$$\leq f(\lambda)_{\mu}(p)$$

Hence

$$f(\lambda)_\mu(p) \geq \vee\{\mu_{A_1}(q_1, x, p_1) \wedge \lambda_\mu(q) \mid q \in Q, f(q) = q_1\}$$
$$= \mu_{A_1}(q_1, x, p_1) \wedge f(\lambda)_\mu(q_1)$$

and

$$\nu_{A_1}(q_1, x, p_1) \vee f(\lambda)_\nu(q_1) = \nu_{A_1}(q_1, x, p_1) \vee (\wedge\{\lambda_\nu(q) \mid q \in Q, f(q) = q_1\})$$
$$= \wedge\{\nu_{A_1}(q_1, x, p_1) \vee \lambda_\nu(q) \mid q \in Q, f(q) = q_1\}$$

Now let $p, q \in Q$ be such that $f(p) = p_1, f(q) = q_1$. Then

$$\begin{aligned}\nu_{A_1}(q_1, x, p_1) \vee \lambda_\nu(q) &= \nu_{A_1}(f(q), x, f(p)) \vee \lambda_\nu(q)\\ &= (\wedge\{\nu_A(q, x, a) \mid a \in Q, f(a) = f(p)\} \vee \lambda_\nu(q)\\ &= \{\wedge\{\nu_A(q, x, a) \vee \lambda_\nu(q) \mid a \in Q, f(a) = f(p)\}\\ &\geq \wedge\{\wedge\{\lambda_\nu(a) \mid a \in Q, f(a) = f(p)\}\\ &\geq f(\lambda)_\nu(p)\end{aligned}$$

Hence

$$f(\lambda)_\nu(p) \leq \wedge\{\nu_{A_1}(q_1, x, p_1) \vee \lambda_\nu(q) \mid q \in Q, f(q) = q_1\}$$
$$= \nu_{A_1}(q_1, x, p_1) \vee f(q)(q_1)$$

Thus $f(\lambda)$ is an intuitionistic fuzzy subsemiautomaton of $T$. □

**Theorem 8.** *Let $\lambda$ be an intuitionistic fuzzy kernel of $S$. Then $\lambda$ is an intuitionistic fuzzy subsemiautomaton of $S$ if and only if*
$\lambda_\mu(p) \geq \mu_A(e, x, p) \wedge \lambda_\mu(e)$ *and* $\lambda_\nu(p) \leq \nu_A(e, x, p) \vee \lambda_\nu(e)$ $\forall p \in Q, x \in \Sigma$.

*Proof.* Suppose that the given condition is satisfied. Then for all
$p, q, r \in Q,\ x \in \Sigma$,

$$\begin{aligned}\lambda_\mu(p) &= \lambda_\mu(p * r * r^{-1})\\ &\geq \lambda_\mu(p * r^{-1}) \wedge \lambda_\mu(r)\\ &\geq \mu_A(q, x, p) \wedge \mu_A(e, x, r) \wedge \lambda_\mu(q) \wedge \lambda_\mu(r)\\ &\geq \mu_A(q, x, p) \wedge \mu_A(e, x, r) \wedge \lambda_\mu(e) \wedge \lambda_\mu(q)\\ &\qquad (by\ the\ given\ condition)\\ &= \mu_A(q, x, p) \wedge \lambda_\mu(q)\\ &\qquad (\because \lambda_\mu(e) \geq \lambda_\mu(q)\ and\ \mu_A(e, x, r) \geq \mu_A(e * q, x, p))\end{aligned}$$

and

$$\begin{aligned}\lambda_\nu(p) &= \lambda_\nu(p * r * r^{-1}) \\ &\leq \lambda_\nu(p * r^{-1}) \vee \lambda_\nu(r) \\ &\leq \nu_A(q,x,p) \vee \nu_A(e,x,r) \vee \lambda_\nu(q) \vee \lambda_\nu(r) \\ &\leq \nu_A(q,x,p) \vee \nu_A(e,x,r) \vee \lambda_\nu(e) \vee \lambda_\nu(q) \\ &\qquad (by\ the\ given\ condition) \\ &= \nu_A(q,x,p) \vee \lambda_\nu(q) \\ &\qquad (\because \lambda_\nu(e) \leq \lambda_\nu(q)\ and\ \nu_A(e,x,r) \leq \nu_A(e * q,x,p))\end{aligned}$$

Thus $\lambda$ is an intuitionistic fuzzy subsemiautomaton of $S$. The converse is immediate. □

**Theorem 9.** *Let $\lambda$ be an intuitionistic fuzzy kernel and $\sigma$ be an intuitionistic fuzzy subsemiautomaton of $S$. Then $\lambda \times \sigma = ((\lambda \times \sigma)_\mu, (\lambda \times \sigma)_\nu)$ is an intuitionistic fuzzy subsemiautomaton of $S$.*

*Proof.* Since $\lambda$ is an intuitionistic fuzzy normal subgroup and $\sigma$ is an intuitionistic fuzzy subgroup of $Q$, it follows that $\lambda \times \sigma$ is an intuitionistic fuzzy subgroup of $Q$ and $\lambda \times \sigma = \sigma \times \lambda$. Now

$$\begin{aligned}(\lambda \times \sigma)_\mu(p) &\geq \lambda_\mu(p * r^{-1}) \wedge \sigma_\mu(r) \\ &\geq (\mu_A(a * b,x,p) \wedge \mu_A(a,x,r) \wedge \lambda_\mu(b)) \wedge (\mu_A(a,x,r) \wedge \sigma_\mu(a)) \\ &= \mu_A(a * b,x,p) \wedge \lambda_\mu(b) \wedge \sigma_\mu(a) \\ &\qquad (since\ \mu_A(a * b,x,p) \leq \mu_A(a,x,r)) \forall a,b,p \in Q, x \in \Sigma.\end{aligned}$$

and

$$\begin{aligned}(\lambda \times \sigma)_\nu(p) &\leq \lambda_\nu(p * r^{-1}) \vee \sigma_\nu(r) \\ &\leq (\nu_A(a * b,x,p) \vee \nu_A(a,x,r) \vee \lambda_\nu(b)) \vee (\nu_A(a,x,r) \vee \sigma_\nu(a)) \\ &= \nu_A(a * b,x,p) \vee \lambda_\nu(b) \vee \sigma_\nu(a) \\ &\qquad (since\ \nu_A(a * b,x,p) \geq \nu_A(a,x,r)) \forall a,b,p \in Q, x \in \Sigma.\end{aligned}$$

Thus for all $p, q \in Q, x \in \Sigma$.

$$\begin{aligned}(\lambda \times \sigma)_\mu(p) &\geq \vee\{\mu_A(a * b,x,p) \wedge \lambda_\mu(b) \wedge \sigma_\mu(a) \mid a,b \in Q, a * b = q\} \\ &= \mu_A(q,x,p) \wedge (\vee\{\lambda_\mu(b) \wedge \sigma_\mu(a) \mid a,b \in Q, a * b = q\} \\ &= \mu_A(q,x,p) \wedge (\sigma \times \lambda)_\mu(q) \\ &= \mu_A(q,x,p) \wedge (\lambda \times \sigma)_\mu(q)\end{aligned}$$

and

$$\begin{aligned}(\lambda \times \sigma)_\nu(p) &\leq \wedge\{\nu_A(a * b,x,p) \vee \lambda_\nu(b) \vee \sigma_\nu(a) \mid a,b \in Q, a * b = q\} \\ &= \nu_A(q,x,p) \vee (\wedge\{\lambda_\nu(b) \vee \sigma_\nu(a) \mid a,b \in Q, a * b = q\} \\ &= \nu_A(q,x,p) \vee (\sigma \times \lambda)_\nu(q) \\ &= \nu_A(q,x,p) \vee (\lambda \times \sigma)_\nu(q)\end{aligned}$$

Hence $\lambda \times \sigma$ is an intuitionistic fuzzy subsemiautomaton of $S$. □

# 4 Conclusion

In this paper, the authors have made an attempt to study some algebraic properties such as an intuitionistic fuzzy kernel and intuitionistic fuzzy subsemiautomaton. We have made a humble beginning in this direction, however, many concepts are yet to be intuitionistic fuzzified.

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