17th Int. Conf. on IFSs, Sofia, 1–2 November 2013
Notes on Intuitionistic Fuzzy Sets
Vol. 19, 2013, No. 3, 56–61

The extension of modal operators' diagram with last operators

Gökhan Çuvalcioğlu

Department of Mathematics, University of Mersin
Mersin, Turkey
e-mail: gcuvalcioglu@mersin.edu.tr

Abstract: Intuitionistic Fuzzy Modal Operator was defined by Atanassov, he and several authors defined some modal operators. These operators classified by naming with one type and two type modal operators on intuitionistic fuzzy sets. Indeed, Atanassov's two operators $X_{a,b,c,d}$, $\circledcirc_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}$ are both one and two type modal operators. In this paper, we defined four operators which are both one and two type modal operators as Atanassov's above operators and called them uni-type modal operators.

Keywords: Diagram of modal operators, Intuitionistic fuzzy operators, Uni-type modal operators.
AMS Classification: 03E72, 47S40.

1 Introduction

The concept of fuzzy sets were defined by Zadeh in [10] as an extension of crisp sets, the usual two-valued sets in ordinary set theory, by expanding the truth value set to the real unit interval $[0, 1]$. In fuzzy set theory, if the membership degree of an element x is $\mu(x)$ then the non-membership degree is $1 - \mu(x)$ and thus is fixed.

Intuitionistic fuzzy sets were introduced by Atanassov in [1] and were formed an extension of fuzzy sets.

In this paper we will use I instead of $[0, 1]$.

Definition 1. *[1] An intuitionistic fuzzy set (IFS) on a set X is an object of the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where $\mu_A(x)$, $(\mu_A : X \to I)$ is called the "degree of membership of x in A", $\nu_A(x)$, $(\nu_A : X \to I)$ is called the "degree of non-membership of x in A", and where μ_A and ν_A satisfy the following condition:

$$\mu_A(x) + \nu_A(x) \leq 1, \textit{ for all } x \in X.$$

Although the sum of the degrees of membership and not being a member of an element in FS theory is 1. But, in IFS theory, this sum is less than 1. Besides this, if $A \in IFS(X)$ then μ, $\nu \in FS(X)$ and $1-\mu \leq \nu$ and $1-\nu \leq \mu$.

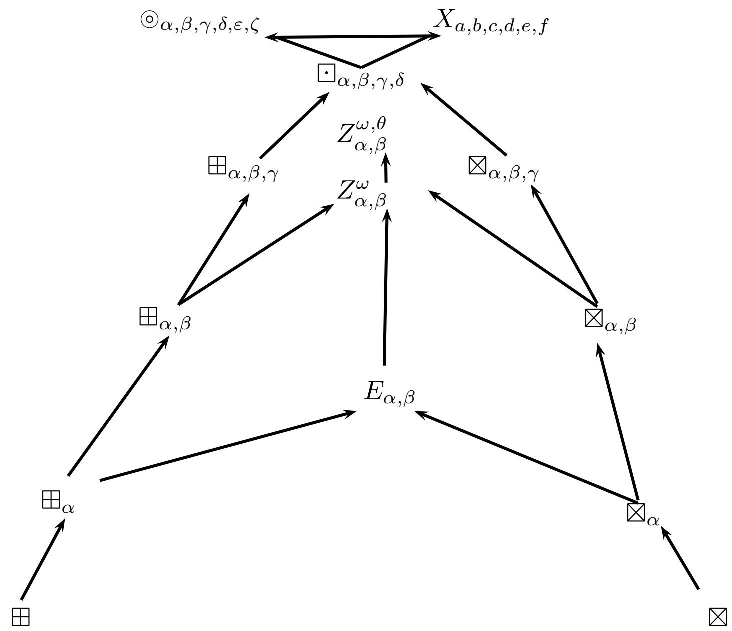
An IFS A is said to be contained in an IFS B (notation $A \sqsubseteq B$) if and only if for all $x \in X$: $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$.It is clear that $A = B$ if and only if $A \sqsubseteq B$ and $B \sqsubseteq A$.

One type intuitionistic fuzzy modal operators were defined by several authors;

Definition 2. *Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \in IFS(X), \alpha, \beta, \gamma, \omega, \theta \in [0,1]$.*

1. $\boxplus A = \{< x, \frac{\mu_A(x)}{2}, \frac{\nu_A(x)+1}{2} >: x \in X\}$, [2]
2. $\boxtimes A = \{< x, \frac{\mu_A(x)+1}{2}, \frac{\nu_A(x)}{2} >: x \in X\}$, [2]
3. $\boxplus_\alpha A = \{< x, \alpha\mu_A(x), \alpha\nu_A(x) + 1 - \alpha >: x \in X\}$, [3]
4. $\boxtimes_\alpha A = \{< x, \alpha\mu_A(x) + 1 - \alpha, \alpha\nu_A(x) >: x \in X\}$, [3]
5. $\boxplus_{\alpha,\beta} A = \{< x, \alpha\mu_A(x), \alpha\nu_A(x) + \beta >: x \in X\}$where $\alpha + \beta \in [0,1]$, [9]
6. $\boxtimes_{\alpha,\beta} A = \{< x, \alpha\mu_A(x) + \beta, \alpha\nu_A(x) >: x \in X\}$ where $\alpha + \beta \in [0,1]$, [9]
7. $\boxplus_{\alpha,\beta,\gamma}(A) = \{< x, \alpha\mu_A(x), \beta\nu_A(x) + \gamma >: x \in X\}$where $maks\{\alpha, \beta\} + \gamma \leqslant 1$, [4]
8. $\boxtimes_{\alpha,\beta,\gamma}(A) = \{< x, \alpha\mu_A(x) + \gamma, \beta\nu_A(x) >: x \in X\}$where $maks\{\alpha, \beta\} + \gamma \leqslant 1$, [4]
9. $E_{\alpha,\beta}(A) = \{< x, \beta(\alpha\mu_A(x) + 1 - \alpha), \alpha(\beta\nu_A(x) + 1 - \beta) >: x \in X\}$, [7]
10. $Z^{\omega}_{\alpha,\beta}(A) = \{< x, \beta(\alpha\mu_A(x) + \omega - \omega.\alpha), \alpha(\beta\nu_A(x) + \omega - \omega.\beta) >: x \in X\}$, [8]
11. $Z^{\omega,\theta}_{\alpha,\beta}(A) = \{< x, \beta(\alpha\mu_A(x) + \omega - \omega.\alpha), \alpha(\beta\nu_A(x) + \theta - \theta.\beta) >: x \in X\}$, [8]
12. $X_{\alpha,\beta,\gamma,\delta}(A) = \{< x, \alpha\mu_A(x) + \gamma, \beta\nu_A(x) + \delta >: x \in X\}$ such that $\max(\alpha, \beta) + \gamma + \delta \leqslant 1$, [5]
13. $\circledcirc_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}(A) = \{< x, \alpha\mu_A(x) - \varepsilon\nu_A(x) + \gamma, \beta\nu_A(x) - \zeta\mu_A(x) + \delta >: x \in X\}$ such that $\max(\alpha - \zeta, \beta - \varepsilon) + \gamma + \delta \leqslant 1$ and $\min(\alpha - \zeta, \beta - \varepsilon) + \gamma + \delta \geq 0$, [6]

Diagram formed by these operators are as follows;

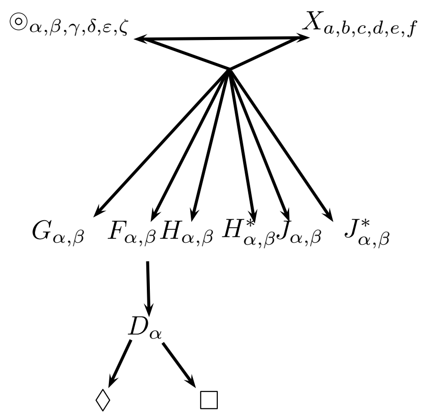


Second type intuitionistic fuzzy modal operators were defined by Atanassov, [1]:

Definition 3. *Let X be universal and $A \in IFS(X)$, $\alpha, \beta \in [0,1]$ then*

1. $D_\alpha(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + (1-\alpha)\pi_A(x) >: x \in X\}$
2. $F_{\alpha,\beta}(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + \beta\pi_A(x) >: x \in X\}$ where $\alpha + \beta \leq 1$
3. $G_{\alpha,\beta}(A) = \{< x, \alpha\mu_A(x), \beta\nu_A(x) >: x \in X\}$
4. $H_{\alpha,\beta}(A) = \{< x, \alpha\mu_A(x), \nu_A(x) + \beta\pi_A(x) >: x \in X\}$
5. $H^*_{\alpha,\beta}(A) = \{< x, \alpha\mu_A(x), \nu_A(x) + \beta(1 - \alpha\mu_A(x) - \nu_A(x)) >: x \in X\}$
6. $J_{\alpha,\beta}(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \beta\nu_A(x) >: x \in X\}$
7. $J^*_{\alpha,\beta}(A) = \{< x, \mu_A(x) + \alpha(1 - \mu_A(x) - \beta\nu_A(x)), \beta\nu_A(x) >: x \in X\}$
8. $\Box(A) = \{< x, \mu_A(x), 1 - \mu_A(x) >: x \in X\}$
9. $\Diamond(A) = \{< x, 1 - \nu_A(x), \nu_A(x) >: x \in X\}$

In the same paper, the author introduces the diagram of these operators,



2 Uni-type modal operators

In this section, we will define new operators which are thinking both one type and two type modal operators on intuitionistic fuzzy sets because of their diagram properties.

Definition 4. *Let X be a universal, $A \in IFS(X)$ and $\alpha, \beta, \omega \in [0,1]$. We define the following operator:*

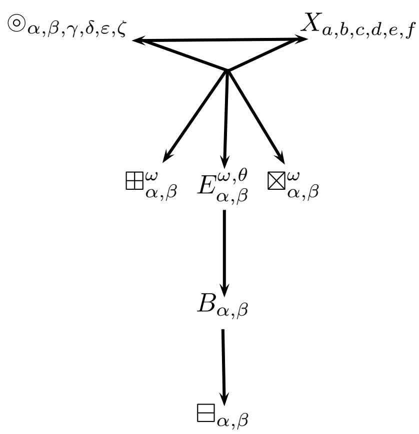
1. $\boxplus^{\omega}_{\alpha,\beta}(A) = \{\langle x, \beta(\mu_A(x) + (1-\alpha)\nu_A(x)), \alpha(\beta\nu_A(x) + \omega - \omega\beta)\rangle : x \in X\}$
2. $\boxtimes^{\omega}_{\alpha,\beta}(A) = \{\langle x, \beta(\alpha\mu_A(x) + \omega - \omega\alpha), \alpha((1-\beta)\mu_A(x) + \nu_A(x))\rangle : x \in X\}$
3. $B_{\alpha,\beta}(A) = \{\langle x, \beta(\mu_A(x) + (1-\alpha)\nu_A(x)), \alpha((1-\beta)\mu_A(x) + \nu_A(x))\rangle : x \in X\}$
4. $\boxminus_{\alpha,\beta}(A) = \{\langle x, \beta(\mu_A(x) + (1-\beta)\nu_A(x)), \alpha((1-\alpha)\mu_A(x) + \nu_A(x))\rangle : x \in X\}$
5. $E^{\omega,\theta}_{\alpha,\beta}(A) = \{\langle x, \beta((1-(1-\alpha)(1-\theta))\mu_A(x) + (1-\alpha)\theta\nu_A(x) + (1-\alpha)(1-\theta)\omega),$
   $\alpha((1-\beta)\theta\mu_A(x) + (1-(1-\beta)(1-\theta))\nu_A(x) + (1-\beta)(1-\theta)\omega)\rangle : x \in X\}$

It is clear that: $\boxplus^{\omega}_{\alpha,\beta}(A), \boxtimes^{\omega}_{\alpha,\beta}(A), B_{\alpha,\beta}(A), \boxminus_{\alpha,\beta}(A) \in IFS(X)$.
Some fundamental properties of these operators are following;

Theorem 1. *Let X be a set, $A \in IFS(X)$ and $\alpha, \beta \in [0,1]$.*

$$B_{\alpha,\alpha}(A) = \boxminus_{\alpha,\alpha}(A)$$

Proof. It is clear from definition. □



Proposition 1. *Let X be a universal, $A \in IFS(X)$ and $\alpha, \beta \in [0,1)$ then*

1. $\boxplus^{\frac{\beta}{1-\alpha}}_{1,\alpha}(A) = \boxplus_{\alpha,\beta}(A)$
2. $\boxtimes^{\frac{\beta}{1-\alpha}}_{\alpha,1}(A) = \boxtimes_{\alpha,\beta}(A)$

Proof. It is clear from definition. □

Proposition 2. *Let X be a set and $A \in IFS(X)$, $\alpha, \beta, \omega \in [0,1]$ then*

1. $E^{\omega,0}_{\alpha,\beta}(A) = Z^{\omega}_{\alpha,\beta}(A)$

2. $E^{\omega,1}_{\alpha,\beta}(A) = B_{\alpha,\beta}(A)$

3. $E^{0,0}_{\alpha,\beta}(A) = G_{\alpha\beta,\alpha\beta}(A)$

4. $E^{1,0}_{\alpha,\beta}(A) = E_{\alpha,\beta}(A)$

5. $E^{0,0}_{1,0}(A) = \emptyset$

6. $E^{0,0}_{0,1}(A) = X$

Proposition 3. *Let X be a set and $A \in IFS(X)$, $\alpha, \beta, \omega \in [0,1]$ then*

1. $E^{1,0}_{\alpha,1}(A) = \boxtimes_{\alpha}(A)$

2. $E^{\omega,0}_{\alpha,1}(A) = \boxtimes_{\alpha,\omega(1-\alpha)}(A)$

3. $E^{\omega,0}_{1,\beta}(A) = \boxplus_{\beta,\omega(1-\beta)}(A)$

4. $E^{\omega,\theta}_{1,1}(A) = A$

5. $E^{1,0}_{1,\beta}(A) = \boxplus_{\beta}(A)$

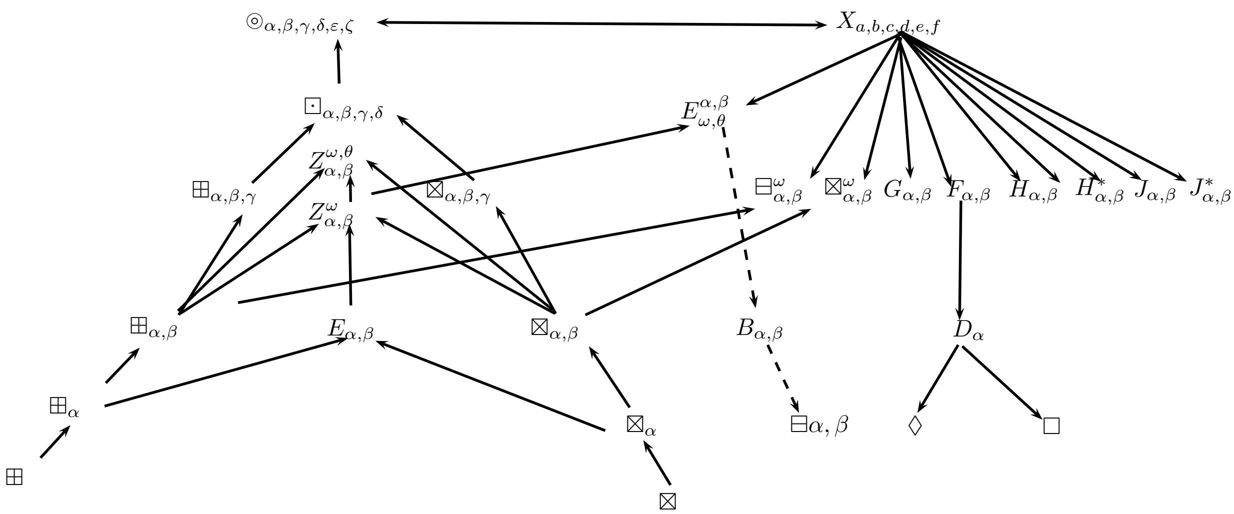
6. $E^{\omega,1}_{\alpha,1}(A) = B_{\alpha,1}(A)$

7. $E^{1,1}_{1,\beta}(A) = B_{1,\beta}(A)$

From the above properties, it is easily shown that the operators $\boxplus^{\omega}_{\alpha,\beta}, \boxtimes^{\omega}_{\alpha,\beta}$, $E^{\omega,\theta}_{\alpha,\beta}$, $B_{\alpha,\beta}$ and $\boxminus_{\alpha,\beta}$ belong to the diagrams of both one type and two type operators. At that time, Atanassov's operators $X_{a,b,c,d}$, $\circledcirc_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}$ have same properties. Therefore, we have to differently naming from the others. So, we can give the following definition:

Definition 5. *Let X be a set and Υ be a modal operator of Intuitionistic Fuzzy Set on X. If Υ is both one type and two type modal operator then it is called uni-type modal operator of Intuitionistic Fuzzy Set on X.*

From that fundemental properties we get the last diagram of all (one/two/uni-type) modal operators on Intuitionistic Fuzzy Sets as following;



References

[1] Atanassov, K. T., Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*, Vol. 20, 1986, 87–96.

[2] Atanassov, K. T., *Intuitionistic Fuzzy Sets*, Phiysica-Verlag, Heidelberg, 1999.

[3] Atanassov, K. T., Remark on Two Operations Over Intuitionistic Fuzzy Sets, *Int. J. of Unceratanity, Fuzziness and Knowledge Syst.*, Vol. 9, 2001, No. 1, 71–75.

[4] Atanassov, K. T., The most general form of one type of intuitionistic fuzzy modal operators, *Notes on Intuitionistic Fuzzy Sets*, Vol. 12, 2006, No. 2, 36–38.

[5] Atanassov, K. T., Some properties of the operators from one type of intuitionistic fuzzy modal operators, *Advanced Studies on Contemporary Mathematics*, Vol. 15, 2007, No. 1, 13–20.

[6] Atanassov, K. T., The most general form of one type of intuitionistic fuzzy modal operators, Part 2, *Notes on Intuitionistic Fuzzy Sets*, Vol. 14, 2008, No. 1, 27–32.

[7] Çuvalcıoğlu, G., Some properties of $E_{\alpha,\beta}$ operator, *Advanced Studies on Contemporary Mathematics*, Vol. 14, 2007, No. 2, 305–310.

[8] Çuvalcıoğlu, G., On the diagram of One Type Modal Operators on Intuitionistic Fuzzy Sets: Last Expanding with $Z^{\omega,\theta}_{\alpha,\beta}$, *Iranian Journal of Fuzzy Systems*, Vol. 10, 2013, No. 1, 89–106.

[9] Dencheva, K., Extension of intuitionistic fuzzy modal operators $\boxplus$ and $\boxtimes$, *Proc. of the Second Int. IEEE Symp. Intelligent Systems*, Varna, 22-24 June 2004, Vol. 3, 21–22.

[10] Zadeh, L. A., Fuzzy Sets, *Information and Control*, Vol. 8, 1965, 338–353.