

Remark on intuitionistic fuzzy numbers

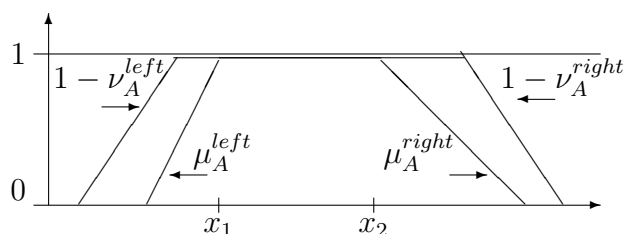
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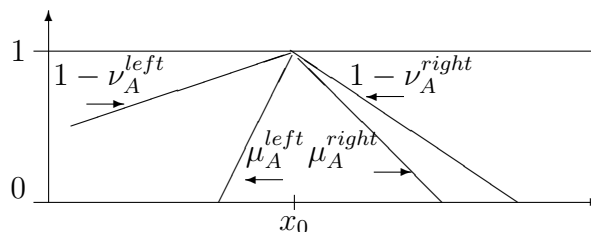
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1. During the last almost 20 years some definitions of the concept of “Intuitionistic Fuzzy Number” (IFN) were introduced (see, e.g., [2, 3, 4, 5, 6, 7, 8, 9, 10, 11]). Here some new definitions of the concept of an IFN will be introduced.

In general, the geometrical forms of the IFNs in the above mentioned papers are the following



and



For both cases we require the functions μ_A and ν_A to satisfy the following conditions:

$$\max_{y \in E} \mu_A(y) = \mu_A(x) = 1,$$

$$\min_{y \in E} \nu_A(y) = \nu_A(x) = 0,$$

for each $x \in [x_1, x_2]$ and

μ_A is increasing function from $-\infty$ to x_1 ;

μ_A is decreasing function from x_2 to $+\infty$;

ν_A is decreasing function from $-\infty$ to x_1 ;

ν_A is increasing function from x_2 to $+\infty$

for the first case, or

$$\max_{y \in E} \mu_A(y) = \mu_A(x_0) = 1,$$

$$\min_{y \in E} \nu_A(y) = \nu_A(x_0) = 0;$$

and

- μ_A is increasing function from $-\infty$ to x_0 ;
- μ_A is decreasing function from x_0 to $+\infty$;
- ν_A is decreasing function from $-\infty$ to x_0 ;
- ν_A is increasing function from x_0 to $+\infty$

for the second case.

Obviously, in both cases the functions μ_A and ν_A can be represented in the form

$$\mu_A = \mu_A^{left} \cup \mu_A^{right},$$

$$\nu_A = \nu_A^{left} \cup \nu_A^{right},$$

where μ_A^{left} and ν_A^{left} are the left, while μ_A^{right} and ν_A^{right} are the right sides of these functions. Therefore, the above conditions can be re-written in the (joint) form:

$$\max_{y \in E} \mu_A(y) = \mu_A(x) = 1,$$

$$\min_{y \in E} \nu_A(y) = \nu_A(x) = 0,$$

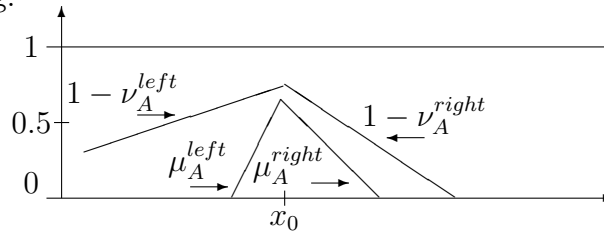
for each $x \in [x_1, x_2]$ and in the particular case, when $x_1 = x_0 = x_2$,

- μ_A^{left} is increasing function;
- μ_A^{right} is decreasing function;
- ν_A^{left} is decreasing function;
- ν_A^{right} is increasing function.

2. Now, we shall generalize this definition, changing the condition for maximal values of μ_A and minimal values for ν_A to the form:

$$\max_{y \in E} \mu_A(y) = \mu_A(x_0) > 0.5 > \nu_A(x_0) = \min_{y \in E} \nu_A(y).$$

Obviously, the first definition is included in the second one. Now, the geometrical interpretation - for simplicity, below we shall work only with the second case discussed above - is the following:



3. The second generalization of the above definitions, again changes the condition for maximal values of μ_A and minimal values for ν_A , but now, to the form:

$$\max_{y \in E} \mu_A(y) = \mu_A(x_0) > \nu_A(x_0) = \min_{y \in E} \nu_A(y).$$

Obviously, the first two definitions are included in the third one.

4. Finally, in the third generalization of the above three definitions we omit the condition for maximal values of μ_A and minimal values for ν_A , keeping only the conditions for increasing and decreasing of the two functions.

Obviously, the first three definitions are included in the fourth one.

The present short communication is only a first step, showing some new possibilities for introducing of the concept of an IFN. In future the properties of the new types of IFNs must be studied and its real applications will be searched.

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