3rd Int. IFS Conf., 29 Aug – 1 Sep 2016, Mersin, Turkey
Notes on Intuitionistic Fuzzy Sets
Print ISSN 1310–4926, Online ISSN 2367–8283
Vol. 22, 2016, No. 4, 1–5

Some intuitionistic fuzzy operators

Mehmet Çitil and Feride Tuğrul

Department of Mathematics, University Of Sutcu Imam
Kahramanmaras, Turkey
e-mails: citil@ksu.edu.tr, feridetugrul@gmail.com

Received: 12 June 2016 **Accepted:** 30 October 2016

Abstract: In this study, uniformly expanding intuitionistic fuzzy operator U with modal operators $\Box$ and $\Diamond$ were investigated. New equalities were obtained and proved with U topological operator [4] by using equalities that were obtained by K.Atanassov in [2, 3] and by us in [5].
Keywords: Intuitionistic fuzzy sets, Modal operators, Operations, Uniformly expanding intuitionistic fuzzy operator.
AMS Classification: 03E72.

1 Introduction

In 1965, Fuzzy Set Theory was defined by L. Zadeh [1]. Then K. T. Atanassov generalized fuzzy sets into Intuitionistic Fuzzy Set in 1986 [2]. Intuitionistic fuzzy set is one of the extensions of fuzzy sets. Intuitionistic Fuzzy Theory are widely used in algebraic structures,robotics, agriculture,control systems, computer, economy and many engineering fields. Furthermore the notion of Intuitionistic Fuzzy Operators (IFO) was introduced first by K. T. Atanassov [2]. Several operators are defined in the Intuitionistic Fuzzy Sets Theory. They are classified in three groups: modal, topological and level operators. The notion of modal operator $\Box$, $\Diamond$ introduced on intuitionistic fuzzy sets were defined by K.Atanassov in 1986 [2]. Modal operators $\Box$, $\Diamond$ defined over the set of all IFS's transform every IFS into a FS. They are similar to the operators 'necessity' and 'possibility' defined in some modal logics. Then U intuitionistic fuzzy topological operator was defined by Krassimir T.Atanassov in 2016 [4]. This operator is called "Uniformly expanding intuitionistic fuzzy operator".

Definition 1. *[1]. Let X be a nonempty set. A fuzzy set A drawn from X is defined as*

$A = \{\langle x, \mu_A(x)\rangle | \ x \in X\}$, *where* $\mu_A(x) : X \to [0,1]$ *is the membership function of the fuzzy set* A.

Definition 2. *[2]. Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | \ x \in X\} ,$$

where the functions

$$\mu_A(x), \nu_A(x) : X \to [0,1]$$

define respectively, the degree of membership and degree of nonmembership of the element $x \in X$, to the set A, which is a subset of X, and for every element $x \in X$,

$$0 \le \mu_A(x) + \nu_A(x) \le 1.$$

Definition 3. *[2, 3]. Let A and $B \in X$ be IFS. For every two IFS's A and B the following operations and relations are defined.*

$$\begin{aligned}
A @ B &= \{\langle x, \tfrac{\mu_A(x)+\mu_B(x)}{2}, \tfrac{\nu_A(x)+\nu_B(x)}{2}\rangle \ | \ x \in X\} \\
A \to B &= \{\langle x, \max(\nu_A(x), \mu_B(x)), \min(\mu_A(x), \nu_B(x))\rangle | \ x \in X\} \\
A \cap B &= \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle | \ x \in X\} \\
A \cup B &= \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle | \ x \in X\} \\
A \oplus B &= \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)\rangle | \ x \in X\} \\
A \otimes B &= \{\langle x, \mu_A(x).\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)\rangle | \ x \in X\} \\
A \ \$ \ B &= \{\langle x, \sqrt{\mu_A(x).\mu_B(x)}, \sqrt{\nu_A(x).\nu_B(x)}\rangle | \ x \in X\} \\
A \# B &= \{\langle x, \frac{2\mu_A(x).\mu_B(x)}{\mu_A(x) + \mu_B(x)}, \frac{2\nu_A(x).\nu_B(x)}{\nu_A(x) + \nu_B(x)}\rangle | \ x \in X\} \\
A * B &= \{\langle x, \frac{\mu_A(x) + \mu_B(x)}{2(\mu_A(x)\mu_B(x) + 1)}, \frac{\nu_A(x) + \nu_B(x)}{2(\nu_A(x)\nu_B(x) + 1)}\rangle | \ x \in X\}
\end{aligned}$$

Definition 4. *[4]. Let us define the operator U over the IFS A by*

$$\begin{aligned}
\sup_y \mu_A(y) &> \inf_y \mu_A(y) \\
\sup_y \nu_A(y) &> \inf_y \nu_A(y)
\end{aligned}$$

$$U(A) = \{\langle \frac{\mu_A(x) - \inf\limits_y \mu_A(y)}{\sup\limits_y \mu_A(y) - \inf\limits_y \mu_A(y)}, \frac{\nu_A(x) - \inf\limits_y \nu_A(y)}{\sup\limits_y \nu_A(y) - \inf\limits_y \nu_A(y)}\rangle | x \in X\}$$

Definition 5. *[2]. Let X be a nonempty set. If A and B are an IFS drawn from X, then;*

$$\begin{aligned} \Box A &= \{\langle x, \mu_A(x), 1-\mu_A(x)\rangle |\ x \in X\} \\ \Diamond A &= \{\langle x, 1-\nu_A(x), \nu_A(x)\rangle |\ x \in X\}\ [1] \end{aligned}$$

Theorem 1. *[4]. Let X be a nonempty set. For every IFS A in X:*

(a) $U(\Box A) = \Box U(A)$

(b) $U(\Diamond A) = \Diamond U(A)$

2 Main result

In this section, new equalities were obtained and proved with U topological operator [4] by using equalities that were obtained by K. Atanassov in [2, 3] and by us in [5].

Theorem 2. *[2, 3]. Let X be a nonempty set. For every IFS A in X;*

(a) $\Box\Box A = \Box A$

(b) $\Box\Diamond A = \Diamond A$

(c) $\Diamond\Box A = \Box A$

(d) $\Diamond\Diamond A = \Diamond A$

Theorem 3. *Let X be a nonempty set. For every IFS A in X;*

(a) $U(\Box\Box A) = \Box U(A)$

(b) $U(\Diamond\Diamond A) = \Diamond U(A)$

(c) $U(\Box\Diamond A) = \Diamond U(A)$

(d) $U(\Diamond\Box A) = \Box U(A)$

Proof. The proof is obtained thanks to Theorem 2. □

Theorem 4. *[2, 3]. Let X be a nonempty set. For every IFS A and B in X;*

(a) $(\Box A \,@\, \Box B) = \Box(A@B)$

(b) $(\Diamond A \,@\, \Diamond B) = \Diamond(A@B)$

Theorem 5. *Let X be a nonempty set. For every IFS A and B in X;*

(a) $U(\Box A \,@\, \Box B) = \Box U(A@B)$

(b) $U(\Diamond A \,@\, \Diamond B) = \Diamond U(A@B)$

(c) $U((\Box A)^c) = (\Box U(A))^c$

(d) $U((\Diamond A)^c) = (\Diamond U(A))^c$

Proof. (a)

$$\begin{aligned} \Box A \,@\, \Box B &= \{\langle x, \tfrac{\mu_A(x)+\mu_B(x)}{2}, \tfrac{1-\mu_A(x)+1-\mu_B(x)}{2}\rangle |\ x \in X\} \\ \Box A \,@\, \Box B &= \{\langle x, \tfrac{\mu_A(x)+\mu_B(x)}{2}, 1-\tfrac{\mu_A(x)+\mu_B(x)}{2}\rangle |\ x \in X\} \end{aligned}$$

$$
\begin{aligned}
U(\Box A \ @\ \Box B) &= \{\langle x, \frac{\frac{\mu_A(x)+\mu_B(x)}{2} - \inf(\frac{\mu_A(x)+\mu_B(x)}{2})}{\sup(\frac{\mu_A(x)+\mu_B(x)}{2}) - \inf(\frac{\mu_A(x)+\mu_B(x)}{2})}, \frac{1-\frac{\mu_A(x)+\mu_B(x)}{2} - \inf(1-\frac{\mu_A(x)+\mu_B(x)}{2})}{\sup(1-\frac{\mu_A(x)+\mu_B(x)}{2}) - \inf(1-\frac{\mu_A(x)+\mu_B(x)}{2})}\rangle\} \\
&= \{\langle x, \frac{\frac{\mu_A(x)+\mu_B(x)}{2} - \inf(\frac{\mu_A(x)+\mu_B(x)}{2})}{\sup(\frac{\mu_A(x)+\mu_B(x)}{2}) - \inf(\frac{\mu_A(x)+\mu_B(x)}{2})}, \frac{1-\frac{\mu_A(x)+\mu_B(x)}{2} - 1 + \sup(\frac{\mu_A(x)+\mu_B(x)}{2})}{1-\inf(\frac{\mu_A(x)+\mu_B(x)}{2}) - 1 + \sup(\frac{\mu_A(x)+\mu_B(x)}{2})}\rangle\} \\
&= \{\langle x, \frac{\frac{\mu_A(x)+\mu_B(x)}{2} - \inf(\frac{\mu_A(x)+\mu_B(x)}{2})}{\sup(\frac{\mu_A(x)+\mu_B(x)}{2}) - \inf(\frac{\mu_A(x)+\mu_B(x)}{2})}, 1 - \frac{\frac{\mu_A(x)+\mu_B(x)}{2} - \inf(\frac{\mu_A(x)+\mu_B(x)}{2})}{\sup(\frac{\mu_A(x)+\mu_B(x)}{2}) - \inf(\frac{\mu_A(x)+\mu_B(x)}{2})}\rangle\}
\end{aligned}
$$

$$U(\Box A \ @\ \Box B) = \Box U(A @ B)$$

$\Box$

Similarly, the other proofs are obvious.

Theorem 6. *[5]. Let X be a nonempty set. For every IFS A and B in X:*

(a) $\Box[(\Diamond A \ @\ \Diamond B)^c] = [\Diamond(A \ @\ B)]^c$

(b) $(\Diamond A \oplus \Diamond B) @ (\Diamond A \otimes \Diamond B) = \Diamond(A @ B)$

(c) $(\Box A \oplus \Box B) @ (\Box A \otimes \Box B) = \Box(A @ B)$

(d) $[(\Box A \oplus \Diamond B)^c \ @\ ((\Box A)^c \otimes \Diamond B)] \cup (\Box A)^c = (\Box(A))^c$

(e) $[(\Diamond A \oplus \Box B) \ @\ ((\Diamond A)^c \otimes \Box B)] \cup (\Diamond A) = \Diamond(A)$

(f) $[(\Diamond A \oplus \Box B)^c \ @\ ((\Diamond A)^c \otimes \Box B)] \cup (\Diamond A)^c = (\Diamond(A))^c$

(g) $[(\Box A \otimes \Diamond B)^c \ @\ ((\Box A)^c \oplus \Diamond B)] \cap (\Box A)^c = (\Box(A))^c$

Theorem 7. *Let X be a nonempty set. For every IFS A and B in X:*

(a) $U(\Box[(\Diamond A \ @\ \Diamond B)^c]) = [\Diamond U(A \ @\ B)]^c$

(b) $U((\Diamond A \oplus \Diamond B) @ (\Diamond A \otimes \Diamond B)) = \Diamond U(A @ B)$

(c) $U((\Box A \oplus \Box B) @ (\Box A \otimes \Box B)) = \Box U(A @ B)$

(d) $U([(\Box A \oplus \Diamond B)^c \ @\ ((\Box A)^c \otimes \Diamond B)] \cup (\Box A)^c) = (\Box U(A))^c$

(e) $U([(\Diamond A \oplus \Box B) \ @\ ((\Diamond A)^c \otimes \Box B)] \cup (\Diamond A)) = \Diamond U(A)$

(f) $U([(\Diamond A \oplus \Box B)^c \ @\ ((\Diamond A)^c \otimes \Box B)] \cup (\Diamond A)^c) = (\Diamond U(A))^c$

(g) $U([(\Box A \otimes \Diamond B)^c \ @\ ((\Box A)^c \oplus \Diamond B)] \cap (\Box A)^c) = (\Box U(A))^c$

Proof. (a)

$$\begin{aligned}
\Diamond A \,@\, \Diamond B &= \left\{\langle x, \tfrac{1-\nu_A(x)+1-\nu_B(x)}{2}, \tfrac{\nu_A(x)+\nu_B(x)}{2}\rangle \mid x \in X\right\} \\
\Diamond A \,@\, \Diamond B &= \left\{\langle x, 1-\tfrac{\nu_A(x)+\nu_B(x)}{2}, \tfrac{\nu_A(x)+\nu_B(x)}{2}\rangle \mid x \in X\right\} \\
(\Diamond A \,@\, \Diamond B)^c &= \left\{\langle x, \tfrac{\nu_A(x)+\nu_B(x)}{2}, 1-\tfrac{\nu_A(x)+\nu_B(x)}{2}\rangle \mid x \in X\right\} \\
\Box[(\Diamond A \,@\, \Diamond B)^c] &= \left\{\langle x, \tfrac{\nu_A(x)+\nu_B(x)}{2}, 1-\tfrac{\nu_A(x)+\nu_B(x)}{2}\rangle \mid x \in X\right\} \\
U(\Box[(\Diamond A \,@\, \Diamond B)^c]) &= \{\langle x, \frac{\frac{\nu_A(x)+\nu_B(x)}{2} - \inf(\frac{\nu_A(x)+\nu_B(x)}{2})}{\sup(\frac{\nu_A(x)+\nu_B(x)}{2}) - \inf(\frac{\nu_A(x)+\nu_B(x)}{2})}, \frac{1-\frac{\nu_A(x)+\nu_B(x)}{2} - \inf(1-\frac{\nu_A(x)+\nu_B(x)}{2})}{\sup(1-\frac{\nu_A(x)+\nu_B(x)}{2}) - \inf(1-\frac{\nu_A(x)+\nu_B(x)}{2})}\rangle\} \\
&= \{\langle x, \frac{\frac{\nu_A(x)+\nu_B(x)}{2} - \inf(\frac{\nu_A(x)+\nu_B(x)}{2})}{\sup(\frac{\nu_A(x)+\nu_B(x)}{2}) - \inf(\frac{\nu_A(x)+\nu_B(x)}{2})}, \frac{1-\frac{\nu_A(x)+\nu_B(x)}{2} -1 + \sup(\frac{\nu_A(x)+\nu_B(x)}{2})}{1-\inf(\frac{\nu_A(x)+\nu_B(x)}{2}) -1+ \sup(\frac{\nu_A(x)+\nu_B(x)}{2})}\rangle\} \\
&= \{\langle x, \frac{\frac{\nu_A(x)+\nu_B(x)}{2} - \inf(\frac{\nu_A(x)+\nu_B(x)}{2})}{\sup(\frac{\nu_A(x)+\nu_B(x)}{2}) - \inf(\frac{\nu_A(x)+\nu_B(x)}{2})}, 1 - \frac{\frac{\nu_A(x)+\nu_B(x)}{2} - \inf(\frac{\nu_A(x)+\nu_B(x)}{2})}{\sup(\frac{\nu_A(x)+\nu_B(x)}{2}) - \inf(\frac{\nu_A(x)+\nu_B(x)}{2})}\rangle\}
\end{aligned}$$

$$U(\Box[(\Diamond A \,@\, \Diamond B)^c]) = [\Diamond U(A \,@\, B)]^c$$

□

Similarly, the other proofs are obvious.

References

[1] Zadeh L. A. (1965) Fuzzy sets, *Information and Control*, 8, 338–353.

[2] Atanassov, K. (1986) Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20(1), 87–96.

[3] Atanassov, K. (1999) *Intuitionistic Fuzzy Sets*, Springer, Heidelberg.

[4] Atanassov, K. (2016). Uniformly expanding intuitionistic fuzzy operator, *Notes on Intuitionistic Fuzzy sets*, 22(1), 48–52.

[5] Eker, E., Tuğrul, F., & Çitil, M. (2015) New equalities on the intuitionistic fuzzy operators and operations, *Notes on Intuitionistic Fuzzy Sets*, 21(4), 124–128.