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SOME PROPERTIES OF INTUITIONISTIC FUZZY LEVEL SUBGROUPS

N.PALANIAPPAN & K.ARJUNAN

Professor of Mathematics,
Alagappa University,
Karaikudi – 630003,
Tamilnadu , India .
palaniappan1950@yahoo.co.in

Department of Mathematics,
Dr.Zakir Hussain College,
Ilayangudi – 630702,
Tamilnadu , India
arjunan_1975@yahoo.co.in

ABSTRACT.

In this paper, we made an attempt to study the algebraic nature of an intuitionistic fuzzy level subgroups .

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INTRODUCTION

After the introdution of fuzzy sets by L.A.Zadeh , several researchers explored on the generalization of the notion of fuzzy set .The concept of intuitionistic fuzzy sets was introduced by K.T.Atanassov [1], as a generalization of the notion of fuzzy set. Choudhury.F.P. and Chakraborty.A.B. and Khare.S.S. [2] defined a fuzzy subgroup and fuzzy homomorphism. Palaniappan.N & Muthuraj.R [3] defined the homomorphism and anti- homomorphism of fuzzy and an anti-fuzzy subgroups. Palaniappan.N & Muthuraj.R [4] defined an anti fuzzy group and lower level subgroups. Salah Abou-Zaid [5] defined on generalized characteristic fuzzy subgroups of a finite group . We introduce the concept of an intuitionistic fuzzy level subgroups and established some results.

1. PRELIMINARIES

1.1 Definition : Let X be a non–empty set. A fuzzy subset A of X is a function $A:X \rightarrow [0,1]$.

1.2 Definition : An intuitionistic fuzzy set (IFS) A in X is defined as an object of the form $A=\{ < x , \mu_A(x) , \nu_A(x) > / x\in X \}$, where $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x\in X$ respectively and for every $x\in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1 .3 Definition : Let G be a group. An intuitionistic fuzzy subset A of G is said to be an intuitionistic fuzzy subgroup of G (IFSG) if

(i) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$
(ii) $\mu_A(x^{-1}) \geq \mu_A(x)$
(iii) $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$
(iv) $\nu_A(x^{-1}) \leq \nu_A(x)$, for all $x ,y\in G$.

1.4 Definition : Let G be a group. An intuitionistic fuzzy subgroup A of G is said to be an intuitionistic fuzzy normal subgroup of G (IFNSG) if

(i) $\mu_A(xy) = \mu_A(yx)$.
(ii) $\nu_A(xy) =\nu_A(yx)$, for all $x ,y\in G$.

1.5 Definition : Let G and G^1 be any two groups, then the function f: G → G^1 is said to be a homomorphism if f(xy) = f(x)f(y) for all x ,y Є G.
1. 6 Definition : Let G and G^1 be any two groups, then the function f: G → G^1 is said to be an isomorphism if f(xy) = f(x)f(y) and f is a bijection , for all x ,y Є G.
1.7 Definition : Let G and G^1 be any two groups, then the function f: G → G^1 is said to be an anti-homomorphism if f(xy) = f(y)f(x) for all x ,y Є G.
1. 8 Definition : Let G and G^1 be any two groups, then the function f: G → G^1 is said to be an anti-isomorphism if f(xy) = f(y)f(x) and f is a bijection , for all x ,y Є G.
1. 9 Definition : Let A be an intuitionistic fuzzy subset of X. For t ∈ [0, 1], the level subset of A is the set, $A_t = \{x \in X : \mu_A(x) \geq t \text{ and } \nu_A(x) \leq t\}$.This is called an intuitionistic fuzzy level subset of A .
1.10 Definition : Let A be an IFSG of a group G. The subgroup A_t, for t ∈ [0, 1] and $t \leq \mu_A(e)$ and $t \geq \nu_A(e)$ are called level subgroups of A.
1.1 Theorem : Let G and G^1 be any two groups . Let f : G ⟶ G^1 be an isomorphism .Then

(i) $f(e) = e^1$ where e and e^1 are the identities of G and G^1 respectively .
(ii) $f(a^{-1}) = [f(a)]^{-1}$, for all a in G.

Proof : It is trivial.

1.2 Theorem . Let G and G^1 be any two groups. Let f : G → G^1 be an anti-isomorphism . Then

(i) $f(e) = e^1$ where e and e^1 are the identities of G and G^1 respectively .
(ii) $f(a^{-1}) = [f(a)]^{-1}$, for all a in G.

Proof : It is trivial.

2. SOME PROPOSITIONS

2.1 Proposition : Let A be an IFSG of a group G. Then for t ∈ [0, 1] such that $t \leq \mu_A(e)$ and $t \geq \mu_A(e)$, A_t is a subgroup of G.
Proof : For all x, y in A_t, we have

$$\mu_A(x) \geq t \text{ and } \nu_A(x) \leq t,$$
$$\mu_A(y) \geq t \text{ and } \nu_A(y) \leq t.$$

Now,

$$\mu_A(xy^{-1}) \geq \min\{\mu_A(x), \mu_A(y)\} \text{ as A is an IFSG of a group G}$$
$$\geq \min\{t, t\}$$
$$= t$$

Therefore $\mu_A(xy^{-1}) \geq t$.
Also, $\nu_A(xy^{-1}) \leq \max\{\nu_A(x), \nu_A(y)\}$ as A is an IFSG of a group G

$$\leq \max\{t, t\}$$
$$= t$$

which implies that $\nu_A(xy^{-1}) \leq t$.
That is $\mu_A(xy^{-1}) \geq t$ and $\nu_A(xy^{-1}) \leq t$.
Therefore $xy^{-1} \in A_t$.
Hence A_t is a subgroup of a group G.
2.2 Proposition : Let A be an IFSG of a group G. For $t_1, t_2 \in [0, 1]$ and $t_1, t_2 \leq \mu_A(e)$ and $t_1, t_2 \geq \nu_A(e)$ with $t_2 < t_1$ of A, the two level subgroups A_{t1}, A_{t2} are equal iff there is no x in G such that $t_1 > \mu_A(x) > t_2$ and $t_2 < \nu_A(x) < t_1$.
Proof : Assume that $A_{t1} = A_{t2}$.
Suppose that there exists a x ∈ G such that $t_1 > \mu_A(x) > t_2$ and $t_2 < \nu_A(x) < t_1$.
Then $A_{t1} \subseteq A_{t2}$.

For $x \in A_{t2}$, but not in A_{t1},

which is contradiction to $A_{t1} = A_{t2}$.

Therefore there is no $x \in G$ such that $t_1 > \mu_A(x) > t_2$ and $t_2 < \nu_A(x) < t_1$.

Conversely,

If there is no $x \in G$ such that $t_1 > \mu_A(x) > t_2$ and $t_2 < \nu_A(x) < t_1$,

then $A_{t1} = A_{t2}$. (by the definition of level set) .

2.3 Proposition : Let G be a group and A be an intuitionistic fuzzy subset of G such that A_t is a subgroup of G. For $t \in [0, 1]$ such that $t \leq \mu_A(e)$ and $t \geq \nu_A(e)$, A is an IFSG of G.

Proof : Let G be a group and x , y in G .

Let $\mu_A(x) = t_1$ and $\mu_A(y) = t_2$, $\nu_A(x) = t_1$ and $\nu_A(y) = t_2$.

Suppose $t_1 < t_2$, then $x, y \in A_{t1}$.

As A_{t1} is a subgroup of G, then $xy^{-1} \in A_{t1}$.

Now $\mu_A(xy^{-1}) \geq t_1 = \min \{t_1, t_2\}$

$= \min \{\mu_A(x), \mu_A(y)\}$

which implies that

$\mu_A(xy^{-1}) \geq \min \{\mu_A(x), \mu_A(y)\}$.

And ,

$\nu_A(xy^{-1}) \leq t_1 < t_2 = \max \{t_1, t_2\}$

$= \max \{\nu_A(x), \nu_A(y)\}$

which implies that

$\nu_A(xy^{-1}) \leq \max \{\nu_A(x), \nu_A(y)\}$.

Hence A is an IFSG of a group G.

2.4 Proposition : Let A be an IFSG of a group G. If $t \in [0, 1]$ and $t \leq \mu_A(e)$ and $t \geq \nu_A(e)$, and if A_{t1}, A_{t2} are level subgroups of A, then $A_{t1} \cap A_{t2}$ is also a level subgroup of A.

Proof : Let $t_1, t_2 \in [0, 1]$.

Case (i) If $t_1 < t_2$, then $A_{t2} \subseteq A_{t1}$.

Therefore $A_{t1} \cap A_{t2} = A_{t2}$, but A_{t2} is a level subgroup of A .

Case (ii) If $t_2 < t_1$, then $A_{t1} \subseteq A_{t2}$.

Therefore $A_{t1} \cap A_{t2} = A_{t1}$, but A_{t1} is a level subgroup of A .

Case (iii) If $t_2 = t_1$, then $A_{t1} = A_{t2}$.

In all the three cases $A_{t1} \cap A_{t2}$ is a level subgroup of A.

This proposition can be extended to any arbitrary elements of level subgroup of A.

2.5 Proposition : Let A be an IFSG of a group G. If $t \in [0, 1]$ and $t \leq \mu_A(e)$ and $t \geq \nu_A(e)$, and if A_{ti}, $i \in I$ are level subgroups of A, then $\cap_{i\in I} A_{ti}$ is also a level subgroup of A.

2.6 Proposition : Let A be an IFSG of a group G. If $t \in [0, 1]$ and $t \leq \mu_A(e)$ and $t \geq \nu_A(e)$, and if A_{t1}, A_{t2} are level subgroups of A, then $A_{t1} \cup A_{t2}$ is also a level subgroup of A.

Proof : Let $t_1, t_2 \in [0, 1]$.

Case (i) If $t_1 < t_2$, then $A_{t2} \subseteq A_{t1}$.

Therefore $A_{t1} \cup A_{t2} = A_{t1}$, but A_{t1} is a level subgroup of A .

Case (ii) If $t_2 < t_1$, then $A_{t1} \subseteq A_{t2}$.

Therefore $A_{t1} \cup A_{t2} = A_{t2}$, but A_{t2} is a level subgroup of A

Case (iii) If $t_2 = t_1$, then $A_{t1} = A_{t2}$.

In all the three cases $A_{t1} \cup A_{t2}$ is a level subgroup of A.

This proposition can be extended to any arbitrary elements of level subgroup of A.

2.7 Proposition : Let A be an IFSG of a group G. If $t \in [0, 1]$ and $t \leq \mu_A(e)$ and $t \geq \nu_A(e)$ and if A_{ti} ,$i \in I$ are level subgroups of A, then $\cup_{i\in I} A_{ti}$ is also a level subgroup of A.

Remark: This result is not true in the case of subgroups.

In the following proposition $\circ$ is the composition operation of functions :

2.8 Proposition : Let A be an intuitionistic fuzzy subgroup (IFSG) of a group H and f is an isomorphism from a group G onto H. Then we have the following:

i) $A\circ f$ is an IFSG of a group G.

ii) If A is an intuitionistic fuzzy normal subgroup(IFNSG) of a group H, then $A\circ f$ is an intuitionistic fuzzy normal subgroup(IFNSG) of a group G.

Proof : Let $x, y \in G$ and A be an IFSG of a group H. Then we have ,

$$\begin{aligned} \mu_A\circ f(xy^{-1}) &= \mu_A(f(xy^{-1})) \\ &= \mu_A(f(x)f(y^{-1})) \text{ as f is an isomorphism} \\ &= \mu_A(f(x)(f(y))^{-1}) \quad \text{by theorem 1.1} \\ &\geq \min \{\mu_A(f(x)), \mu_A(f(y)) \} \text{ as A is an IFSG of a group H} \\ &\geq \min \{\mu_A\circ f(x), \mu_A\circ f(y) \} \end{aligned}$$

which implies that

$$\mu_A\circ f(xy^{-1}) \geq \min \{\mu_A\circ f(x), \mu_A\circ f(y) \}.$$

And

$$\begin{aligned} \nu_A\circ f(xy^{-1}) &= \nu_A(f(xy^{-1})) \\ &= \nu_A(f(x)f(y^{-1})) \text{ as f is an isomorphism} \\ &= \nu_A(f(x)(f(y))^{-1}) \quad \text{by theorem 1.1} \\ &\leq \max \{ \nu_A(f(x)), \nu_A(f(y)) \} \text{ as A is an IFSG of a group H} \\ &\leq \max \{\nu_A\circ f(x), \nu_A\circ f(y) \} \end{aligned}$$

which implies that

$$\nu_A\circ f(xy^{-1}) \leq \max \{\nu_A\circ f(x), \nu_A\circ f(y) \}.$$

Therefore $A\circ f$ is an IFSG of a group G.

Hence (i) is proved .

Let $x, y \in G$ and A be an IFNSG of a group H. Then we have ,

$$\begin{aligned} \mu_A\circ f(xy) &= \mu_A(f(xy)) \\ &= \mu_A(f(x)f(y)) \text{ as f is an isomorphism} \\ &= \mu_A(f(y)f(x)) \text{ as A is an IFNSG of a group H} \\ &= \mu_A(f(yx)) \text{ as f is an isomorphism} \\ &= \mu_A\circ f(yx) \end{aligned}$$

which implies that

$$\mu_A\circ f(xy) = \mu_A\circ f(yx).$$

Now,

$$\begin{aligned} \nu_A\circ f(xy) &= \nu_A(f(xy)) \\ &= \nu_A(f(x)f(y)) \text{ as f is an isomorphism} \\ &= \nu_A(f(y)f(x)) \text{ as A is an IFNSG of a group H} \\ &= \nu_A(f(yx)) \text{ as f is an isomorphism} \\ &= \nu_A\circ f(yx) \end{aligned}$$

which implies that

$$\nu_A\circ f(xy) = \nu_A\circ f(yx).$$

Hence $A\circ f$ is an IFNSG of a group G.

2.9 Proposition : Let A be an intuitionistic fuzzy subgroup (IFSG) of a group H and f is an anti-isomorphism from a group G onto H. Then we have the following:

(i) $A\circ f$ is an IFSG of a group G.

(ii) If A is an intuitionistic fuzzy normal subgroup(IFNSG) of a group H, then $A\circ f$ is an intuitionistic fuzzy normal subgroup(IFNSG) of a group G.

Proof : Let x, y $\in$ G and A be an IFSG of a group H. Then we have ,

$\mu_A \circ f(xy^{-1}) = \mu_A(f(xy^{-1}))$

$= \mu_A(f(y^{-1})f(x))$ as f is an anti-isomorphism

$= \mu_A((f(y))^{-1}f(x))$ by theorem 1.2

$\geq \min\{\mu_A(f(x)), \mu_A(f(y))\}$ as A is an IFSG of a group H

$\geq \min\{\mu_A \circ f(x), \mu_A \circ f(y)\}$

which implies that

$\mu_A \circ f(xy^{-1}) \geq \min\{\mu_A \circ f(x), \mu_A \circ f(y)\}$.

And

$\nu_A \circ f(xy^{-1}) = \nu_A(f(xy^{-1}))$

$= \nu_A(f(y^{-1})f(x))$ as f is an anti-isomorphism

$= \nu_A((f(y))^{-1} f(x))$ by theorem 1.2

$\leq \max\{\nu_A(f(x)), \nu_A(f(y))\}$ as A is an IFSG of a group H

$\leq \max\{\nu_A \circ f(x), \nu_A \circ f(y)\}$

which implies that

$\nu_A \circ f(xy^{-1}) \leq \max\{\nu_A \circ f(x), \nu_A \circ f(y)\}$.

Therefore A$\circ$f is an IFSG of a group G.

(i) is proved .

Let x, y $\in$ G and A be an IFSG of a group H. Then we have ,

$\mu_A \circ f(xy) = \mu_A(f(xy))$

$= \mu_A(f(y)f(x))$ as f is an anti-isomorphism

$= \mu_A(f(x)f(y))$ as A is an IFNSG of a group H

$= \mu_A(f(yx))$ as f is an anti-isomorphism

$= \mu_A \circ f(yx)$

which implies that

$\mu_A \circ f(xy) = \mu_A \circ f(yx)$.

Now,

$\nu_A \circ f(xy) = \nu_A(f(xy))$

$= \nu_A(f(y)f(x))$ as f is an anti-isomorphism

$= \nu_A(f(x)f(y))$ as A is an IFNSG of a group H

$= \nu_A(f(yx))$ as f is an anti-isomorphism

$= \nu_A \circ f(yx)$

which implies that

$\nu_A \circ f(xy) = \nu_A \circ f(yx)$.

Hence A$\circ$f is an IFNSG of a group G.

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