NIFS 3 (1997) 5, 123-125

INTUITIONISTIC FUZZY INTERPRETATION OF KUN'S AXIOM

Krassimir T. Atanassov

CLBME - Bulg. Academy of Sci., and MRL, P.O.Box 12, Sofia-1113, Bulgaria

e-mail: krat@bgcict.acad.bg

Devoted to Prof. Janusz Kacprzyk
for his 50-th birthday

In [1] V. Kostjuk formalize the Kun's axiom *"if A and B are two theories from which follow contradictory consequences, then the expression containing the conjunction of A and B is false"* in the form

$$((A \supset C) \& (B \supset \neg C)) \supset \neg(A \& B)).$$

He assert that this axiom will have more correct form, if it contains modalities and he offers the axiom:

$$(\Box(A \supset C) \& \Box(B \supset \neg C)) \supset \neg\Diamond(A \& B)).$$

We shall show that these axioma can be interpreted as Intuitionistic Fuzzy Tautologies (IFTs).

First, following [2,3] we shall give some short remarks on the intuitionistic fuzzy logic related to the below text.

To each proposition (see, e.g., [4]) we can assign its truth value: truth – denoted by 1, or falsity – 0. In the case of fuzzy logic this truth value is a real number in the interval [0, 1] and may be called "truth degree" of a particular proposition. Here we add one more value – "falsity degree" – which will be in the interval [0, 1] as well. Thus two real numbers, $\mu(p)$ and $\nu(p)$, are assigned to the proposition p with the following constraint to hold:

$$\mu(p) + \nu(p) \leq 1.$$

Let this assignment be provided by an evaluation function V defined over a set of propositions S in such a way that:

$$V(p) = \langle \mu(p),\ \nu(p) \rangle.$$

Hence the function $V : S \to [0,1] \times [0,1]$ gives the truth and falsity degrees of all propositions in S.

When the values $V(p)$ and $V(q)$ of the propositions p and q are known, the evaluation function V can be extended also for the operations "&", "∨" through the definition :

$$V(\neg p) = \langle \nu(p), \mu(p) \rangle,$$
$$V(p\&q) = \langle min(\mu(p), \mu(q)), max(\nu(p), \nu(q)) \rangle,$$
$$V(p \supset q) = \langle max(\nu(p), \mu(q)), min(\mu(p), \nu(q)) \rangle,$$

As with the operations over IFS, it will be convenient to define for the propositions $p, q \in S$:

$$\neg V(p) = V(\neg p),$$
$$V(p) \wedge V(q) = V(p\&q),$$
$$V(p) \to V(q) = V(p \supset q).$$

For the needs of the discussion below we shall define the notion of IFT through:

$$\text{"}A \text{ is an IFT" } iff \text{ if } V(A) = \langle a, b \rangle, \text{ then } a \geq b.$$

Now, we shall prove the following

THEOREM: For every three propositional forms A, B and C:
(a) $((A \supset C)\&(B \supset \neg C)) \supset \neg(A\&B))$,
(b) $(\Box(A \supset C)\&\Box(B \supset \neg C)) \supset \neg\Diamond(A\&B))$
are IFTs.

Proof: Let $V(A) = <\mu_A, \nu_A>$, $V(B) = <\mu_B, \nu_B>$, $V(C) = <\mu_C, \nu_C>$
(a) $V(((A \supset C)\&(B \supset \neg C)) \supset \neg(A\&B)))$
$= ((\langle a, b \rangle \to \langle e, f \rangle) \wedge (\langle c, d \rangle \to \langle f, e \rangle)) \to \neg\langle a, b \rangle \wedge \langle c, d \rangle$
$= (\langle max(b, e), min(a, f) \rangle \wedge \langle max(d, f), min(c, e) \rangle)$
$\quad \to \neg\langle min(a, c), max(b, d) \rangle$
$= \langle min(max(b, e), max(d, f)), max(min(a, f), min(c, e)) \rangle$
$\quad \to \langle max(b, d), min(a, c) \rangle$
$= \langle max(b, d, min(a, f), min(c, e)), min(a, c, max(b, e), max(d, f)) \rangle.$

Let

$$X = max(b, d, min(a, f), min(c, e)) - min(a, c, max(b, e), max(d, f)).$$

If $a \leq f$, then

$$X \geq max(b, d, a) - a \geq a - a = 0$$

If $a > f$, then

$$X \geq max(b, d, f) - max(d, f) \geq 0.$$

Therefore, the axiom is an IFT.

(b) $V(\Box(A \supset C)\&\Box(B \supset \neg C)) \supset \neg\Diamond(A\&B)))$

$$= (\Box\langle max(b,e), min(a,f)\rangle \wedge \Box\langle max(d,f), min(c,e)\rangle)$$
$$\rightarrow \neg\Diamond\langle min(a,c), max(b,d)\rangle$$
$$= (\langle max(b,e), 1 - max(b,e)\rangle \wedge \langle max(d,f), 1 - max(d,f)\rangle)$$
$$\rightarrow \neg\langle 1 - max(b,d), max(b,d)\rangle$$
$$= \langle min(max(b,e), max(d,f)), max(1 - max(b,e), 1 - max(d,f)\rangle$$
$$\rightarrow \langle max(b,d), 1 - max(b,d)\rangle$$
$$= \langle max(b,d,1 - max(b,e), 1 - max(d,f)), min(1 - max(b,d), max(b,e), max(d,f))\rangle.$$

Let

$$X = max(b,d,1 - max(b,e), 1 - max(d,f)) - min(1 - max(b,d), max(b,e), max(d,f)).$$

If $d \geq e$, then

$$X \geq max(b,d) - max(b,e) \geq 0$$

Let $d < e$ and let $b \geq e$, then

$$X \geq b - max(b,e) = b - b = 0.$$

Let $d < e$ and $b < e$, then

$$X = max(b,d,1 - e, 1 - max(d,f)) - min(1 - max(b,d), e, max(d,f)).$$

If $d \geq f$, then

$$X \geq 1 - d - 1 + max(b,d) = max(b,d) - d \geq 0.$$

If $d < f$, then

$$X = 1 - f - e \geq 0.$$

Therefore, the axiom is an IFT.

References:

[1] Kostjuk V., Elements of Modal Logic, Kiev, Naukova Dumka, 1978 (in Russian).

[2] Atanassov K. Two variants of intuitonistic fuzzy propositional calculus. Preprint IM-MFAIS-5-88, Sofia, 1988.

[3] Atanassov K., Gargov G. C. R. Acad. bulg. sci., Tome 43, 1990, No 3, 9-12.

[4] Mendelson E., Introduction to Mathematical Logic, Princeton, NJ, 1964.