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A Note on a Geometric Interpretation of the Intuitionistic Fuzzy Set Operators $P_{\alpha,\beta}$ and $Q_{\alpha,\beta}$

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Abstract

In the present paper a geometric interpretation of two new operators over intuitionistic fuzzy sets is given.

1 Introduction

The theory of fuzzy sets was first introduced by Zadeh as an appropriate mathematical instrument for the description of uncertainty observed in nature (see [4], [5]). One of the extensions of the fuzzy sets are the intuitionistic fuzzy sets introduced by K. Atanassov (see [1]). We will briefly remind some of the basic definitions and notions.

Let E be a universe set (i.e. the set of all the (*relevant*) elements that will be considered). Then let $A \subset E$. We call the set

$$A^* \stackrel{\text{def}}{=} \{x, \mu_A(x), \nu_A(x) | x \in E\}$$

where the functions

$$\mu_A(x) \to [0,1]$$

and

$$\nu_A(x) \to [0,1]$$

reflect the degree of membership (belongingness) and non-membership (non-belongingness) of the element x from E to the set A, respectively, and for every x it is fulfilled that:

$$0 \le \mu_A(x) + \nu_A(x) \le 1$$

The function $\pi_A(x)$ which is given by

$$\pi_A(x) \stackrel{\text{def}}{=} 1 - \mu_A(x) - \nu_A(x), \tag{1}$$

defines the degree of uncertainty of the membership of the element x to the set A. Obviously, in the case of $(\forall x \in E)\pi_A(x) = 0$ we have a fuzzy set. The following geometric interpretation is usually used for representational purposes (see [1]):

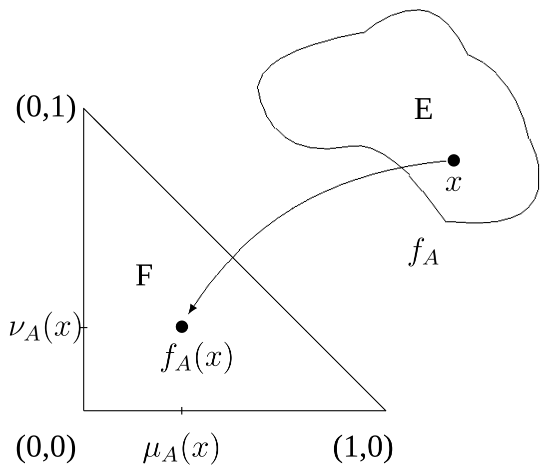

Figure 1

However, we must note that in this case the membership and non-membership functions are privileged in terms of representation. Some other geometric representations have been proposed in order to visually reflect the relations between the three degrees associated with x. While they are functionally equivalent (due to (1)), there is a subtle difference. For instance, if in the above representation we happen to know $\mu_A(x)$ and $\pi_A(x)$ we will still have to calculate the value of $\nu_A(x)$ using:

$$\nu_A(x) \stackrel{\text{def}}{=} 1 - \mu_A(x) - \pi_A(x), \tag{1'}$$

in order to find the point in the triangle corresponding to x. An alternative representation (proposed in [2]) is the following:

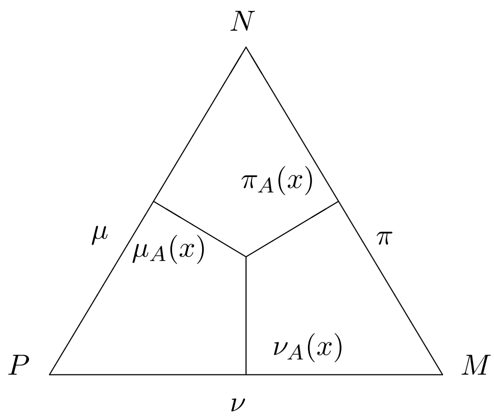

Figure 2

It is easy to see that a point is easily found by any two of its degrees without the need of additional calculation.

2 Geometric representation and properties of $P_{\alpha,\beta}$ and $Q_{\alpha,\beta}$

In [1] were defined the following two operators:

$$P_{\alpha,\beta}(A^*) = \{< x, \max(\alpha, \mu_A(x)), \min(\beta, \nu_A(x)) > | x \in E\}$$

$$Q_{\alpha,\beta}(A^*) = \{< x, \min(\alpha, \mu_A(x)), \max(\beta, \nu_A(x)) > | x \in E\}$$

where $\alpha > 0, \beta > 0, \alpha + \beta < 1$.

We will now give a brief description of the geometric interpretation of the work of the two operators in the case of Fig 2.

Let us consider the result of $P_{\alpha,\beta}$

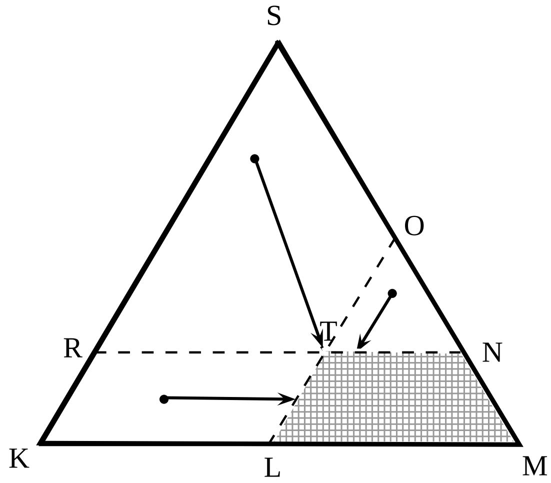

Figure 3

Case 1) The associated degrees of $x \in E$ lie in $KLTR$ then their values are replaced by the values of a point from LT

Case 2) The associated degrees of $x \in E$ lie in $LMNT$ then they preserve their previous values.

Case 3) The associated degrees of $x \in E$ lie in TNO then their values are replaced by the values of a point from TN

Case 4) The associated degrees of $x \in E$ lie in $TSOR$ then their values are replaced by the values of the T.

From 1), 2), 3) and 4) we may conclude that the result of the application of $P_{\alpha,\beta}$ is the shift of all degrees into $LMNT$.

Let us consider the result of $Q_{\alpha,\beta}$

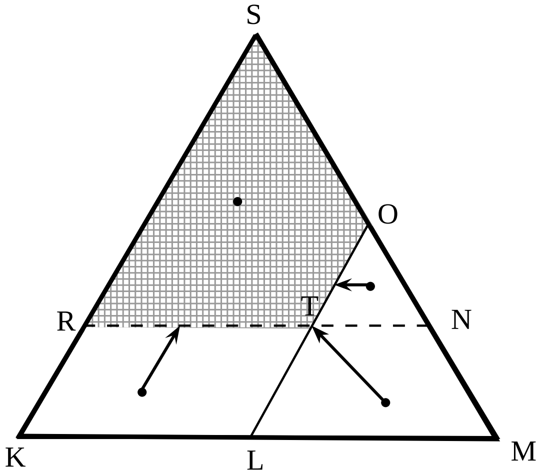

Figure 4

Case 1 The associated degrees of $x \in E$ lie in $KLTR$ then their values are replaced by the values of a point from RT

Case 2 The associated degrees of $x \in E$ lie in $LMNT$ then their values are replaced by the values of the T.

Case 3 The associated degrees of $x \in E$ lie in TNO then their values are replaced by the values of a point from TO

Case 4 The associated degrees of $x \in E$ lie in $RTSO$ then they preserve their previous values.

From 1), 2), 3) and 4) we may conclude that the result of the application of $P_{\alpha,\beta}$ is the shift of all degrees into $RTOS$.

3 Conclusion

In conclusion, we must note that the application of any of the operators over a data set in pattern recognition problem will move only the points from classes with non-strict membership. This may prove useful in manipulating the available data with different threshold values of the parameters α and β.

References

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