Notes on Intuitionistic Fuzzy Sets
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On compactness in temporal intuitionistic fuzzy Šostak topology

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Abstract: In this paper, we introduce concepts of temporal and overall intuitionistic fuzzy continuous mapping, (α_t, β_t)-temporal intuitionistic fuzzy (almost, nearly) compactness, (α_t, β_t)-overall intuitionistic fuzzy (almost, nearly) compactness, (α^*, β^*)-continuous intuitionistic fuzzy (almost, nearly) compactness in temporal intuitionistic fuzzy Šostak topological space and we investigate some properties of these concepts.
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1 Introduction

The concept of fuzzy set was introduced by Zadeh in 1965 and has been well understood and used in various aspects of science and technology such as engineering and medicine. The theory of fuzzy set is one of the most important inventions of our time. On the other hand, as a natural generalization of fuzzy set, intuitionistic fuzzy set (IFS for short) was introduced by Atanassov in 1983. His definition was found to be useful to deal with vagueness of knowledge. In the concept of intuitionistic fuzzy set, each element has two degrees named degree of membership and degree of non-membership to IFS respectively [1]. The concept of fuzzy topological space was defined by Chang in 1968 as a collection of fuzzy sets. Fuzzifying of topology concept was made by Šostak in 1985. In his definition, openness and closeness of fuzzy sets are graded among 0 and 1. In 1996, D. Çoker and M. Demirci introduced the concept of intuitionistic fuzzy set in Šostak's sense and gave fundamental definitions and properties of it. The concept of compactness has been defined in various ways by many researchers ([1, 6, 8, 11, 12, 14]).

Temporal intuitionistic fuzzy set (TIFS) was defined by Atanassov in 1991. In his definition, membership and non-membership degrees of an element change with both of the element and time moment. This is one of the most important extensions of IFS. Because, real world situations are generally spatio-temporal [13]. Thus, by the theory of TIFS, real world situations like weather, medicine, economy, image-video processing can be handled more realistic and effective. As stated in [13]; it is well-known that time is monotone and time is a fundamental issue for modeling dynamic information. In recent years, some fundamental concepts have been defined by several authors [13, 16]. In 2014, Çuvalcıoğlu and S. Yılmaz defined level operators on TIFSs [15]. The concept of temporal intuitionistic fuzzy is very untouched area and the most fundamental concepts have not been defined yet. One of these concepts is topology and topological concepts of TIFSs. Šostak's mean temporal intuitionistic fuzzy topology was defined by Kutlu and Bilgin [10].

This study is organized as follows: In section 2, we give basic definitions of intuitionistic fuzzy sets and temporal intuitionistic fuzzy sets. In section 3, we introduce the concept of temporal (overall) intuitionistic fuzzy continuous mapping in ST-IFS and investigate some fundamental properties of temporal (overall) intuitionistic fuzzy continuous. In section 4, we define (α_t, β_t)-temporal intuitionistic fuzzy (almost, nearly) compactness, (α_t, β_t)-overall intuitionistic fuzzy (almost, nearly) compactness, (α^*, β^*)-continuous intuitionistic fuzzy (almost, nearly) compactness and investigate some properties of these concepts. Also we give relationship between these new compactness concepts.

2 Preliminaries

Definition 2.1 [2] An intuitionistic fuzzy set in a non-empty set X given by a set of ordered triples $A=\{(x,\mu_A(x),\eta_A(x)) \mid x\in X\}$ where $\mu_A(x):X\to I$, $\eta_A(x):X\to I$ and $I=[0,1]$, are functions such that $0\le\mu(x)+\eta(x)\le 1$ for all $x\in X$. For $x\in X$, $\mu_A(x)$ and $\eta_A(x)$ represent the degree of membership and degree of non-membership of x to A respectively. For each $x\in X$; intuitionistic fuzzy index of x in A can be defined as follows $\pi_A(x)=1-\mu_A(x)-\eta_A(x)$. π_A is the called degree of hesitation or indeterminacy.

By $IFS(X)$, we denote to the set of all intuitionistic fuzzy sets.

Definition 2.2 [3] Let $A,B\in IFS(X)$. Then,

(i) $A\subseteq B \Leftrightarrow \mu_A(x)\le\mu_B(x)$ and $\eta_A(x)\ge\eta_B(x)$ for $\forall x\in X$,

(ii) $A=B\Leftrightarrow A\subseteq B$ and $B\subseteq A$,

(iii) $A^c=\{(x,\eta_A(x),\mu_A(x)) \mid x\in X\}$,

(iv) $\bigcap A_i = \{(x, \wedge\mu_{A_i}(x), \vee\eta_{A_i}(x)) \mid x \in X\}$,

(v) $\bigcup A_i = \{(x, \vee\mu_{A_i}(x), \wedge\eta_{A_i}(x)) \mid x \in X\}$,

(vi) $\underset{\sim}{0} = \{(x,0,1) \mid x \in X\}$ and $\underset{\sim}{1} = \{(x,1,0) \mid x \in X\}$.

Definition 2.3 [2,5]. Let a and b be two real numbers in $[0,1]$ satisfying the inequality $a+b \le 1$. Then, the pair $\langle a,b\rangle$ is called an intuitionistic fuzzy pair. Let $\langle a_1,b_1\rangle$ and $\langle a_2,b_2\rangle$ be two intuitionistic fuzzy pair (briefly IF-pair). Then define

(i) $\langle a_1,b_1\rangle \le \langle a_2,b_2\rangle \Leftrightarrow a_1 \le a_2$ and $b_1 \ge b_2$,

(ii) $\langle a_1,b_1\rangle = \langle a_2,b_2\rangle \Leftrightarrow a_1 = a_2$ and $b_1 = b_2$,

(iii) If $\{\langle a_i,b_i\rangle; i \in J\}$ is a family of intuitionistic fuzzy pairs, then $\vee\langle a_i,b_i\rangle = \langle \vee a_i, \wedge b_i\rangle$ and $\wedge\langle a_i,b_i\rangle = \langle \wedge a_i, \vee b_i\rangle$,

(iv) The complement of $\langle a,b\rangle$ is defined by $\overline{\langle a,b\rangle} = \langle b,a\rangle$,

(v) $1^{\sim} = \langle 1,0\rangle$ and $0^{\sim} = \langle 0,1\rangle$.

Definition 2.4 [5]. An intuitionistic fuzzy topology in Šostak's sense (briefly, S-IFS) on a nonempty set *X* is an IFF τ defined with $\tau(A) = (\mu_\tau(A), \eta_\tau(A))$ on X satisfying the following axioms:

(T1) $\tau(\underset{\sim}{0}) = 1^{\sim}$ and $\tau(\underset{\sim}{1}) = 1^{\sim}$,

(T2) $\tau(A_1 \cap A_2) \ge \tau(A_1) \wedge \tau(A_2)$ for any $A_1, A_2 \in IFS(X)$,

(T3) $\tau(\bigcup A_i) \ge \bigwedge_{i\in J}(\tau(A_i))$ for any $\{A_i \mid i \in J\} \subseteq IFS(X)$.

The pair (X,τ) is called an intuitionistic fuzzy topological space in Šostak sense. For any $A \in IFS(X)$, the number $\mu_\tau(A)$ is called the openness degree of *A*, while $\eta_\tau(A)$ is called non-openness degree of *A*.

Definition 2.5 [4]. Let E be an universe and *T* be a non-empty time-moment set. We call the elements of *T* "time moments". Based on the definition of IFS, a temporal intuitionistic fuzzy set (TIFS) *A* is defined as the following:

$$A(T) = \{(x, \mu_A(x,t), \eta_A(x,t)) \mid (x,t) \in E \times T\}$$

where:

(a) $A \subseteq E$ is a fixed set

(b) $\mu_A(x,t) + \eta_A(x,t) \le 1$ for every $(x,t) \in E \times T$

(c) $\mu_A(x,t)$ and $\eta_A(x,t)$ are the degrees of membership and non-membership, respectively, of the element $x \in E$ at the time moment $t \in T$

By $TIFS^{(X,T)}$, we denote to the set of all TIFSs over nonempty set *X* and time-moment set *T*. For brevity, we write *A* instead of *A*(*T*). The hesitation degree of a TIFS is defined as $\pi_A(x,t) = 1 - \mu_A(x,t) - \eta_A(x,t)$. Obviously, every ordinary IFS can be regarded as TIFS for which *T* is a singleton set. All operations and operators on IFS can be defined for TIFSs.

Definition 2.6 [3]. Let

$$A(T') = \{(x, \mu_A(x,t), \eta_A(x,t)) \mid (x,t) \in X \times T'\}$$

and

$$B(T'')=\{(x,\mu_B(x,t),\eta_B(x,t))\mid(x,t)\in X\times T''\}$$

where T' and T'' have finite number of distinct time-elements or they are time intervals. Then,

$$A(T')\cap B(T'')=\{(x,\ \min(\bar{\mu}_A(x,t),\bar{\mu}_B(x,t)),\max(\bar{\eta}_A(x,t),\bar{\eta}_B(x,t))\mid(x,t)\in X\times(T'\cup T'')\},$$

$$A(T')\cup B(T'')=\{(x,\ \max(\bar{\mu}_A(x,t),\bar{\mu}_B(x,t)),\min(\bar{\eta}_A(x,t),\bar{\eta}_B(x,t))\mid(x,t)\in X\times(T'\cup T'')\}.$$

Also from definition of subset in IFS theory, Subsets of TIFS can be defined as the following:

$A(T')\subseteq B(T'')\Leftrightarrow \bar{\mu}_A(x,t)\le\bar{\mu}_B(x,t)$ and $\bar{\eta}_A(x,t)\ge\bar{\eta}_B(x,t)$ for every $(x,t)\in X\times(T'\cup T'')$

where

$$\bar{\mu}_A(x,t)=\begin{cases}\mu_A(x,t), & if\ t\in T'\\ 0, & if\ t\in T''-T'\end{cases}\qquad\text{and}\qquad \bar{\mu}_B(x,t)=\begin{cases}\mu_B(x,t), & if\ t\in T''\\ 0, & if\ t\in T'-T''\end{cases}$$

$$\bar{\eta}_A(x,t)=\begin{cases}\eta_A(x,t), & if\ t\in T'\\ 1, & if\ t\in T''-T'\end{cases}\quad\text{and}\quad \bar{\eta}_B(x,t)=\begin{cases}\eta_B(x,t), & if\ t\in T''\\ 1, & if\ t\in T''-T'\end{cases}$$

It is obviously seen that if $T'=T''$; $\bar{\mu}_A(x,t)=\mu_A(x,t)$, $\bar{\mu}_B(x,t)=\mu_B(x,t)$, $\bar{\eta}_A(x,t)=\eta_A(x,t)$, $\bar{\eta}_B(x,t)=\eta_B(x,t)$.

Let J be an arbitrary index set. Then we define that $T=\bigcup_{i\in J}T_i$ where T_i is a time set for each $i\in J$. Thus, we can extend the definition of union and intersection of TIFSs family $F=\{A_i(T_i)=(x,\mu_{A_i}(x,t),\eta_{A_i}(x,t))\mid x\in X\times T_i, i\in J\}$ as follows:

$$\bigcup_{i\in J}A(T_i)=\{(x,\ \max_{i\in J}(\bar{\mu}_{A_i}(x,t)),\min_{i\in J}(\bar{\eta}_{A_i}(x,t))\mid(x,t)\in X\times T\},$$

$$\bigcap_{i\in J}A(T_i)=\{(x,\ \min_{i\in J}(\bar{\mu}_{A_i}(x,t)),\max_{i\in J}(\bar{\eta}_{A_i}(x,t))\mid(x,t)\in X\times T\},$$

where

$$\bar{\mu}_{A_j}(x,t)=\begin{cases}\mu_{A_j}(x,t), & if\ t\in T_j\\ 0, & if\ t\in T-T_j,\end{cases}$$

and

$$\bar{\eta}_{A_j}(x,t)=\begin{cases}\eta_{A_j}(x,t), & if\ t\in T_j\\ 1, & if\ t\in T-T_j.\end{cases}$$

Definition 2.7 [10]. $\underset{\sim}{0}^t$ and $\underset{\sim}{1}^t\in TIFS(X,T)$ are defined as: $\underset{\sim}{0}^t=\{(x,0,1)\mid(x,t)\in X\times T\}$ and $\underset{\sim}{1}^t=\{(x,1,0)\mid(x,t)\in X\times T\}$ for each time moment *t*, i.e. $\mu_{\underset{\sim}{0}^t}(x,t)=0$, $\eta_{\underset{\sim}{0}^t}(x,t)=1$ and $\mu_{\underset{\sim}{1}^t}(x,t)=1$, $\eta_{\underset{\sim}{1}^t}(x,t)=0$ for each $(x,t)\in X\times T$.

Definition 2.8 [10]. An temporal intuitionistic fuzzy topology in Šostak's sense (briefly, ST-TIFS) on a non-empty set *X* is an IFF τ_t defined with $\tau_t(A)=(\mu_{\tau_t}(A),\eta_{\tau_t}(A))$ on *X* satisfying the following axioms for each time moment *t*:

I. $\tau_t(\underset{\sim}{0}^t)=1^{\sim}$ and $\tau_t(\underset{\sim}{1}^t)=1^{\sim}$,

II. $\tau_t(A_1 \cap A_2) \geq \tau_t(A_1) \wedge \tau_t(A_2)$ for any sets $A_1, A_2 \in TIFS^{(X,T)}$,

III. $\tau_t(\bigcup A_i) \geq \bigwedge_{i\in J}(\tau_t(A_i))$ for $\{A_i \mid i \in J\} \subseteq TIFS^{(X,T)}$.

The pair (X, τ_t) is called temporal intuitionistic fuzzy topological space in Šostak sense. For any $A \in TIFS^{(X,T)}$, the number $\mu_{\tau_t}(A)$ is called instant openness degree of *A* at time-moment *t*, while $\eta_{\tau_t}(A)$ is called instant non-openness degree of *A* at time-moment *t*. In this definition, it is worth to note that the instant openness and the instant non-openness degree change with depending on both time and TIFS.

It is worth to note that for singleton time set (X, τ_t) is an intuitionistic fuzzy topology in Šostak's sense.

Definition 2.9 [10]. IFF τ_t^* defined with $\tau_t^*(A) = \left(\mu_{\tau_t^*}(A), \eta_{\tau_t^*}(A)\right)$, if it satisfies the following axioms for each time moment *t*:

I. $\tau_t^*(\underset{\sim}{0}^t) = 1^\sim$ and $\tau_t^*(\underset{\sim}{1}^t) = 1^\sim$,

II. $\tau_t^*(A_1 \cup A_2) \geq \tau_t(A_1) \wedge \tau_t(A_2)$ for any sets $A_1, A_2 \in TIFS^{(X,T)}$,

III. $\tau_t^*\left(\bigcap_{i\in J} A_i\right) \geq \bigwedge_{i\in J}(\tau_t^*(A_i))$ for $\{A_i \mid i \in J\} \subseteq TIFS^{(X,T)}$.

Then, the number $\mu_{\tau_t^*}(A)$ is called instant closeness degree of *A* at time moment *t*, while $\eta_{\tau_t^*}(A)$ is called instant non-closeness degree of *A* at time moment *t*.

Proposition 2.10 [10]. Let (X, τ_t) be a ST-TIFS on *X* and *T* be a time-moment set. Then $(X, \wedge\tau_t)$ defined by $\wedge\tau_t(A) = \left(\min_{t\in T} \mu_{\tau_t}(A), \max_{t\in T} \eta_{\tau_t}(A)\right)$ is an intuitionistic fuzzy topology on $TIFS^{(X,T)}$ in Šostak's sense.

Definition 2.11 [10]. Let (X, τ_t) be a ST-TIFS and $A \in TIFS^{(X,T)}$. Then we define instant closure and instant interior of *A* at time moment *t* according to τ_t respectively as:

$$\mathrm{cl}^t(A) = \bigcap\left\{K \in TIFS(X) \mid \tau_t^*(K) > \tilde{0}, A \subseteq K\right\}$$

and

$$\mathrm{int}^t(A) = \bigcup\left\{K \in TIFS(X) \mid \tau_t(K) > \tilde{0}, K \subseteq A\right\}.$$

On the other hand (α, β)-instant closure and (α, β)-instant interior of *A* are defined by:

$$\mathrm{cl}^t_{(\alpha,\beta)}(A) = \bigcap\left\{K \in TIFS(X) \mid \tau_t^*(K) \geq \langle\alpha, \beta\rangle, A \subseteq K\right\}$$

and

$$\mathrm{int}^t_{(\alpha,\beta)}(A) = \bigcup\left\{K \in TIFS(X) \mid \tau_t(K) \geq \langle\alpha, \beta\rangle, K \subseteq A\right\}$$

where $\alpha \in (0,1]$, $\beta \in [0,1)$ with $\alpha + \beta \leq 1$.

Proposition 2.12 [10]. Let (X, τ_t) be a ST-TIFS on *X* and time-moment set *T*. Then

$$\mathrm{cl}(A) = \underset{t\in T}{\wedge}\left(\mathrm{cl}^t(A)\right),$$

$$\mathrm{int}(A) = \underset{t\in T}{\vee}\left(\mathrm{int}^t(A)\right)$$

are closure and interior of *A* according to $(X, \wedge\tau_t)$.

Proposition 2.13. Let (X,τ) be a ST-TIFS and $A,B\in TIFS^{(X,T)}$. Then the following statements are satisfied for each $t\in T$;

(a) $A\subseteq B \;\Rightarrow\; \text{int}^t(A)\subseteq \text{int}^t(B)$,

(b) $A\subseteq B \;\Rightarrow\; cl^t(A)\subseteq cl^t(B)$,

(c) $\overline{\text{int}^t(A)}=cl^t(\bar{A})$,

(d) $\overline{cl^t(A)}=\text{int}^t(\bar{A})$,

(e) $\text{int}^t(A)=\overline{cl^t(\bar{A})}$,

(f) $cl^t(A)=\overline{\text{int}^t(\bar{A})}$.

(g) $\text{int}^t(\tilde{1}_t)=\tilde{1}_t$

(h) $\text{int}^t(A)\subseteq A$

(i) $\text{int}^t(\text{int}^t(A))=\text{int}^t(A)$

(j) $\text{int}^t(A\cap B)\subseteq \text{int}^t(A)\cap\text{int}^t(B)$

(k) $cl^t(\tilde{0}_t)=\tilde{0}_t$

(l) $A\subseteq cl^t(A)$

(m) $cl^t(cl^t(A))=cl^t(A)$

(n) $cl^t(A)\cup cl^t(B)\subseteq cl^t(A\cup B)$

Proposition 2.14. Let (X,τ) be a ST-TIFS and $A,B\in TIFS^{(X,T)}$. Then the following statements are satisfied for each $t\in T$ and $(\alpha,\beta)\in(0,1]\times[0,1)$;

(a) $cl^t_{\langle\alpha,\beta\rangle}(A)\supseteq A$;

(b) $\text{int}^t_{\langle\alpha,\beta\rangle}(A)\subseteq A$;

(c) $A\subseteq B$ ve $\langle\alpha,\beta\rangle\le\langle r,s\rangle$ ise $cl_{\langle\alpha,\beta\rangle}(A)\subseteq cl^t_{\langle r,s\rangle}(A)$;

(d) $A\subseteq B$ ve $\langle\alpha,\beta\rangle\le\langle r,s\rangle$ ise $\text{int}^t_{\langle\alpha,\beta\rangle}(A)\subseteq \text{int}^t_{\langle r,s\rangle}(A)$;

(e) $cl^t_{\langle\alpha,\beta\rangle}\left(cl^t_{\langle\alpha,\beta\rangle}(A)\right)=cl^t_{\langle\alpha,\beta\rangle}(A)$;

(f) $\text{int}^t_{\langle\alpha,\beta\rangle}\left(\text{int}^t_{\langle\alpha,\beta\rangle}(A)\right)=\text{int}^t_{\langle\alpha,\beta\rangle}(A)$;

(g) $cl^t_{\langle\alpha,\beta\rangle}(A\cup B)=cl^t_{\langle\alpha,\beta\rangle}(A)\cup cl^t_{\langle\alpha,\beta\rangle}(B)$;

(h) $\text{int}^t_{\langle\alpha,\beta\rangle}(A\cap B)=\text{int}^t_{\langle\alpha,\beta\rangle}(A)\cap\text{int}^t_{\langle\alpha,\beta\rangle}(B)$;

(i) $\overline{cl^t_{\langle\alpha,\beta\rangle}(A)}=\text{int}^t_{\langle\alpha,\beta\rangle}(\bar{A})$;

(j) $\overline{\text{int}^t_{\langle\alpha,\beta\rangle}(A)}=cl^t_{\langle\alpha,\beta\rangle}(\bar{A})$.

Proposition 2.15. Let (X,τ_t) be a ST-TIFS and $A\in TIFS^{(X,T)}$. Then the following statements are satisfied for each $t\in T$ and $(\alpha,\beta)\in(0,1]\times[0,1)$;

(i) $A\subseteq cl^t(A)\subseteq cl^t_{(\alpha,\beta)}(A)$

(ii) $\text{int}^t_{(\alpha,\beta)}(A)\subseteq\text{int}^t(A)\subseteq A$

(iii) If $\tau^*_t(A)>0^-$, $cl^t(A)=\bigcap_{(\alpha,\beta)\in I_0\times I_1} cl^t_{(\alpha,\beta)}(A)$

(iv) If $\tau_t(A) > 0^{\sim}$, $\text{int}^t(A) = \bigcup_{(\alpha,\beta)\in I_0\times I_1} \text{int}^t_{(\alpha,\beta)}(A)$

Proof. (i) Let us denote the set $\{K \in TIFS^{(X,T)}; \tau_t^*(K) \geq \langle\alpha,\beta\rangle, A \subseteq K\}$ for

$$\text{cl}^t_{(\alpha,\beta)}(A) = \bigcap\{K \in TIFS^{(X,T)}; \tau_t^*(K) \geq \langle\alpha,\beta\rangle, A \subseteq K\}$$

by $C^*_{\langle\alpha,\beta\rangle,t}$ and denote $\{K \in TIFS^{(X,T)}; \tau_t^*(K) > 0^{\sim}, A \subseteq K\}$ the set

$$\text{cl}^t(A) = \bigcap\{K \in TIFS^{(X,T)}; \tau_t^*(K) > 0^{\sim}, A \subseteq K\}$$

for $\{K \in TIFS^{(X,T)}; \tau_t^*(K) > 0^{\sim}, A \subseteq K\}$ by C_t^* . Since $\tau_t^*(K) \geq \langle\alpha,\beta\rangle > 0^{\sim}$ for each $K \in C^*_{\langle\alpha,\beta\rangle,t}$, it is clearly understood that $K \in C_t^*$. Hence we obtain that $C^*_{\langle\alpha,\beta\rangle,t} \subseteq C_t^*$. Therefore we can get $\bigcap_{K\in C_t} K \subseteq \bigcap_{K\in C^*_{\langle\alpha,\beta\rangle,t}} K$ i.e. $cl^t(A) \subseteq cl^t_{\langle\alpha,\beta\rangle}$ from the last statement

(ii) Let us denote the set $\{L \in TIFS^{(X,T)}; \tau_t(L) \geq \langle\alpha,\beta\rangle, L \subseteq A\}$ for

$$\text{int}^t_{(\alpha,\beta)}(A) = \bigcup\{L \in TIFS(X); \tau_t(L) \geq \langle\alpha,\beta\rangle, L \subseteq A\}$$

by $G^*_{\langle\alpha,\beta\rangle,t}$ and denote the set $\{L \in TIFS^{(X,T)}; \tau_t(L) > 0^{\sim}, L \subseteq A\}$ for

$$\text{int}^t(A) = \bigcup\{L \in TIFS^{(X,T)}; \tau_t(L) > 0^{\sim}, L \subseteq A\}$$

by G_t^* . Since $\tau_t(L) \geq \langle\alpha,\beta\rangle > 0^{\sim}$ for each $L \in G^*_{\langle\alpha,\beta\rangle,t}$, it is clearly understood that $L \in G_t^*$. Hence we obtain that $G^*_{\langle\alpha,\beta\rangle,t} \subseteq G_t^*$. Therefore we can get $\bigcup_{L\in G^*_{\langle\alpha,\beta\rangle,t}} L \subseteq \bigcup_{L\in G_t^*} L$ i.e. $\text{int}^t_{\langle\alpha,\beta\rangle}(A) \subseteq \text{int}^t(A)$ from the last statement.

(iii) From (i), we get $cl^t(A) \subseteq cl^t_{\langle\alpha,\beta\rangle}$ for each $\langle\alpha,\beta\rangle \in I_0 \times I_1$. So we can easily get $cl^t(A) \subseteq \bigcap_{\langle\alpha,\beta\rangle\in I_0\times I_1} cl^t_{\langle\alpha,\beta\rangle}$ (*). On the other hand, it is obtained that $cl^t(A) = A$ from $\tau_t^*(A) > 0^{\sim}$ and Definition 2.11. Since $\tau_t^*(A) > 0^{\sim}$ we can find at least one $\langle\alpha_0,\beta_0\rangle \in I_0 = (0,1] \times I_1 = [0,1)$ that satisfies the condition $\tau_t^*(A) \geq \langle\alpha_0,\beta_0\rangle$. Then, it is obtained that $cl^t_{\langle\alpha_0,\beta_0\rangle}(A) = A$ from Definition 2.11. Since $cl^t_{\langle\alpha_0,\beta_0\rangle}(A) \supseteq \bigcap_{\langle\alpha,\beta\rangle\in I_0\times I_1} cl^t_{\langle\alpha,\beta\rangle}(A)$, it is obtained that $cl^t(A) \supseteq \bigcap_{\langle\alpha,\beta\rangle\in I_0\times I_1} cl^t_{\langle\alpha,\beta\rangle}(A)$ (**). From (*) and (**), it is obtained that $cl^t(A) = \bigcap_{\langle\alpha,\beta\rangle\in I_0\times I_1} cl^t_{\langle\alpha,\beta\rangle}(A)$.

(iv) It can be easily shown as (iii). □

3 Continuity in temporal intuitionistic fuzzy Šostak topology

Definition 3.1. Let (X,τ_t) and (Y,ϕ_t) be ST-TIFSs respectively for non-empty sets *X*, *Y*, time sets T' and T'' . Let $f : X \to Y$ be a function. Then,

(i) The preimage of $B \in TIFS^{(Y,T'')}$ under f at time moment *t* is defined as $f^{-1}(B) = \{(x, \overline{\mu}_B(f(x),t), \overline{\eta}_B(f(x),t)) : x \in X\}$ where

$$\overline{\mu}_B(f(x),t) = \begin{cases} \mu_B(f(x),t) & , \quad t \in T'' \\ 0 & , \quad t \in T' - T'' \end{cases}$$

and

$$\overline{\eta}_B(f(x),t)=\begin{cases}\eta_B(f(x),t) & , \quad t\in T'' \\ 1 & , \quad t\in T'-T''.\end{cases}$$

(ii) The image of $A\in TIFS^{(X,T')}$ under f at time moment t is defined as $f(A)=\{(y,f(\overline{\mu}_A)(y,t),f_-(\overline{\eta}_A)(y,t)):y\in Y\}$ where

$$f(\overline{\mu}_A)(y,t)=\begin{cases}f(\mu_A)(y,t), & t\in T' \\ 0 & , \quad t\in T''-T'\end{cases}$$

and

$$f_-(\overline{\eta}_A)(y,t)=\begin{cases}1-f(1-\eta_A)(y,t), & t\in T' \\ 1 & , \quad t\in T''-T'.\end{cases}$$

If $T'=T''$, It is clearly understood that $f^{-1}(B)=\{(x,\mu_B(f(x),t),\eta_B(f(x),t)):x\in X\}$ and $f(A)=\{(y,f(\mu_A)(y,t),f_-(\eta_A)(y,t)):y\in Y\}$.

Let (X,τ_t) and (Y,ϕ_t) be ST-TIFSs for non-empty sets *X*, *Y* and time set *T*. If $\tau_t(f^{-1}(B))\geq\phi_t(B)$ for $t\in T$ and each $B\in TIFS^{(Y,T)}$, f is called temporal intuitionistic fuzzy continuous function at time moment *t*. If *f* is temporal intuitionistic fuzzy continuous function at each time moment, *f* is called overall intuitionistic fuzzy continuous function.

On the other hand, If $\phi_t(f(A))\geq\tau_t(A)$ for $t\in T$ and each $A\in TIFS^{(X,T')}$, *f* is called temporal intuitionistic fuzzy open function at time moment *t*. If *f* is temporal intuitionistic fuzzy open function at each time moment, *f* is called overall intuitionistic fuzzy open function.

Example 3.2. Let $X=\{a,b,c,d\}$ and $T=\{t_1,t_2,t_3\}$. We define $A\in TIFS^{(X,T)}$ as follows:

A	t_1	t_2	t_3
a	$\langle 0.1,0.2\rangle$	$\langle 0.5,0.3\rangle$	$\langle 0.7,0.1\rangle$
b	$\langle 0.5,0.1\rangle$	$\langle 0.3,0.2\rangle$	$\langle 0.6,0.4\rangle$
c	$\langle 0.3,0.3\rangle$	$\langle 0.5,0.3\rangle$	$\langle 0.1,0.1\rangle$
d	$\langle 0.4,0.5\rangle$	$\langle 0.3,0.2\rangle$	$\langle 0,0\rangle$

On the other hand, we define (X,τ_t) and (X,ϕ_t) such as $\tau_t:TIFS^{(X,T)}\rightarrow[0,1]\times[0,1]$ and $\phi_t:TIFS^{(X,T)}\rightarrow[0,1]\times[0,1]$ defined respectively as:

τ_t	t_1	t_2	t_3
$\tilde{0}_t$, $\tilde{1}_t$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$
A	$\langle 0.4,0.3\rangle$	$\langle 0.8,0.1\rangle$	$\langle 0.3,0.2\rangle$
otherwise	$\langle 0,0\rangle$	$\langle 0,0\rangle$	$\langle 0,0\rangle$

and

ϕ_t	t_1	t_2	t_3
$\tilde{0}_t$, $\tilde{1}_t$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$
A	$\langle 0.7,0.1\rangle$	$\langle 0.2,0.5\rangle$	$\langle 0.6,0.4\rangle$
otherwise	$\langle 0,0\rangle$	$\langle 0,0\rangle$	$\langle 0,0\rangle$

Let $I_m : X \to X$ be defined as identity function. Then it is clear that I_m is temporal intuitionistic fuzzy continuous at time moment t_2. But I_m is not temporal intuitionistic fuzzy continuous at time moments t_1 and t_3. On the other hand, I_m is temporal intuitionistic fuzzy open at time moment t_1. But I_m is not temporal intuitionistic fuzzy continuous at time moments t_2 and t_3.

Theorem 3.3. Let (X,τ_t) and (Y,ϕ_t) be ST-TIFSs accordingly non-empty sets X, Y and time set T respectively, and $f : X \to Y$ be a overall intuitionistic fuzzy continuous function. Then $f : X \to Y$ is intuitionistic fuzzy continuous respectively $(X,\wedge\overline{\tau}_t)$ and $(Y,\wedge\overline{\phi}_t)$.

Proof. Since f is overall intuitionistic fuzzy continuous, it obtained that $\tau_t\left(f^{-1}(B)\right)\geq\phi_t(B)$ for each $t\in T$ and $B\in TIFS^{(Y,T)}$. So the inequalities $\mu_{\tau_t}\left(f^{-1}(B)\right)\geq\mu_{\phi_t}(B)$ and $\eta_{\tau_t}\left(f^{-1}(B)\right)\leq\eta_{\phi_t}(B)$ for each $t\in T$. Therefore $\underset{t\in T}{\wedge}\mu_{\tau_t}\left(f^{-1}(B)\right)\geq\underset{t\in T}{\wedge}\mu_{\phi_t}(B)$ and $\underset{t\in T}{\vee}\eta_{\tau_t}\left(f^{-1}(B)\right)\leq\underset{t\in T}{\vee}\eta_{\phi_t}(B)$. Thus $\wedge\tau_t\left(f^{-1}(B)\right)\geq\wedge\phi_t(B)$. From last inequality, it is understood that f is intuitionistic fuzzy continuous respectively $(X,\wedge\overline{\tau}_t)$ and $(Y,\wedge\overline{\phi}_t)$. □

Theorem 3.4. Let (X,τ_t) and (Y,ϕ_t) be ST-TIFSs accordingly non-empty sets X, Y and time set T respectively, and $f : X \to Y$ be a overall intuitionistic fuzzy continuous open. Then $f : X \to Y$ is intuitionistic fuzzy open respectively $(X,\wedge\overline{\tau}_t)$ and $(Y,\wedge\overline{\phi}_t)$.

Proof. It can be proven as Theorem 3.3. Since temporal intuitionistic fuzzy sets can be seen as intuitionistic fuzzy sets for singleton time sets the properties of image, preimage of TIFS under $f : X \to Y$ are protected as the properties of image, preimage in intuitionistic fuzzy set theory for singleton time moment or singleton time sets. In addition to these properties, we give some fundamental properties of overall intuitionistic fuzzy continuous functions and overall intuitionistic fuzzy open functions. □

Proposition 3.5. Let (X,τ_t) and (Y,ϕ_t) be ST-TIFSs accordingly non-empty sets *X*, *Y* and time set T respectively, and $f : X \to Y$ be a overall intuitionistic fuzzy continuous function. Then;

- **a.** $f\left(cl(A)\right)\subseteq cl\left(f(A)\right)$ for each $A\in TIFS^{(X,T)}$,
- **b.** $cl\left(f^{-1}(B)\right)\subseteq f^{-1}\left(cl(B)\right)$ for each $B\in TIFS^{(Y,T)}$,
- **c.** $f^{-1}\left(\text{int}(B)\right)\subseteq\text{int}\left(f^{-1}(B)\right)$ for each $B\in TIFS^{(Y,T)}$.

Proof.

a. $f^{-1}\left(cl\left(f(A)\right)\right)=f^{-1}\left(\bigcap\left\{K\in TIFS^{(Y,T)}:\wedge\phi_t^*(K)>0^{\sim},f(A)\subseteq K\right\}\right)$

$=\bigcap\left\{f^{-1}(K)\in TIFS^{(X,T)}:\wedge\phi_t^*(K)>0^{\sim},A\subseteq f^{-1}(K)\right\}$

$\supseteq\bigcap\left\{f^{-1}(K)\in TIFS^{(X,T)}:\wedge\tau_t^*\left(f^{-1}(K)\right)>0^{\sim},A\subseteq f^{-1}(K)\right\}\supseteq\bigcap\left\{G\in TIFS^{(X,T)}:\wedge\tau_t^*(G)>0^{\sim},\ A\subseteq G\right\}$

$=cl(A)$. Then, $cl\left(f(A)\right)\supseteq f\left(cl(A)\right)$

b. Since $cl\left(f^{-1}(B)\right)\subseteq f^{-1}\left(f\left(\left(cl\left(f^{-1}(B)\right)\right)\right)\right)$ for each $B\in TIFS(Y,T)$, it is obtained that $cl\left(f^{-1}(B)\right)\subseteq f^{-1}\left(f\left(\left(cl\left(f^{-1}(B)\right)\right)\right)\right)\subseteq f^{-1}\left(cl\left(f\left(f^{-1}(B)\right)\right)\right)$ from (a.). Hence it is obtained that $cl\left(f^{-1}(B)\right)\subseteq f^{-1}\left(cl(B)\right)$.

c. $f^{-1}\left(\text{int}(B)\right)=f^{-1}\left(\bigcup\left\{L:L\in TIFS^{(Y,T)}:\wedge\varphi_t(L)>0^{\sim},L\subseteq B\right\}\right)$

$=f^{-1}\left(\bigcup\left\{L:L\in TIFS^{(Y,T)}:\wedge\varphi_t(L)>0^{\sim},L\subseteq B\right\}\right)$

$\subseteq\left(\bigcup\left\{f^{-1}(L):L\in TIFS^{(Y,T)}:\wedge\tau_t\left(f^{-1}(L)\right)>0^{\sim},f^{-1}(L)\subseteq f^{-1}(B)\right\}\right)$

$\subseteq\left(\bigcup\left\{G:G\in TIFS^{(X,T)}:\wedge\tau_t(G)>0^{\sim},G\subseteq f^{-1}(B)\right\}\right)=\text{int}\left(f^{-1}(B)\right)$. □

Proposition 3.6. Let (X,τ_t) and (Y,ϕ_t) be ST-TIFSs accordingly non-empty sets X, Y and time set T respectively, and $f:X\rightarrow Y$ be a overall intuitionistic fuzzy open function. Then $f\left(\text{int}(A)\right)\subseteq\text{int}\left(f(A)\right)$ for each $A\in TIFS^{(X,T)}$.

Proof. $f\left(\text{int}(A)\right)=f\left(\bigcup\left\{L:L\in TIFS^{(X,T)}:\wedge\tau_t(L)>0^{\sim},L\subseteq A\right\}\right)$

$=\left(\bigcup\left\{f(L):L\in TIFS^{(X,T)}:\wedge\tau_t(L)>0^{\sim},f(L)\subseteq f(A)\right\}\right)$

$\subseteq\left(\bigcup\left\{f(L):L\in TIFS^{(X,T)}:\wedge\varphi_t\left(f(L)\right)>0^{\sim},f(L)\subseteq f(A)\right\}\right)\subseteq\left(\bigcup\left\{G\in TIFS^{(Y,T)}:\wedge\varphi_t(G)>0^{\sim},G\subseteq f(A)\right\}\right)$

$=\text{int}\left(f(A)\right)$. □

Corollary 3.7: Let (X,τ_t) and (Y,ϕ_t) be ST-TIFSs accordingly non-empty sets X, Y and time set T respectively, and $f:X\rightarrow Y$ be a temporal intuitionistic fuzzy continuous function at time moment t. Then the following statements are satisfied for each $\langle\alpha,\beta\rangle\in I_0\times I_1$.

- **a.** $f\left(cl^t_{(\alpha,\beta)}(A)\right)\subseteq cl^t_{(\alpha,\beta)}\left(f(A)\right)$ for each $A\in TIFS^{(X,T)}$,
- **b.** $cl^t_{(\alpha,\beta)}\left(f^{-1}(B)\right)\subseteq f^{-1}\left(cl^t_{(\alpha,\beta)}(B)\right)$ for each $B\in TIFS^{(Y,T)}$,
- **c.** $f^{-1}\left(\text{int}^t_{(\alpha,\beta)}(B)\right)\subseteq\text{int}^t_{(\alpha,\beta)}\left(f^{-1}(B)\right)$ for each $B\in TIFS^{(Y,T)}$.

Corollary 3.8. Let (X,τ_t) and (Y,ϕ_t) be ST-TIFSs accordingly non-empty sets X, Y and time set T respectively, and $f:X\rightarrow Y$ be a temporal intuitionistic fuzzy open function at time moment t. Then $f\left(\text{int}^t_{\langle\alpha,\beta\rangle}(A)\right)\subseteq\text{int}^t_{\langle\alpha,\beta\rangle}\left(f(A)\right)$ for each $\langle\alpha,\beta\rangle\in I_0\times I_1$ and $A\in TIFS^{(X,T)}$.

Corollary 3.9. Let (X,τ_t) and (Y,ϕ_t) be ST-TIFSs accordingly non-empty sets X, Y and time set T respectively, and $f:X\rightarrow Y$ be a temporal intuitionistic fuzzy continuous function at time moment t. Then;

- a. $f\left(cl^t(A)\right)\subseteq cl^t\left(f(A)\right)$ for each $A\in TIFS^{(X,T)}$,
- b. $cl^t\left(f^{-1}(B)\right)\subseteq f^{-1}\left(cl^t(B)\right)$ for each $B\in TIFS^{(Y,T)}$,
- c. $f^{-1}\left(\text{int}^t(B)\right)\subseteq\text{int}^t\left(f^{-1}(B)\right)$ for each $B\in TIFS^{(Y,T)}$.

Corollary 3.10. Let (X,τ_t) and (Y,ϕ_t) be ST-TIFSs accordingly non-empty sets *X*, *Y* and time set *T* respectively, and $f: X \to Y$ be a temporal intuitionistic fuzzy open function at time moment *t*. Then $f(\text{int}(A)) \subseteq \text{int}(f(A))$ for each $A \in TIFS^{(X,T)}$.

4 Some compactness definitions of temporal intuitionistic fuzzy Šostak topology

In this Section, we give some definitions of compactness of a ST-TIFS. Fuzzy and intuitionistic fuzzy types of these definitions were given in [1, 7, 8, 14]. We extend these definitions to ST-TIFS and define some of the extensions that cannot be defined in previous studies because of temporality.

Let (X,τ_t) be a ST-TIFS on non-empty set X and time-moment set *T*.

Definition 4.1. Let $(\alpha^{t_0},\beta^{t_0}) \in (0,1]\times[0,1)$ and $\alpha^{t_0}+\beta^{t_0} \leq 1$ for $t_0 \in T$. Then, (X,τ_t) is called $(\alpha^{t_0},\beta^{t_0})$ – temporal IF-compact ST-TIFS at time moment t_0 if and only if every family of the set $G_{(\alpha^{t_0},\beta^{t_0})} = \{G \in TIFS(X,T) : \tau_{t_0}(G) \geq (\alpha^{t_0},\beta^{t_0})\}$ which is satisfied condition $\bigcup_{G\in G_{(\alpha^{t_0},\beta^{t_0})}} G = 1_{t_0}^{\sim}$ has finite subfamily $G^*_{(\alpha^{t_0},\beta^{t_0})}$ which is satisfied the condition $\bigcup_{G\in G^*_{(\alpha^{t_0},\beta^{t_0})}} G = 1_{t_0}^{\sim}$ where $1_{t_0}^{\sim} = \{(x,1,0) : (x,t_0) \in X\times T\}$

It is obvious that this definition corresponds to the definition of compactness in [14] for singleton time sets. But, in the former definition, we give definition of compactness of a ST-TIFS in specific time moment and openness and non-openness degrees in a ST-TIFS change depending on time moment, so compactness of a ST-TIFS change depending on time. In more explicit words, compactness of a ST-TIFS depends both of selected $(\alpha^{t_0},\beta^{t_0})$ ordered pairs and time moment t_0. Now, we give two new definitions of compactness which are generalized according to time set.

Definition 4.2. If we can find ordered pairs $(\alpha^t,\beta^t) \in (0,1]\times[0,1)$ for each $t \in T$ which are satisfied these conditions:

(a) $\alpha^t+\beta^t \leq 1$ for each $t \in T$

(b) (X,τ_t) is (α^t,β^t) – temporal IF-compact ST-TIFS for $t \in T$

Then (X,τ_t) is called (α^t,β^t) -overall IF-compact ST-TIFS.

Definition 4.3. Let $\alpha^*: T \to (0,1]$ and $\beta^*: T \to [0,1)$ be continuous maps which are satisfied these two conditions:

a. $\alpha^*(t)+\beta^*(t) \leq 1$ for each $t \in T$,

b. (X,τ_t) is $(\alpha^*(t),\beta^*(t))$ – temporal IF-compact for each $t \in T$.

Then (X,τ_t) is called (α^*,β^*) -continuous IF-compact ST-TIFS.

Proposition 4.4. The relationship between these three definitions is described as follows:
(α^*,β^*) – continuous IF-compactness ST-TIFS $\Rightarrow$ (α^t,β^t) -overall IF-compact ST-TIFS $\Rightarrow$ $(\alpha^{t_0},\beta^{t_0})$ – temporal IF-compact ST-TIFS at time moment t_0.

Proof. Let (X,τ_t) be a ST-TIFS on non-empty set X and time set T. Let assume that (X,τ_t) is a $\langle \alpha^*,\beta^* \rangle-$ continuous intuitionistic fuzzy compact ST-TIFS. So, the following statements are provided from the definition.

a. $\alpha^*(t)+\beta^*(t)\le 1$ for each $t\in T$,

b. (X,τ_t) is $(\alpha^*(t),\beta^*(t))-$ temporal IF-compact for each $t\in T$.

If we choose $\alpha^*(t)=\alpha^t$ and $\beta^*(t)=\beta^t$ for each $t\in T$, it is understood that (X,τ_t) is a (α^t,β^t) - overall intuitionistic fuzzy compact ST-TIFS. The second claim of the proposition is clear from the definitions of (α^t,β^t) - overall intuitionistic fuzzy compactness and $(\alpha^{t_0},\beta^{t_0})-$ temporal intuitionistic fuzzy compactness. □

The other definitions of intuitionistic fuzzy compactness which are given in [14] can be extended to ST-TIFS as follows:

Definition 4.5. Let (X,τ_t) be a ST-TIFS on non-empty X and time-moment set T. Then,

(i) If $(\alpha^{t_0},\beta^{t_0})\in(0,1]\times[0,1)$ and $\alpha^{t_0}+\beta^{t_0}\le 1$ for $t_0\in T$. Then, (X,τ_t) called $(\alpha^{t_0},\beta^{t_0})-$ temporal nearly IF-compact ST-TIFS at time moment t_0 if and only if every family of the set $G_{(\alpha^t,\beta^t)}=\{G\in TIFS^{(X,T)}:\tau_{t_0}(G)\ge(\alpha^{t_0},\beta^{t_0})\}$ which satisfied condition $\bigcup_{G\in G_{(\alpha^{t_0},\beta^{t_0})}} G=1^{\sim}_{t_0}$ has finite subfamily $G^*_{(\alpha^{t_0},\beta^{t_0})}$ which is satisfied the condition $\bigcup_{G\in G^*_{(\alpha^{t_0},\beta^{t_0})}} \mathrm{int}_{(\alpha^{t_0},\beta^{t_0})}\left(cl_{(\alpha^{t_0},\beta^{t_0})}(G)\right)=1^{\sim}_{t_0}$.

(ii) If we can find ordered pairs $(\alpha^t,\beta^t)\in(0,1]\times[0,1)$ for each $t\in T$ which are satisfied these conditions:

(a) $\alpha^t+\beta^t\le 1$ for each $t\in T$,

(b) (X,τ_t) is $(\alpha^t,\beta^t)-$ temporal nearly IF-compact ST-TIFS for (α^t,β^t),

Then (X,τ_t) is called (α^*,β^*)-overall nearly IF-compact ST-TIFS.

(iii) Let $\alpha^*:T\to(0,1]$ and $\beta^*:T\to[0,1)$ are continuous maps which are satisfied these two conditions:

(a) $\alpha^*(t)+\beta^*(t)\le 1$ for each $t\in T$

(b) (X,τ_t) is $(\alpha^*(t),\beta^*(t))-$ temporal nearly IF-compact for each $t\in T$.

Then (X,τ_t) is called (α^*,β^*)-continuous nearly IF-compact ST-TIFS.

Definition 4.6. Let (X,τ_t) be a ST-TIFS on non-empty X and time set T. Then,

(I) Let $(\alpha^t,\beta^t)\in(0,1]\times[0,1)$ and $\alpha^t+\beta^t\le 1$. Then, (X,τ_t) called $(\alpha^{t_0},\beta^{t_0})-$ temporal almost IF-compact ST-TIFS at time moment t if and only if every family of the set $G_{(\alpha^t,\beta^t)}=\{G\in TIFS^{(X,T)};\tau_{t_0}(G)\ge(\alpha^{t_0},\beta^{t_0})\}$ which satisfied condition $\bigcup_{G\in G_{(\alpha^{t_0},\beta^{t_0})}} G=1^{\sim}_{t_0}$ has finite subfamily $G^*_{(\alpha^{t_0},\beta^{t_0})}$ which is satisfied the condition $\bigcup_{G\in G^*_{(\alpha^{t_0},\beta^{t_0})}}\left(cl_{(\alpha^{t_0},\beta^{t_0})}(G)\right)=1^{\sim}_{t_0}$.

(II) If we can find ordered pairs $(\alpha^t,\beta^t)\in(0,1]\times[0,1)$ for each $t\in T$ which are satisfied these conditions:

(a) $\alpha^t + \beta^t \le 1$ for each $t \in T$,

(b) (X,τ_t) is (α^t,β^t) – temporal almost IF-compact ST-TIFS for (α^t,β^t),

then (X,τ_t) is called (α^*,β^*)-overall almost IF-compact ST-TIFS.

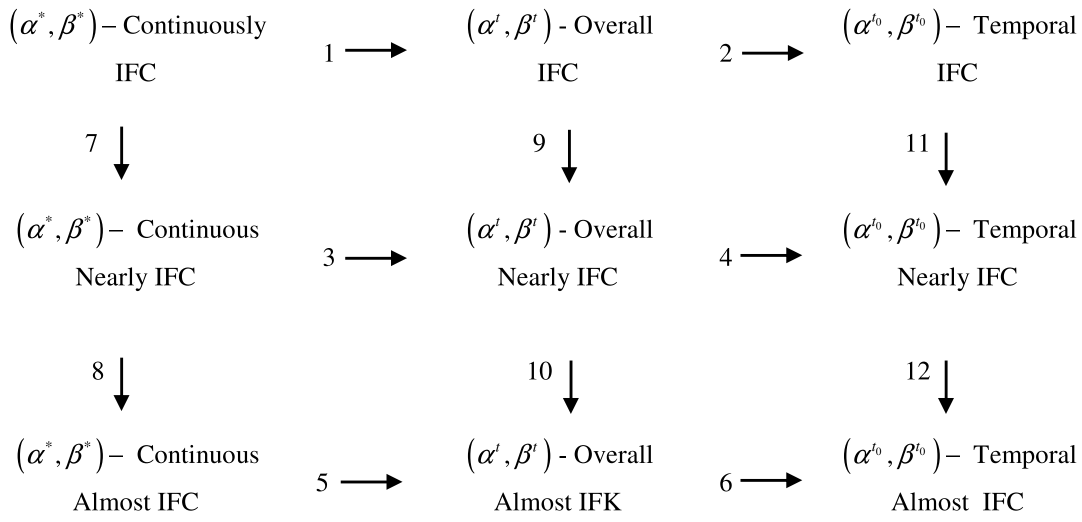
(III) Let $\alpha^*: T \to (0,1]$ and $\beta^*: T \to [0,1)$ are continuous maps which are satisfied these two conditions:

(a) $\alpha^*(t) + \beta^*(t) \le 1$ for each $t \in T$

(b) (X,τ_t) is $(\alpha^*(t),\beta^*(t))$ – temporal almost IF-compact for each $t \in T$

Then (X,τ_t) is called (α^*,β^*)-continuous almost IF-compact ST-TIFS .

Theorem 4.7. The Following diagram shows relationship between compactness definitions of a ST-TIFS (X,τ_t) : (IFC: Intuitionistic Fuzzy Compactness)



Proof. 1st and 2nd relationships have been provided in Proposition 4.4. Now we provide 3rd relation. Since (X,τ_t) is a $\langle\alpha^*,\beta^*\rangle$ – continuous nearly IFC, each family $G_{\langle\alpha^*(t),\beta^*(t)\rangle} = \{G \in TIFS^{(X,T)} : \tau_t(G) > \langle\alpha^*(t),\beta^*(t)\rangle\}$ which satisfy the condition $\bigcup_{G \in G_{\langle\alpha^*(t),\beta^*(t)\rangle}} G = 1_t^\sim$ for $\alpha^*: T \to I_0$, $\beta^*: T \to I_1$ functions and $t \in T$ has a subfamily $G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}$ which satisfy that $\bigcup_{G \in G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}} \text{int}_{\langle\alpha^*(t),\beta^*(t)\rangle}\left(cl_{\langle\alpha^*(t),\beta^*(t)\rangle}(G)\right) = 1_t^\sim$. If we can choose $\alpha^*(t) = \alpha^t$ and $\beta^*(t) = \beta^t$ for each $t \in T$ it is obtained that $\bigcup_{G \in G^*_{\langle\alpha^t,\beta^t\rangle}} \text{int}_{\langle\alpha^t,\beta^t\rangle}\left(cl_{\langle\alpha^t,\beta^t\rangle}(G)\right) = 1_t^\sim$. So it is understood that (X,τ_t) is a $\langle\alpha^t,\beta^t\rangle$-overall nearly IFC. On the other hand 4th relation is clear from Definition 4.5. 5th and 6th relations can be provided as 3th and 4th relations. Now, we provide 7th relation. Since (X,τ_t) is a $\langle\alpha^*,\beta^*\rangle$ – continuous IFC ST-TIFS for $\alpha^*: T \to I_0$, $\beta^*: T \to I_1$ functions and each $t \in T$, each

$G_{\langle\alpha^*(t),\beta^*(t)\rangle} = \{G\in TIFS^{(X,T)} : \tau_t(G) > \langle\alpha^*(t),\beta^*(t)\rangle\}$ family which satisfies that $\bigcup_{G\in G_{\langle\alpha^*(t),\beta^*(t)\rangle}} G = 1_t^{\sim}$ has a subfamily $G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}$ which satisfies that condition $\bigcup_{G\in G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}} G = 1_t^{\sim}$. Since $\tau_t(G) > \langle\alpha^*(t),\beta^*(t)\rangle$, it is obtained that $G = \mathrm{int}_{\langle\alpha^*(t),\beta^*(t)\rangle}(G)$. Since $G = \mathrm{int}_{\langle\alpha^*(t),\beta^*(t)\rangle}(G) \subseteq \mathrm{int}_{\langle\alpha^*(t),\beta^*(t)\rangle}\left(cl_{\langle\alpha^*(t),\beta^*(t)\rangle}(G)\right)$, it is obtained that $1_t^{\sim} = \bigcup_{G\in G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}} G \subseteq \bigcup_{G\in G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}} \mathrm{int}_{\langle\alpha^*(t),\beta^*(t)\rangle}\left(cl_{\langle\alpha^*(t),\beta^*(t)\rangle}(G)\right)$. Hence it is clear that $1_t^{\sim} = \bigcup_{G\in G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}} \mathrm{int}_{\langle\alpha^*(t),\beta^*(t)\rangle}\left(cl_{\langle\alpha^*(t),\beta^*(t)\rangle}(G)\right)$. So each family $G_{\langle\alpha^*(t),\beta^*(t)\rangle}$ $= \{G\in TIFS^{(X,T)} : \tau_t(G) > \langle\alpha^*(t),\beta^*(t)\rangle\}$ which satisfies that $\bigcup_{G\in G_{\langle\alpha^*(t),\beta^*(t)\rangle}} G = 1_t^{\sim}$ has a subfamily $G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}$ which satisfies that $1_t^{\sim} = \bigcup_{G\in G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}} \mathrm{int}_{\langle\alpha^*(t),\beta^*(t)\rangle}\left(cl_{\langle\alpha^*(t),\beta^*(t)\rangle}(G)\right)$. Thus it is understood that (X,τ_t) is a $\langle\alpha^*,\beta^*\rangle-$continuous nearly IFC ST-TIFS. Now, we provide 9th relation. Let (X,τ_t) be a $\langle\alpha^*,\beta^*\rangle-$continuous nearly IFC ST-TIFS for $\alpha^* : T\to I_0$, $\beta^* : T\to I_1$ continuous functions and each $t\in T$. Each $G_{\langle\alpha^*(t),\beta^*(t)\rangle} = \{G\in TIFS^{(X,T)} : \tau_t(G) > \langle\alpha^*(t),\beta^*(t)\rangle\}$ for $\langle\alpha^*(t),\beta^*(t)\rangle$-IF pair which satisfies the condition $\bigcup_{G\in G_{\langle\alpha^*(t),\beta^*(t)\rangle}} G = 1_t^{\sim}$ has a subfamily $G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}$ which satisfies that $\bigcup_{G\in G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}} \mathrm{int}_{\langle\alpha^*(t),\beta^*(t)\rangle}\left(cl_{\langle\alpha^*(t),\beta^*(t)\rangle}(G)\right) = 1_t^{\sim}$. Since $\mathrm{int}_{\langle\alpha^*(t),\beta^*(t)\rangle}\left(cl_{\langle\alpha^*(t),\beta^*(t)\rangle}(G)\right) \subseteq cl_{\langle\alpha^*(t),\beta^*(t)\rangle}(G)$, it is obtained that $\bigcup_{G\in G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}} cl_{\langle\alpha^*(t),\beta^*(t)\rangle}(G) = 1_t^{\sim}$. Hence it is understood that (X,τ_t) is a $\langle\alpha^*,\beta^*\rangle-$continuous almost IFC ST-TIFS. 9th, 10th, 11th and 12th relationships can be proven as 7th and 8th. □

We can get more general compactness definitions by replacing $\mathrm{int}_{(\alpha^{t_0},\beta^{t_0})}$ and $cl_{(\alpha^{t_0},\beta^{t_0})}$ operators with int^t and cl^t operators respectively in Definition 2.11. We call these new compactness definitions as:

(1) temporal IF-compact , overall IF compact, continuous IF compact ;
(2) temporal nearly IF-compact , overall nearly IF compact, continuous nearly IF compact
(3) temporal almost IF-compact, overall almost IF-compact and continuous almost IF-compact,

respectively.

In this section, we purpose some definitions of compactness for a TIFS in ST-TIFS. Let (X,τ_t) be a ST-TIFS on X and time set T, $t_0\in T$. It has to remind again that following definitions are equivalent to definitions in [14] for singleton time set.

Definition 4.7. Let $(\alpha^t,\beta^t)\in(0,1]\times[0,1)$ and $\alpha^t+\beta^t\le 1$ for any $t\in T$.If the set $C_{(\alpha^t,\beta^t)}(A) = \{C\in TIFS^{(X,T)}; \tau_t(C)\ge(\alpha^t,\beta^t)\}$ satisfies the condition: $A\subseteq \bigcup_{C\in C_{(\alpha^t,\beta^t)}(A)} C$, then $C_{(\alpha^t,\beta^t)}(A)$ is called "temporal (α^t,β^t)-cover of A" at time moment t. Subfamily $C^*_{(\alpha^t,\beta^t)}(A)$ of $C_{(\alpha^t,\beta^t)}(A)$ preserves the condition $A\subseteq \bigcup_{C\in C_{(\alpha^t,\beta^t)}(A)} C$, then $C^*_{(\alpha^t,\beta^t)}(A)$ is called temporal subfamily of $C_{(\alpha^t,\beta^t)}(A)$ at time moment t.

Definition 4.8. Let (X,τ_t) be a ST-TIFS and $A\in TIFS^{(X,T)}$. Then,

(I) Let $(\alpha^{t_0},\beta^{t_0})\in(0,1]\times[0,1)$ and $\alpha^{t_0}+\beta^{t_0}\le 1$ for $t_0\in T$. If every $(\alpha^{t_0},\beta^{t_0})-$ temporal cover of A contain finite $(\alpha^{t_0},\beta^{t_0})-$temporal subcover at time moment t_0, A is called $(\alpha^{t_0},\beta^{t_0})-$ temporal compact at time moment t.

(II) If we can find ordered pairs $(\alpha^t,\beta^t)\in(0,1]\times[0,1)$ which are satisfied $\alpha^t+\beta^t\le 1$ for each $t\in T$ which makes A is $(\alpha^t,\beta^t)-$temporal compact at time moment t, A is called (α^t,β^t) -overall IF-compact TIFS.

(III) Let $\alpha^*:T\to(0,1]$ and $\beta^*:T\to[0,1)$ are continuous maps which are satisfied these two conditions:

a. $\alpha^*(t)+\beta^*(t)\le 1$ for each $t\in T$

b. A is $(\alpha^*(t),\beta^*(t))-$temporal IF-compact for each $t\in T$

Then, A is called (α^*,β^*)-continuous IF-compact TIFS.

Proposition 4.9. Relationship between these three definitions is described as follows:
$(\alpha^*,\beta^*)-$continuous compactness TIFS $\Rightarrow$ (α^t,β^t)-overall compact TIFS $\Rightarrow$ $(\alpha^{t_0},\beta^{t_0})-$ temporal compact TIFS

Proposition 4.10. Let (X,τ_t) and (Y,ϕ_t) be ST-TIFSs where non-empty sets X, Y, time set T, respectively and $f:X\to Y$ be a surjective temporal intuitionistic fuzzy continuous mapping. IF (X,τ_t) is $(\alpha^t,\beta^t)-$ continuous ($(\alpha^t,\beta^t)-$overall, $(\alpha^t,\beta^t)-$temporal) intuitionistic fuzzy compact then (Y,ϕ_t) is continuous (overall, temporal) intuitionistic fuzzy compact.

Proof. Since (X,τ_t) is a $(\alpha^*,\beta^*)-$continuous IFC ST-TIFS, there exists $\alpha^*:T\to I_0$ ve $\beta^*:T\to I_1$ continuous functions. Now, we must find a finite subfamily $G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}$ which satisfies that $\bigcup_{G\in G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}} G=1_t^{\sim}$ for every family $G_{\langle\alpha^*(t),\beta^*(t)\rangle}=\{G\in TIFS^{(Y,T)}:\phi_t(G)>\langle\alpha^*(t),\beta^*(t)\rangle\}$ where $t\in T$ and $\langle\alpha^*(t),\beta^*(t)\rangle$-IF pair which satisfies the condition $\bigcup_{G\in G_{\langle\alpha^*(t),\beta^*(t)\rangle}} G=1_t^{\sim}$. Since f is a surjective and IF continuous function, it is obtained that $f^{-1}\left(\bigcup_{G\in G_{\langle\alpha^*(t),\beta^*(t)\rangle}} G\right)=\bigcup_{G\in G_{\langle\alpha^*(t),\beta^*(t)\rangle}} f^{-1}(G)$ and $\tau_t(f^{-1}(G))\ge\phi_t(G)>\langle\alpha^*(t),\beta^*(t)\rangle$. Since (X,τ_t) is a $(\alpha^*,\beta^*)-$continuous IFC ST-TIFS the family $G'_{\langle\alpha^*(t),\beta^*(t)\rangle}=\{f^{-1}(G):G\in G_{\langle\alpha^*(t),\beta^*(t)\rangle},\ \tau_t(f^{-1}(G))>\langle\alpha^*(t),\beta^*(t)\rangle\}$ which satisfies the condition $\bigcup_{f^{-1}(G)\in G'_{\langle\alpha^*(t),\beta^*(t)\rangle}} G=1_t^{\sim}$ has got a subfamily $G''_{\langle\alpha^*(t),\beta^*(t)\rangle}$ which satisfies the condition $\bigcup_{G\in G''_{\langle\alpha^*(t),\beta^*(t)\rangle}} G=1_t^{\sim}$. So it is obtained that $f\left(\bigcup_{f^{-1}(G)\in G''_{\langle\alpha^*(t),\beta^*(t)\rangle}} f^{-1}(G)\right) = \bigcup_{f^{-1}(G)\in G''_{\langle\alpha^*(t),\beta^*(t)\rangle}} f\left(f^{-1}(G)\right) = \bigcup_{f^{-1}(G)\in G''_{\langle\alpha^*(t),\beta^*(t)\rangle}} G = 1_t^{\sim}$. Thus $G^*_{\langle\alpha^*(t),\beta^*(t)\rangle} = \left\{G : f^{-1}(G)\in G''_{\langle\alpha^*(t),\beta^*(t)\rangle},\ \tau_t\left(f^{-1}(G)\right) > \langle\alpha^*(t),\beta^*(t)\rangle\right\}$ is finite subfamily of $G_{\langle\alpha^*(t),\beta^*(t)\rangle} = \left\{G\in TIFS^{(Y,T)} : \phi_t(G) > \langle\alpha^*(t),\beta^*(t)\rangle\right\}$ which satisfies that $\bigcup_{G\in G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}} G = 1_t^{\sim}$. Therefore, it is understood that (Y,ϕ_t) is a (α^*,β^*) – continuous IFC ST-TIFS. The other claims can be proven as above. □

Proposition 4.11. Let (X,τ_t) and (Y,ϕ_t) be ST-TIFSs where X, Y are non-empty sets, T' and T'' are time sets, respectively and $f : X \to Y$ intuitionistic fuzzy continuous mapping. If $A\in TIFS^{(X,T)}$ is (α^t,β^t) – continuous ((α^t,β^t) – overall, (α^t,β^t) – temporal) intuitionistic fuzzy compact then $f(A)$ is (α^t,β^t) – continuous ((α^t,β^t) – overall, (α^t,β^t) – temporal) intuitionistic fuzzy compact.

Proof. Since $A\in TIFS^{(X,T)}$ is a (α^*,β^*) – continuous IFC TIFS, there exists $\alpha^* : T\to I_0$ and $\beta^* : T\to I_1$ continuous functions. Now we must show that $f(A)\in TIFS^{(Y,T)}$ is a (α^*,β^*) – continuous IFC. The family $G_{\langle\alpha^*(t),\beta^*(t)\rangle} = \left\{G\in TIFS^{(Y,T)} : \phi_t(G) > \langle\alpha^*(t),\beta^*(t)\rangle\right\}$ for each $t\in T$ and $\langle\alpha^*(t),\beta^*(t)\rangle$-IF pair which satisfies that $f(A)\subseteq \bigcup_{G\in G_{\langle\alpha^*(t),\beta^*(t)\rangle}} G$. We can obtain that

$A\subseteq f^{-1}\left(f(A)\right)\subseteq f^{-1}\left(f(A)\right)\subseteq f^{-1}\left(\bigcup_{G\in G_{\langle\alpha^*(t),\beta^*(t)\rangle}} G\right)\subseteq \bigcup_{G\in G_{\langle\alpha^*(t),\beta^*(t)\rangle}} f^{-1}(G)$. Since f is a intuitionistic fuzzy continuous, it is obtained that $\tau_t\left(f^{-1}(G)\right)\geq\phi_t(G) > \langle\alpha^*(t),\beta^*(t)\rangle$. Thus we can find finite subfamily $G''_{\langle\alpha^*(t),\beta^*(t)\rangle}$ of

$$G'_{\langle\alpha^*(t),\beta^*(t)\rangle} = \left\{f^{-1}(G)\in TIFS^{(X,T)} : G\in G_{\langle\alpha^*(t),\beta^*(t)\rangle},\ \tau_t\left(f^{-1}(G)\right) > \langle\alpha^*(t),\beta^*(t)\rangle\right\}$$

which satisfies that $A\subseteq \bigcup_{G\in G'_{\langle\alpha^*(t),\beta^*(t)\rangle}} f^{-1}(G)$ such that $A\subseteq \bigcup_{f^{-1}(G)\in G''_{\langle\alpha^*(t),\beta^*(t)\rangle}} f^{-1}(G)$. From the last expression,

it is obtained that $f(A)\subseteq f\left(\bigcup_{f^{-1}(G)\in G''_{\langle\alpha^*(t),\beta^*(t)\rangle}} f^{-1}(G)\right) = \bigcup_{f^{-1}(G)\in G''_{\langle\alpha^*(t),\beta^*(t)\rangle}} f\left(f^{-1}(G)\right)\subseteq \bigcup_{f^{-1}(G)\in G''_{\langle\alpha^*(t),\beta^*(t)\rangle}} G$. Therefore it is understood that the finite subfamily $G^*_{\langle\alpha^*(t),\beta^*(t)\rangle} = \left\{G\in TIFS^{(Y,T)} : f^{-1}(G)\in G'_{\langle\alpha^*(t),\beta^*(t)\rangle},\ \phi_t(G) > \langle\alpha^*(t),\beta^*(t)\rangle\right\}$ of $G_{\langle\alpha^*(t),\beta^*(t)\rangle}$ satisfies that $f(A)\subseteq \bigcup_{G\in G^*_{\langle\alpha^*(t),\beta^*(t)\rangle}} G$. Thus it is understood that $f(A)$ is a (α^*,β^*) – continuous IFC TIFS. □

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