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On probabilities on IF-sets and MV-algebras

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Abstract

A new proof is presented for the general form of probability on the generated MV-algebras by the help of the representation theorem for probabilities of the family of IF-events generating the MV-algebra..

Keywords: IF-sets, MV-algebras, probability

Definition 1: Let (Ω,S) be the measurable space where S is a σ - algebra. Let T be the tribe of all S – measurable functions $f:\Omega \longrightarrow \langle 0,1\rangle$. Define the set F as follows:

$$F = \{(\mu_A, \nu_A), \mu_A, \nu_A \in T, \mu_A + \nu_A \le 1\}$$

Also we will use the following definitions:

$\langle a,b\rangle + \langle c,d\rangle = \langle a+c, b+d\rangle$

$\langle a_n, b_n\rangle \nearrow \langle a,b\rangle \Leftrightarrow a_n \nearrow a \wedge b_n \nearrow b$

$(\mu_{A_n}, \nu_{A_n}) \nearrow (\mu_A, \nu_A) \Leftrightarrow \mu_{A_n} \nearrow \mu_A \wedge \nu_{A_n} \searrow \nu_A$

$\mu_A \oplus \mu_B = \min(\mu_A + \mu_B, 1)$

$\mu_A \otimes \mu_B = \max(\mu_A + \mu_B - 1, 0)$

$(\mu_A, \nu_A) \oplus (\mu_B, \nu_B) = (\mu_A \oplus \mu_B, \nu_A \otimes \nu_B)$

$(\mu_A, \nu_A) \otimes (\mu_B, \nu_B) = (\mu_A \otimes \mu_B, \nu_A \oplus \nu_B)$

Definition 2: IF probability on F is a mapping $P: F \longrightarrow I$ (I is the family of all compact intervals in R) satisfiing the following conditions:

(*i*) $P((0,1)) = \langle 0,0\rangle, \quad P((1,0)) = \langle 1,1\rangle$

(*ii*) $P((\mu_A, \nu_A)) + P((\mu_B, \nu_B)) = P((\mu_A, \nu_A) \oplus (\mu_B, \nu_B)) + P((\mu_A, \nu_A) \otimes (\mu_B, \nu_B))$

(*iii*) $(\mu_{A_n}, \nu_{A_n}) \nearrow (\mu_A, \nu_A) \Rightarrow P((\mu_{A_n}, \nu_{A_n})) \nearrow P((\mu_A, \nu_A))$

Theorem 1: To any probability $P: F \longrightarrow I$ there exists real numbers α and β such that $0 \le \alpha \le \beta \le 1$ and:

$$P((\mu_A, \nu_A)) = \left\langle (1-\alpha)\int_\Omega \mu_A dP + \alpha \int_\Omega (1-\nu_A) dP, (1-\beta)\int_\Omega \mu_A dP + \beta \int_\Omega (1-\nu_A) dP \right\rangle.$$

Proof: see [3].

Definition 3: MV algebra is the system $(M, \oplus, \otimes, \neg, 0, u)$, if:

$\oplus$, $\otimes$ are binary operations, $\oplus$ is comutative and associative, $\neg$ is unary operation, 0 and u are from set M and for any $a \in M$: $a \oplus 0 = a$, $a \oplus u = u$, $\neg(\neg a) = a$, $\neg 0 = u$, $a \oplus (\neg a) = u$, $\neg(\neg a \oplus b) \oplus b = \neg(a \oplus \neg b) \oplus a$, $a \otimes b = \neg(\neg a \oplus \neg b)$.

Lemma: Let M be the set $M = \{(\mu_A, \nu_A), \mu_A, \nu_A$ are S-measurable, $\mu_A, \nu_A : \Omega \longrightarrow \langle 0,1 \rangle\}$

and operations $\oplus$, $\otimes$, $\neg$ are defined as follows:

$$A \oplus B = (\mu_A \oplus \mu_B, \nu_A \otimes \nu_B)$$
$$A \otimes B = (\mu_A \otimes \mu_B, \nu_A \oplus \nu_B)$$
$$\neg(\mu_A, \nu_A) = (1-\mu_A, 1-\nu_A)$$

Then $(M, \oplus, \otimes, \neg, 0, 1)$ is an MV algebra where $0 = (0,1)$ and $1 = (1,0)$.

Proof. See [2]

We can define probability on set M analogously to Definition 1. If we consider the set $J = \{[a,a];\ a$ is a real number$\}$, then the mapping $P: F \longrightarrow I$ called a state.

Theorem 2: To any state $p: F \longrightarrow \langle 0,1 \rangle$ exists exactly one state $\bar{p}: M \longrightarrow \langle 0,1 \rangle$ such that $\bar{p} \mid F = p$.

Proof. Define $\bar{p}((\mu_A, \nu_A)) = p((\mu_A, 0)) - p((0, 1-\nu_A))$. The proof of all properties of $\bar{p}$ is straightforward.

Theorem 3: To any IF probability $P: F \longrightarrow I$ there exists exactly one probability $\bar{P}: M \longrightarrow I$ such that $\bar{P} \mid F = P$.

Proof. For any probability $P: F \longrightarrow I$ we can put

$$P(A) = \langle P^{(1)}(A), P^{(2)}(A) \rangle,$$

where $P^{(1)}$, $P^{(2)}$ are states on F. For the states we have exactly one state $\bar{P}^{(1)}, \bar{P}^{(2)}$ on M extending $P^{(1)}$, $P^{(2)}$. Therefore the mapping

$$\bar{P}(A) = \langle \bar{P}^{(1)}(A), \bar{P}^{(2)}(A) \rangle$$

is a probability on M extending P. .

Theorem 6: To any probability on M $P : M \longrightarrow I$ there exists real numbers α and β, such that $0 \le \alpha \le \beta \le 1$ and

$$P((\mu_A, \nu_A)) = \left\langle (1-\alpha)\int_\Omega \mu_A dP + \alpha \int_\Omega (1-\nu_A) dP, (1-\beta)\int_\Omega \mu_A dP + \beta \int_\Omega (1-\nu_A) dP \right\rangle.$$

Proof. It is a consequence of Theorems 1 and 3.

References

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