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Generalized Net Models of Queueing Disciplines in Finite Buffer Queueing Systems

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Abstract: Generalized net models of different queueing disciplines in a queueing system are proposed. In each of the models, the buffer has finite capacity and is represented by two Generalized net transitions. The buffer cells are represented by places of the net. The two simple queueing disciplines considered are FIFO and LIFO while the apparatus of the Generalized nets allows for other queueing disciplines such as priority queueing and weighted-fair queueing to be modeled through changes in the predicates of the transitions condition and the characteristics of specific tokens which are used to specify the type of queueing. More general models in which the requests can change their parameters and position within the buffer are also proposed.
Keywords: Generalized net, Queueing system, Queueing discipline.
AMS Classification: 68Q85.

1 Introduction

The idea of modelling of the processes in buffers (entering and leaving of requests/pieces of information) by Petri nets lead to the introduction of the Buffer-Transition Nets of Hans Fuss [3]. They represent exactly the buffer functioning, following the procedures of queueing theory (see, e.g., [4–6]).

Here, Generalized Net (GN, see [1,2]) models of these processes will be described, but except for the two standard procedures, new ones will be introduced.

Let us have the source of data G (generator of pieces of information/requests), buffer B with n cells $b_1, ..., b_n$ and Server denoted by U (see Fig. 1).

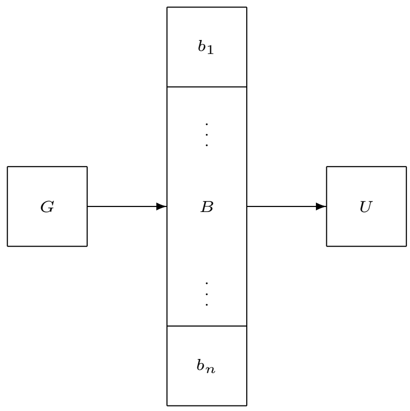

Figure1. Conceptual representation of a buffer.

All notations related to GNs used below are taken from [1, 2].

For brevity, here we use only reduced GNs without priorities of the transitions, places and tokens, without temporal components and capacities of the places corresponding to the buffer cells equal to 1, while the capacities of the rest places are not specified, i.e., they can be ∞. The arcs capacities and transition types are not defined.

2 Generalized net models of simple queueing disciplines

In [7] a GN model of a queueing system in service networks with FIFO queueing discipline is proposed. Here we propose an alternative GN representation. The simple queueing disciplines can be represented by a GN containing only two transitions (see Fig. 2), where as above, n is the number of buffer cells (capacity of the buffer), that are represented here by GN-places.

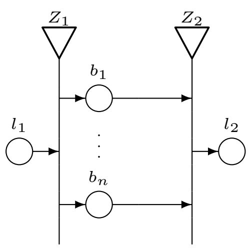

Figure 2. GN representation of simple queueing disciplines.

The transitions of the first GN-model are the following:

$$Z_1 = \langle \{l_1\}, \{b_1, ..., b_n\}, r_1 \rangle,$$

where

$$r_1 = \begin{array}{c|ccccc} & b_1 & \dots & b_j & \dots & b_n \\ \hline l_1 & r_{1,1} & \dots & r_{1,j} & \dots & r_{1,n} \end{array};$$

$r_{1,j}$ = "b_j is the first empty place from top to bottom among places $b_1, b_2, ..., b_n$", where here and below $1 \leq j \leq n$.

$$Z_2 = \langle \{b_1, ..., b_n\}, \{l_2\}, r_2 \rangle,$$

where

$$r_2 = \begin{array}{c|c} & l_2 \\ \hline b_1 & s_{1,2} \\ \vdots & \vdots \\ b_j & s_{j,2} \\ \vdots & \vdots \\ b_n & s_{n,2} \end{array};$$

$s_{j,2}$ = "the token in place b_j has stayed for the more time in the place compared to the tokens in all places $b_1, b_2, ..., b_n$ ", for $j = 1, 2, ..., n$.

In this way, we describe the queueing discipline "first enters – first comes out" (FIFO).

The second GN model, describing the other simple queueing discipline – "last enters – first comes out" (LIFO) has the same graphical representation as in Fig. 2 and the difference is only in the form of the predicates, that now are, respectively

$r_{i,j}$ = "b_j is the first empty place from top to bottom among places $b_1, b_2, ..., b_n$", for $j = 1, 2, ..., n$,

$s_{j,2}$ = " the token in place b_j has stayed in the place for less time compared to the tokens in places $b_1, b_2, ..., b_n$ ", for $j = 1, 2, ..., n$.

The existence of the GN-transition predicates gives possibility to describe more complex queueing systems, changing again only the forms of the predicates, as follows:

$r_{i,j}$ = "b_j is the first empty place from top to bottom among places $b_1, b_2, ..., b_n$ and satisfies condition C_1", for $j = 1, 2, ..., n$,

$s_{j,2}$ = "the token in place b_j has stayed in the place for more (less) time in the place compared to the tokens in places $b_1, b_2, ..., b_n$ and satisfies condition C_2",

where C_1 and C_2 are some conditions given at the beginning.

In this way, we have described the third and the fourth GN-models.

For example, if C_1 and C_2 are some tautologies, we obtain the first two cases of standard procedures (first and second GN-models, respectively).

In the above four GN models are not interested in the characteristics of the tokens entering l- and b-places. They are given by the generator of data and the tokens keep them without changes during their transfer in the GN. Also, in order to determine the duration of staying of the tokens in places $b_1, b_2, ..., b_n$, here and in all GN models below we consider that upon entering the places corresponding to the buffer cells, the tokens obtain as characteristic the time-moment of entering.

3 Generalized net models of more general queueing disciplines

The above GN models are static, in sense that in them the conditions for the transfer of tokens are fixed from the beginning. Here, we propose a fifth and a sixth GN models in which the additional criteria for transfer of tokens can be changed during the functioning of the net.

Let tokens γ_1 and γ_2 enter places c_1 and c_2 of the fifth GN which is shown in Fig. 3 with initial characteristic

$$x_0^{\gamma_i} = \text{“criterion } C_i \text{ for choice of a token”}, i = 1, 2$$

respectively.

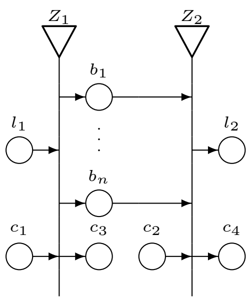

Figure 3. GN representation of more general queueing disciplines.

Now, the transitions have the forms:

$$Z_1 = \langle \{l_1, c_1\}, \{b_1, ..., b_n, c_3\}, r_1 \rangle,$$

where

$$r_1 = \begin{array}{c|cccccc} & b_1 & \dots & b_j & \dots & b_n & c_3 \\ \hline l_1 & r_{1,1} & \dots & r_{1,j} & \dots & r_{1,n} & false \\ c_1 & false & \dots & false & \dots & false & true \end{array};$$

$r_{1,j}$ = “b_j is the first empty place from top to bottom among places $b_1, b_2, ..., b_n$” & $x_0^{\gamma_1}$.

$$Z_2 = \langle \{b_1, ..., b_n, c_2\}, \{l_2, c_4\}, r_2 \rangle,$$

where

$$r_2 = \begin{array}{c|cc} & l_2 & c_4 \\ \hline b_1 & s_{1,2} & false \\ \vdots & \vdots & \vdots \\ b_j & s_{j,2} & false \\ \vdots & \vdots & \vdots \\ b_n & s_{n,2} & false \\ c_2 & false & true \end{array};$$

$s_{j,2}$ = “the token in place b_j has stayed in the place for more time compared to the tokens in places $b_1, b_2, ..., b_n$ ” & $x_0^{\gamma_2}$.

In this way, we have described again the queueing discipline "first enters – first comes out" (FIFO).

The sixth GN model, describes the other simple queueing discipline. It has the same graphical representation as in Fig. 3 and the predicates are

$r_{1,j} =$ "b_j is the first empty place from top to bottom among places $b_1, b_2, ..., b_n$" & $x_0^{\gamma_1}$,

$s_{j,2} =$ " the token in place b_j has stayed in the place for less time compared to the tokens in the rest of the places $b_1, b_2, ..., b_n$" & $x_0^{\gamma_2}$, for $j = 1, 2, ..., n$.

In the special case when tokens γ_1 and γ_2 enter the GN in the first time-moment and they do not change their initial characteristics during the GN-functioning, we obtain the third and the fourth GN models (from the previous section). In the general case, the queueing discipline can be changed in each time-moment of GN-functioning.

Now, we will discuss a GN model in which the tokens can change their places within the buffer (the places $b_1, b_2, ..., b_n$), according to some criterion D. This GN is shown in Fig. 4. In some moments, token δ will enter place d_1 with initial characteristic

$$x_0^\delta = \text{"criterion } D \text{ for change of token places"}.$$

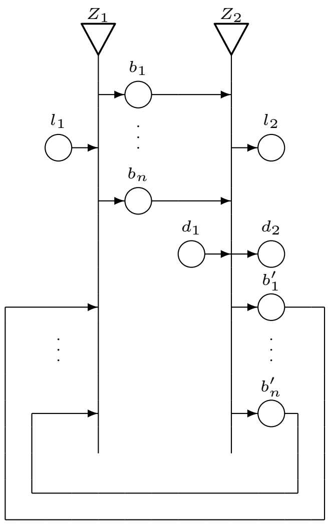

Figure 4. GN representation of queueing discipline with requests, changing their positions within the buffer.

As above, the present model has two forms, corresponding to the simple queueing disciplines from the previous section (FIFO, LIFO and the "C-criterion"). We will mention only the differences in the respective predicates.

The GN-transitions have the forms:

$$Z_1 = \langle \{l_1, b'_1, ..., b'_n\}, \{b_1, ..., b_n\}, r_1 \rangle,$$

where

$$r_1 = \begin{array}{c|ccccc} & b_1 & \dots & b_j & \dots & b_n \\ \hline l_1 & r_{1,1} & \dots & r_{1,j} & \dots & r_{1,n} \\ b'_1 & t_{1,1} & \dots & t_{1,j} & \dots & t_{1,n} \\ \vdots & \vdots & \dots & \ddots & \dots & \vdots \\ b'_i & t_{i,1} & \dots & t_{i,j} & \dots & t_{i,n} \\ \vdots & \vdots & \dots & \ddots & \dots & \vdots \\ b'_n & t_{n,1} & \dots & t_{n,j} & \dots & t_{n,n} \end{array};$$

$r_{1,j}$ = "b_j is the first empty place from top to bottom among places $b_1, b_2, ..., b_n$",
$t_{i,j}$ = "following criterion D from x_0^δ, the current token has to be transferred transfer from place b'_i to place b_j", where $1 \leq i \leq n$ and $1 \leq j \leq n$.

$$Z_2 = \langle \{b_1, ..., b_n, d_1\}, \{l_2, d_2, b'_1, ..., b'_n\}, r_2 \rangle,$$

where

$$r_2 = \begin{array}{c|ccccccc} & l_2 & d_2 & b'_1 & \dots & b'_j & \dots & b'_n \\ \hline b_1 & s_{1,2} & false & u_{1,1} & \dots & false & \dots & false \\ \vdots & \vdots & \vdots & \dots & \ddots & \dots & \vdots & \\ b_j & s_{j,2} & false & false & \dots & u_{j,j} & \dots & false \\ \vdots & \vdots & \vdots & \dots & \ddots & \dots & \vdots & \\ b_n & s_{n,2} & false & false & \dots & false & \dots & u_{n,n} \\ d_1 & false & true & false & \dots & false & \dots & false \end{array};$$

$s_{j,2}$ = "the token in place b_j has stayed in the place for the longest (shortest or the C-criterion) time among the tokens in places $b_1, b_2, ..., b_n$,
$u_{j,j}$ = "following criterion D from x_0^δ, the current token has to be transferred from place b_j to another place".

Of course, this GN can be extended with places c_1, c_2, c_3, c_4 and tokens γ_1, γ_2 as the fifth and sixth GN-models described in the this section.

In the present form, the criterion D can be used for the whole period of the GN-functioning. In this case, token δ enters the GN one time and its initial characteristic is used permanently. If we want the GN-model to change criterion D dynamically, then more than one token δ has to enter the net and each of them has to have different initial characteristics.

In the next modification of the first GN model some of the tokens that are transferred in the GN and represent the data that enters the buffer, can leave the GN in a time-moment, because for instance the duration of life of these tokens has finished (see [2]). This net is shown in Fig. 5.

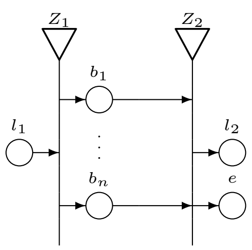

Figure 5. GN representation of a queueing discipline
with tokens having duration of life.

The first transition of this net will coincide with transition Z_1 of the first GN (see Section 2), while the second transition of the new net has the form:

$$Z_2 = \langle \{b_1, ..., b_n\}, \{l_2, e\}, r_2 \rangle,$$

where

$$r_2 = \begin{array}{c|cc} & l_2 & e \\ \hline b_1 & s_{1,2} & v_1 \\ \vdots & \vdots & \vdots \\ b_j & s_{j,2} & v_j \\ \vdots & \vdots & \vdots \\ b_n & s_{n,2} & v_n \end{array};$$

$s_{j,2} =$ " the token in place b_j has stayed in the place for the longest (shortest or C-criterion) time in the place among the tokens in all place $b_1, b_2, ..., b_n$", for $j = 1, 2, ..., n$,
$v_j =$ "the duration of staying of the current token in place b_j has finished".

In this form, the GN is static, because the duration of staying is fixed in advance. If we like this duration to be changed during the functioning of the net, the GN has to be changed as shown in Fig. 6 and its description is similar to the above one.

In this GN, we can add places $c_1, c_2, c_3, c_4, d_1, d_2$ and tokens $\gamma_1, \gamma_2, \delta$, too, if we want to model a more complex queueing discipline.

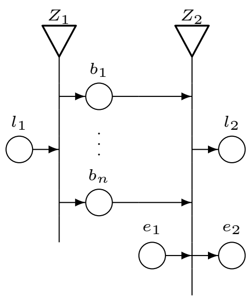

Figure 6. GN representation of a queueing discipline with tokens having time-adjusted duration of life.

Finally, we will propose a GN in which the tokens staying in the buffer can obtain new characteristics. One possible form of the net is shown in Fig. 7.

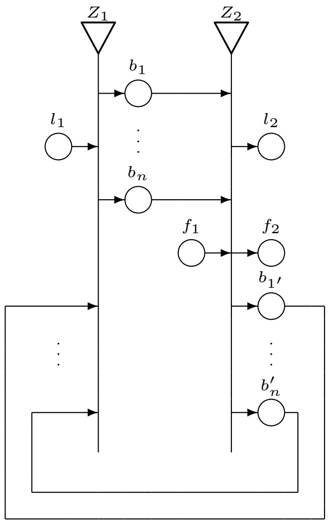

Figure 7. GN representation of a queueing discipline
with requests changing their characteristics.

The transitions of this GN have the following forms.

$$Z_1 = \langle \{l_1, b'_1, ..., b'_n\}, \{b_1, ..., b_n\}, r_1 \rangle,$$

where

$$r_1 = \begin{array}{c|ccccc} & b_1 & \dots & b_j & \dots & b_n \\ \hline l_1 & r_{1,1} & \dots & r_{1,j} & \dots & r_{1,n} \\ b'_1 & true & \dots & false & \dots & false \\ \vdots & \vdots & \dots & \ddots & \dots & \vdots \\ b'_j & false & \dots & true & \dots & false \\ \vdots & \vdots & \dots & \ddots & \dots & \vdots \\ b'_n & false & \dots & false & \dots & true \end{array};$$

$r_{1,j}$ = "b_j is the first empty place from top to bottom among places $b_1, b_2, ..., b_n$", for $j = 1, 2, ..., n$.

$$Z_2 = \langle \{b_1, ..., b_n, f_1\}, \{l_2, f_2, b'_1, ..., b'_n\}, r_2 \rangle,$$

where

$$r_2 = \begin{array}{c|ccccccc} & l_2 & f_2 & b'_1 & \dots & b'_j & \dots & b'_n \\ \hline b_1 & s_{1,2} & false & true & \dots & false & \dots & false \\ \vdots & \vdots & \vdots & \dots & \ddots & \dots & \vdots & \\ b_j & s_{j,2} & false & false & \dots & true & \dots & false \\ \vdots & \vdots & \vdots & \dots & \ddots & \dots & \vdots & \\ b_n & s_{n,2} & false & false & \dots & false & \dots & true \\ f_1 & false & true & false & \dots & false & \dots & false \end{array};$$

$s_{j,2}$ = "b_j is the first from above full cell of the buffer".

As in the previous cases, we can add places $c_1, c_2, c_3, c_4, d_1, d_2, e$ and tokens $\gamma_1, \gamma_2, \delta$ in this GN, too, if we want to model more complex queueing discipline.

4 Conclusion

In future, other modifications of the above GNs will be introduced. We shall propose GN models of other queueing systems such as queueing systems with feedback, with catastrophes, failures and repairs, etc.

In [8], different GSs are described as tools for the modelling of Data Mining (DM) processes. Having in mind that the queueing systems can be included in the DM tools, the present paper is a continuation of the research discussed there.

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