Fixed point theorems
in intuitionistic fuzzy contraction mappings
in intuitionistic fuzzy generalized metric spaces

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Abstract: In this paper, we introduce intuitionistic fuzzy contraction mappings in intuitionistic fuzzy generalized metric spaces. The presented theorems, extend, generalize and improve the corresponding result which given in the literature. Some fixed point theorems in intuitionistic fuzzy generalized metric space in the sense of George and Veeramani [2].

Keywords: Intuitionistic fuzzy metric spaces, Intuitionistic fuzzy contraction mapping, Intuitionistic fuzzy generalized metric spaces.

AMS Classification: 47H10, 54H25.

1   Introduction

In 1965, the concept of fuzzy set was introduced by Zadeh [14] in domain X and [0, 1]. In 1986, Atanassov [1] introduced the notion of an intuitionistic fuzzy sets. Afterward, Park [8] gave the notion of an intuitionistic fuzzy metric space and generalized the notion of a fuzzy metric space due to George and Veeramani. In 2008, Saadati et al. [10] modified the idea of an intuitionistic fuzzy metric space and presented the new notion of an intuitionistic fuzzy metric space.

On the other hand, in 1981, Heilpern [3] developed fixed point theory in fuzzy metric spaces, introduced the concept of fuzzy contraction mappings and proved some fixed point theorems for fuzzy contraction mappings. Afterward, in 2006, Rafi and Noorani [9] introduced the concept of intuitionistic fuzzy contraction mappings and proved the existence fixed point in intuitionistic fuzzy generalized metric spaces for an intuitionistic fuzzy contraction mappings. We introduce intuitionistic fuzzy contraction mappings in intuitionistic fuzzy generalized metric spaces. The presented theorems, extend, generalize and improve the corresponding result which given in the literature.

Definition 1.1. A binary operation $\Diamond : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is said to be a continuous t-conorm if it satisfies the following conditions:

i) $\Diamond$ is associative and commutative,

ii) $\Diamond$ is continuous,

iii) $a \Diamond 0 = a$ for all $a \in [0,1]$,

iv) $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Definition 1.2. A 5-tuple $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ is said to be an intuitionistic fuzzy generalized metric space (shortly IFGM-space), if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and $\mathcal{M}, \mathcal{N}$ are fuzzy sets on $X^3 \times (0, \infty)$ satisfying for all $x, y, z, a \in X$ and $s, t > 0$, the following conditions

(IFGM-1) $\mathcal{M}(x, y, z, t) + \mathcal{N}(x, y, z, t) = 1$,

(IFGM-2) $\mathcal{M}(x, y, z, t) > 0$,

(IFGM-3) $\mathcal{M}(x, y, z, t) = 1$ if and only if $x = y = z$,

(IFGM-4) $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$, where p is a permutation function,

(IFGM-5) $\mathcal{M}(x, y, z, a, t) * \mathcal{M}(a, z, z, s) \leq \mathcal{M}(x, y, z, t + s)$,

(IFGM-6) $\mathcal{M}(x, y, z, .) : (0, \infty) \rightarrow [0, 1]$ is continuous,

(IFGM-7) $\mathcal{N}(x, y, z, t) < 1$,

(IFGM-8) $\mathcal{N}(x, y, z, t) = 0$ if and only if $x = y = z$,

(IFGM-9) $\mathcal{N}(x, y, z, t) = \mathcal{N}(p\{x, y, z\}, t)$, where p is a permutation function,

(IFGM-10) $\mathcal{N}(x, y, z, a, t) \Diamond \mathcal{N}(a, z, z, s) \geq \mathcal{N}(x, y, z, t + s)$,

(IFGM-11) $\mathcal{N}(x, y, z, .) : (0, \infty) \rightarrow [0, 1]$ is continuous.

The above definition, the triangular inequality (IFGM-5) and (IFGM-10) are replaced by

$$\mathcal{M}(x, y, z, a, t) * \mathcal{M}(a, z, z, s) \leq \mathcal{M}(x, y, z, \max\{t, s\})$$

and

$$\mathcal{N}(x, y, z, a, t) \Diamond \mathcal{N}(a, z, z, s) \geq \mathcal{N}(x, y, z, \min\{t, s\}).$$

Then the 5-tuple $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ is called a non-Archimedean intuitionistic fuzzy generalized metric space. It is easy to check that the triangular inequality (NA) implies (IFGM-5) and (IFGM-10), that is every non-Archimedean intuitionistic fuzzy generalized metric space is itself an intuitionistic fuzzy generalized metric space.

Definition 1.3. Let $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ be an intuitionistic fuzzy generalized metric space. Then

a)   A sequence $\{x_n\}$ in X is said to be converges to a point $x \in X$, if for all $t > 0$,

$$\lim_{n \to \infty} \mathcal{M}(x, x, x_n, t) = 1 \text{ and } \lim_{n \to \infty} \mathcal{N}(x, x, x_n, t) = 0.$$

b)   A sequence $\{x_n\}$ in X is said to be a Cauchy sequence if for all $\varepsilon > 0$, there exists $n_0 \in N$ such that for all $m, n \geq n_0$, we have $\mathcal{M}(x_n, x_n, x_m, t) > 1 - \varepsilon$ and $\mathcal{N}(x_n, x_n, x_m, t) > \varepsilon$.

c) X is complete if every Cauchy sequence is converges in X.

Lemma 1.4. Let $\{x_n\}$ be a sequence in a intuitionistic fuzzy generalized metric space $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$, if there exists a constant $k \in (0, 1)$ such that $\mathcal{M}(x_n, x_{n+1}, x_{n+1}, kt) \geq \mathcal{M}(x_{n-1}, x_n, x_n, t)$ and $\mathcal{N}(x_n, x_{n+1}, x_{n+1}, kt) \leq \mathcal{N}(x_{n-1}, x_n, x_n, t)$ for all $t > 0$. Then $\{x_n\}$ is Cauchy sequence in X.

2   Main results

Definition 2.1.   Let $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ be an intuitionistic fuzzy generalized metric space. A mapping $T : X \to X$ is an intuitionistic fuzzy generalized contractive mapping, if there exists $k \in (0, 1)$, such that

$$\left(\frac{1}{\mathcal{M}(Tx, Ty, Tz, t)} - 1 \right) \leq k \left(\frac{1}{\mathcal{M}(x, y, z, t)} - 1 \right)$$

and $\mathcal{N}(Tx, Ty, Tz, t) \geq k, \mathcal{N}(x, y, z, t)$, for each $x, y, z \in X$ and $t > 0$ (k is called the contractive constant of T).

Proposition 2.2. Let (X, d) be a metric space. The mapping $f : X \to X$ is metric contractive on (X, d) with contractive constant k if and only if f is intuitionistic fuzzy generalized contractive, with contractive constant k, on the intuitionistic fuzzy generalized metric space $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$, induced by d.

Proposition 2.3. Let $(X, \mathcal{M}_d, \mathcal{N}_d, *, \Diamond)$ be an intuitionistic fuzzy generalized metric space induced by the metric d on X. The sequence $\{x_n\}$ in X is contractive in (X, d) if and only if $\{x_n\}$ is intuitionistic fuzz generalized contractive in $(X, \mathcal{M}_d, \mathcal{N}_d, *, \Diamond)$.

Definition 2.4.   Let $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ be an intuitionistic fuzzy generalized metric space. A sequence is called $\{x_n\}$ G-Cauchy iff for each $t > 0$ and $p \in N$,

$$\lim_{n \to \infty} \mathcal{M}(x_{n+p}, x_{n+p}, x_n, t) = 1, \quad \lim_{n \to \infty} \mathcal{N}(x_{n+p}, x_{n+p}, x_n, t) = 0.$$

A intuitionistic fuzzy generalized metric space in which every G-Cauchy sequence is convergent is called G-Complete.

Theorem 2.5. Let $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ be a complete non-Archimedean intuitionistic fuzzy generalized metric space where the continuous t-norm $*$ is defined as min and continuous t-conorm $\Diamond$ is defined as max and $T : X \to X$ be a self mapping on X such that for each $x, y, z \in X$, $t > 0$,

$$\left(\frac{1}{\mathcal{M}(Tx,Ty,Tz,t)} - 1\right) \leq \begin{Bmatrix} \alpha\left(\frac{1}{\mathcal{M}(x,y,z,t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}(x,Tx,z,t)} - 1\right) + \gamma\left(\frac{1}{\mathcal{M}(x,Ty,Tz,t)} - 1\right) \\ \delta\left(\frac{1}{\mathcal{M}(y,Ty,Tz,t)} - 1\right) + \eta\left(\frac{1}{\mathcal{M}(Tx,Ty,z,t)} - 1\right) \end{Bmatrix} \quad (2.5.1)$$

$$\mathcal{N}(Tx,Ty,Tz,t) \geq \begin{Bmatrix} \alpha\,\mathcal{N}(x,y,z,t) + \beta\,\mathcal{N}(x,Tx,z,t) + \gamma\,\mathcal{N}(x,Ty,Tz,t) \\ \delta\,\mathcal{N}(y,Ty,Tz,t) + \eta\,\mathcal{N}(Tx,Ty,z,t) \end{Bmatrix}. \quad (2.5.2)$$

where $\alpha, \beta, \gamma, \delta, \eta \in [0,1]$ and $k = \alpha + \beta + \gamma + \delta + \eta < 1$. Then T has a unique fixed point.

Proof: Let $x \in X$ and $t > 0$ be arbitrary and consider a sequence pickard iterations x_n, defined inductively by $x_0 = x$, $x_1 = x_0, \ldots, x_{n+1} = T(x_n)$ for each $n \in N$, we will show that x_n is fuzzy contractive. From (2.5.1) and (2.5.2) by replacing $x = x_n$, $y = x_{n+1}$ and $z = x_{n+1}$, we get

$$\left(\frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1\right) = \left(\frac{1}{\mathcal{M}(Tx_{n-1}, Tx_n, Tx_n, t)} - 1\right)$$

$$\leq \begin{Bmatrix} \alpha\left(\frac{1}{\mathcal{M}(x_{n-1}, x_n, x_n, t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}(x_{n-1}, Tx_{n-1}, x_n, t)} - 1\right) + \\ \gamma\left(\frac{1}{\mathcal{M}(x_{n-1}, Tx_n, Tx_n, t)} - 1\right) + \delta\left(\frac{1}{\mathcal{M}(x_n, Tx_n, Tx_n, t)} - 1\right) + \\ \eta\left(\frac{1}{\mathcal{M}(Tx_{n-1}, Tx_n, x_n, t)} - 1\right) \end{Bmatrix}$$

$$\leq \begin{Bmatrix} \alpha\left(\frac{1}{\mathcal{M}(x_{n-1}, x_n, x_n, t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}(x_{n-1}, x_n, x_n, t)} - 1\right) + \\ \gamma\left(\frac{1}{\mathcal{M}(x_{n-1}, x_{n+1}, x_{n+1}, t)} - 1\right) + \delta\left(\frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1\right) \\ + \eta\left(\frac{1}{\mathcal{M}(x_n, x_{n+1}, x_n, t)} - 1\right) \end{Bmatrix}$$

$$\mathcal{N}(x_n, x_{n+1}, x_{n+1}, t) = \mathcal{N}(Tx_{n-1}, Tx_n, Tx_n, t)$$

$$\geq \begin{Bmatrix} \alpha\,\mathcal{N}(x_{n-1}, x_n, x_n, t) + \beta\,\mathcal{N}(x_{n-1}, Tx_{n-1}, x_n, t) + \\ \gamma\,\mathcal{N}(x_{n-1}, Tx_n, Tx_n, t) + \delta\,\mathcal{N}(x_n, Tx_n, Tx_n, t) + \eta\,\mathcal{N}(Tx_{n-1}, Tx_n, x_n, t) \end{Bmatrix}$$

$$\geq \begin{Bmatrix} \alpha\,\mathcal{N}(x_{n-1}, x_n, x_n, t) + \beta\,\mathcal{N}(x_{n-1}, x_n, x_n, t) + \\ \gamma\,\mathcal{N}(x_{n-1}, x_{n+1}, x_{n+1}, t) + \delta\,\mathcal{N}(x_n, x_{n+1}, x_{n+1}, t) + \eta\,\mathcal{N}(x_n, x_{n+1}, x_n, t) \end{Bmatrix}.$$

By our choice of *t*-norm * and *t*-conom ◊ and triangular inequality in the above, we have

$$\left(\frac{1}{\mathcal{M}(x_{n-1}, x_{n+1}, x_{n+1}, t)} - 1\right) \leq \left(\frac{1}{\min\{\mathcal{M}(x_{n+1}, x_{n+1}, x_n, t), \mathcal{M}(x_n, x_{n-1}, x_{n-1}, t)\}} - 1\right)$$

$$= \max\left(\frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1, \frac{1}{\mathcal{M}(x_n, x_{n-1}, x_{n-1}, t)} - 1\right)$$

and

$$\mathcal{N}(x_{n-1}, x_{n+1}, x_{n+1}, t) \geq \min\{\mathcal{N}(x_{n+1}, x_{n+1}, x_n, t), \mathcal{N}(x_n, x_{n-1}, x_{n-1}, t)\}$$

$$= \min\{\mathcal{N}(x_n, x_{n+1}, x_{n+1}, t), \mathcal{N}(x_n, x_{n-1}, x_{n-1}, t)\}$$

$$\left(\frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1\right)$$

$$\leq (\alpha + \beta + \delta + \eta)\max\left\{\frac{1}{\mathcal{M}(x_{n-1}, x_n, x_n, t)} - 1, \frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1\right\}$$

$$+\gamma \max\left\{\frac{1}{\mathcal{M}\,(x_n, x_n,\, x_{n+1}, t)} - 1, \frac{1}{\mathcal{M}\,(x_n, x_{n-1}, x_{n-1}, t)} - 1\right\}$$

and

$$\mathcal{N}(x_n, x_{n+1}, x_{n+1}, t) \geq (\alpha + \beta + \delta + \eta)\min\{\mathcal{N}(x_{n-1}, x_n, x_n, t), \mathcal{N}(x_n, x_{n+1}, x_{n+1}, t)\}$$
$$+\gamma \min\{\mathcal{N}(x_n, x_n, x_{n+1}, t), \mathcal{N}(x_n, x_{n-1}, x_{n-1}, t)\},$$

hence,

$$\left(\frac{1}{\mathcal{M}\,(x_n, x_{n+1}, x_{n+1}, t)} - 1\right) \leq k \max\left\{\frac{1}{\mathcal{M}\,(x_{n-1}, x_n, x_n, t)} - 1, \frac{1}{\mathcal{M}\,(x_n, x_{n+1}, x_{n+1}, t)} - 1\right\}$$

and

$$\mathcal{N}(x_n, x_{n+1}, x_{n+1}, t) \geq k \min\{\mathcal{N}(x_{n-1}, x_n, x_n, t), \mathcal{N}(x_n, x_{n+1}, x_{n+1}, t)\},$$

where $k < 1$, this implies

$$\left(\frac{1}{\mathcal{M}\,(x_n,\ x_{n+1},\ x_{n+1}, t)} - 1\right) \leq k\left(\frac{1}{\mathcal{M}\,(x_{n-1},\ x_n,\ x_n, t)} - 1\right)$$

and

$$\mathcal{N}(x_n, x_{n+1}, x_{n+1}, t) \geq k\, \mathcal{N}(x_{n-1}, x_n, x_n, t).$$

So, sequence $\{x_n\}$ is intuitionistic fuzzy generalized contractive sequence.

Since $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ is a complete intuitionistic fuzzy generalized metric space.

So, sequence $\{x_n\}$ converges to u for some $u \in X$. Now, we shall show u is fixed point of T. From (2.5.1) and (2.5.2), we have

$$\left(\frac{1}{\mathcal{M}\,(Tu, Tx_n, Tx_n, t)} - 1\right) \leq$$
$$\left\{\begin{array}{c}\alpha\left(\frac{1}{\mathcal{M}\,(u, x_n,\, x_n, t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}\,(u, Tu, x_n, t)} - 1\right) + \gamma\left(\frac{1}{\mathcal{M}\,(u,\, Tx_n,\, Tx_n, t)} - 1\right) + \\ \delta\left(\frac{1}{\mathcal{M}\,(x_n,\, Tx_n, Tx_n, t)} - 1\right) + \eta\left(\frac{1}{\mathcal{M}\,(Tu,\, Tx_n, x_n, t)} - 1\right)\end{array}\right\}$$

$$\mathcal{N}(Tu, Tx_n, Tx_n, t) \geq \left\{\begin{array}{c}\alpha\,\mathcal{N}(u, x_n, x_n, t) + \beta\,\mathcal{N}(u, Tu, x_n, t) + \gamma\,\mathcal{N}(u, Tx_n, Tx_n, t) + \\ \delta\,\mathcal{N}(x_n,\, Tx_n, Tx_n, t) + \eta\,\mathcal{N}(Tu, Tx_n, x_n, t)\end{array}\right\}.$$

Taking the limit as $n \to \infty$, we obtain,

$$\left(\frac{1}{\mathcal{M}\,(Tu, u, u, t)} - 1\right) \leq \left\{\beta\left(\frac{1}{\mathcal{M}\,(u, Tu, u, t)} - 1\right) + \eta\left(\frac{1}{\mathcal{M}\,(Tu, u, u, t)} - 1\right)\right\}$$
$$\leq k\left(\frac{1}{\mathcal{M}\,(u, u, Tu, t)} - 1\right)$$

and

$$\mathcal{N}(Tu, u, u, t) \geq \{\beta\,\mathcal{N}(u, Tu, u, t) + \eta\,\mathcal{N}(Tu, u, u, t)\} \geq k\,\mathcal{N}(u, u, Tu, t).$$

Since $k < 1$, we have $\mathcal{M}\,(u, u, Tu, t) = 1$ and $\mathcal{N}(u, u, Tu, t) = 0$. Thus $Tu = u$.

Uniqueness: Suppose there exist $v \in X$ such that $Tv = v$ and $v \neq u$. Now, we consider

$$\mathcal{M}(u, v, v, t) = \mathcal{M}(Tu, Tv, Tv, t)$$

$$\leq \left\{ \begin{array}{c} \alpha\left(\frac{1}{\mathcal{M}(u,v,v,t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}(u,Tu,v,t)} - 1\right) + \gamma\left(\frac{1}{\mathcal{M}(u,Tv,Tv,t)} - 1\right) + \\ \delta\left(\frac{1}{\mathcal{M}(v,Tv,Tv,t)} - 1\right) + \eta\left(\frac{1}{\mathcal{M}(Tu,Tv,v,t)} - 1\right) \end{array} \right\}$$

$$\leq \left\{ \begin{array}{c} \alpha\left(\frac{1}{\mathcal{M}(u,v,v,t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}(u,u,v,t)} - 1\right) + \gamma\left(\frac{1}{\mathcal{M}(u,v,v,t)} - 1\right) + \\ \delta\left(\frac{1}{\mathcal{M}(v,v,v,t)} - 1\right) + \eta\left(\frac{1}{\mathcal{M}(u,v,v,t)} - 1\right) \end{array} \right\}$$

$$\leq (\alpha + \beta + \gamma + \eta)\left(\frac{1}{\mathcal{M}(u,v,v,t)} - 1\right)$$

$$\leq k\left(\frac{1}{\mathcal{M}(u,v,v,t)} - 1\right)$$

and

$$\mathcal{N}(u, v, v, t) = \mathcal{N}(Tu, Tv, Tv, t)$$

$$\geq \left\{ \begin{array}{c} \alpha\,\mathcal{N}(u,v,v,t) + \beta\,\mathcal{N}(u,Tu,v,t) + \gamma\,\mathcal{N}(u,Tv,Tv,t) + \\ \delta\,\mathcal{N}(v,Tv,Tv,t) + \eta\,\mathcal{N}(Tu,Tv,v,t) \end{array} \right\}$$

$$\geq \left\{ \begin{array}{c} \alpha\,\mathcal{N}(u,v,v,t) + \beta\,\mathcal{N}(u,u,v,t) + \gamma\,\mathcal{N}(u,v,v,t) + \\ \delta\,\mathcal{N}(v,v,v,t) + \eta\,\mathcal{N}(u,v,v,t) \end{array} \right\}$$

$$\geq (\alpha + \beta + \gamma + \eta)\,\mathcal{N}(u,v,v,t) \geq k\,\mathcal{N}(u,v,v,t).$$

Since $k < 1$, we have $\mathcal{M}(u, v, v, t) = 1$ and $\mathcal{N}(u, v, v, t) = 0$. Then $u = v$. Therefore u is fixed point of T. $\qquad\square$

Theorem 2.6. Let $(X, \mathcal{M}, \mathcal{N}, *, \Diamond)$ be a G-complete intuitionistic fuzzy generalized metric space, where the continuous t-norm $*$ is defined as min and continuous t-conorm is defined as max and $T : X \to X$ be self mapping on X such that for each $x, y, z \in X$, $t > 0$, $k \in (0, 1)$.

$$\left(\frac{1}{\mathcal{M}(Tx,Ty,Tz,t)} - 1\right) \leq \left\{ \begin{array}{c} \alpha\left(\frac{1}{\mathcal{M}(x,y,z,t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}(x,Tx,z,t)} - 1\right) + \gamma\left(\frac{1}{\mathcal{M}(x,Ty,Tz,t)} - 1\right) + \\ \delta\left(\frac{1}{\mathcal{M}(y,Ty,Tz,2t)} + \frac{1}{\mathcal{M}(Tx,Ty,z,2t)} - 2\right) \end{array} \right\}$$

$$\mathcal{N}(Tx, Ty, Tz, t) \geq \left\{ \begin{array}{c} \alpha\,\mathcal{N}(x,y,z,t) + \beta\,\mathcal{N}(x,Tx,z,t) + \gamma\,\mathcal{N}(x,Ty,Tz,t) + \\ \delta\{\mathcal{N}(y,Ty,Tz,2t) + \mathcal{N}(Tx,Ty,z,2t)\} \end{array} \right\}.$$

Where $\alpha, \beta, \gamma, \delta \in [0, 1]$ and $k = \alpha + \beta + \gamma + \delta < 1$. Then T has a unique fixed point.

Proof: The proof is very similar as the Theorem (2.5). Instead of this equation (2.5.1) and (2.5.2) we have

$$\left(\frac{1}{\mathcal{M}(x_{n-1}, x_{n+1}, x_{n+1}, 2t)} - 1\right) \leq \left(\frac{1}{\min\{\mathcal{M}(x_{n+1},x_{n+1},x_n,t),\mathcal{M}(x_n,x_{n-1},x_{n-1},t)\}} - 1\right)$$

$$= \max\left(\frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1, \frac{1}{\mathcal{M}(x_n,x_{n-1}, x_{n-1},t)} - 1\right) \text{ and}$$

$$\mathcal{N}(x_{n-1}, x_{n+1}, x_{n+1}, 2t) \geq \min\{\mathcal{N}(x_{n+1}, x_{n+1}, x_n, t), \mathcal{N}(x_n, x_{n-1}, x_{n-1}, t)\}$$

$$= \min\{\mathcal{N}(x_n, x_{n+1}, x_{n+1}, t), \mathcal{N}(x_n, x_{n-1}, x_{n-1}, t)\}.$$

Proceed as the proof of the Theorem (2.5). Then, the sequence $\{x_n\}$ is intuitionistic fuzzy generalized contractive, Thus, by Definition (2.4) is G-Cauchy. Since X is G-complete, $\{x_n\}$ converges to u for some $u \in X$. Instead of (2.5), we find

$$\left(\frac{1}{\mathcal{M}\,(Tu,Tx_n,Tx_n,t)} - 1\right) \le$$

$$\left\{\begin{matrix}\alpha\left(\frac{1}{\mathcal{M}\,(u,\,x_n,\,x_n,t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}\,(u,Tu,x_n,t)} - 1\right) + \gamma\left(\frac{1}{\mathcal{M}\,(u,Tx_n,Tx_n,t)} - 1\right) + \\ \delta\left(\frac{1}{\mathcal{M}(x_n,Tx_n,Tx_n,2t)} + \frac{1}{\mathcal{M}\,(Tu,Tx_n,x_n,2t)} - 2\right)\end{matrix}\right\}$$

$$\mathcal{N}(Tu,\,Tx_n,\,Tx_n,\,t) \ge \left\{\begin{matrix}\alpha\,\mathcal{N}(u,x_n,x_n,t) + \beta\,\mathcal{N}(u,Tu,x_n,t) + \gamma\,\mathcal{N}(u,Tx_n,Tx_n,t) + \\ \delta\,(\mathcal{N}(x_n,Tx_n,Tx_n,2t) + \mathcal{N}(Tu,Tx_n,x_n,2t))\end{matrix}\right\}.$$

Taking the limit as $n \to \infty$, we obtain

$$\left(\frac{1}{\mathcal{M}\,(Tu,u,u,t)} - 1\right)$$

$$\le \left\{\beta\left(\frac{1}{\mathcal{M}\,(u,Tu,u,t)} - 1\right) + \delta\left(\frac{1}{\mathcal{M}\,(Tu,u,u,2t)} - 1\right)\right\} \le k\left(\frac{1}{\mathcal{M}\,(u,u,Tu,t)} - 1\right),$$

$$\mathcal{N}(Tu,u,u,t) \ge \{\beta\,\mathcal{N}(u,Tu,u,t) + \delta\,\mathcal{N}(Tu,u,u,2t)\} \ge k\,\mathcal{N}(u,u,Tu,t).$$

Since $k < 1$, we have $\mathcal{M}\,(u,u,Tu,t) = 1$ and $\mathcal{N}(Tu,u,u,t) = 0$.

Thus $Tu = u$. It is found that fixed point is unique. $\qquad\square$

Remark 2.7. A similar proof, it is found that the generalized contraction condition (2.5.1) and (2.5.2) are equivalent to following:

$$\left(\frac{1}{\mathcal{M}\,(Tx,Ty,Tz,t)} - 1\right) \le k\max\left\{\begin{matrix}\left(\frac{1}{\mathcal{M}\,(x,y,z,t)} - 1\right),\left(\frac{1}{\mathcal{M}\,(x,Tx,z,t)} - 1\right),\left(\frac{1}{\mathcal{M}\,(x,Ty,Tz,t)} - 1\right), \\ \left(\frac{1}{\mathcal{M}\,(y,Ty,Tz,t)} - 1\right),\left(\frac{1}{\mathcal{M}\,(Tx,Ty,z,t)} - 1\right)\end{matrix}\right\}$$

$$\left(\frac{1}{\mathcal{M}\,(Tx,Ty,Tz,t)} - 1\right) \le k\max\left\{\begin{matrix}\left(\frac{1}{\mathcal{M}\,(x,y,z,t)} - 1\right),\left(\frac{1}{\mathcal{M}\,(x,Tx,z,t)} - 1\right),\left(\frac{1}{\mathcal{M}\,(x,Ty,Tz,t)} - 1\right), \\ \left(\frac{1}{\mathcal{M}\,(y,Ty,Tz,2t)} - 1\right),\left(\frac{1}{\mathcal{M}\,(Tx,Ty,z,2t)} - 1\right)\end{matrix}\right\}$$

$$\mathcal{N}(Tx,Ty,Tz,t) \ge k\min\left\{\begin{matrix}\mathcal{N}(x,y,z,t),\mathcal{N}(x,Tx,z,t),\mathcal{N}(x,Ty,Tz,t), \\ \mathcal{N}(y,Ty,Tz,t),\mathcal{N}(Tx,Ty,z,t)\end{matrix}\right\}$$

$$\mathcal{N}(Tx,Ty,Tz,t) \ge k\min\left\{\begin{matrix}\mathcal{N}(x,y,z,t),\mathcal{N}(x,Tx,z,t),\mathcal{N}(x,Ty,Tz,t), \\ \mathcal{N}(y,Ty,Tz,2t),\mathcal{N}(Tx,Ty,z,2t)\end{matrix}\right\}.$$

Respectively, where $k \in [0,1]$. $\qquad\square$

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