

On intuitionistic fuzzy slightly β -continuous functions

R. Renuka and V. Seenivasan

Department of Mathematics, University College of Engineering, Panruti

A Constituent College of Anna University Chennai

Panruti–607 106, Tamilnadu, India

e-mails: renuka.autpc@gmail.com and seenujsc@yahoo.co.in

Abstract: In this paper the concept of intuitionistic fuzzy slightly β -continuous functions are introduced and studied. Intuitionistic fuzzy slightly β -continuity generalize intuitionistic fuzzy β -continuity. Besides giving characterizations and basic properties of this function, preservation theorems of intuitionistic fuzzy slightly β -continuous are also obtained. We also study relationships between intuitionistic fuzzy slightly β -continuity and separation axioms. Moreover, we investigate and the relationships among intuitionistic fuzzy slightly β -continuity and compactness and connectedness.

Keywords: Intuitionistic fuzzy *beta*-open set, Intuitionistic fuzzy *beta*-continuous, Intuitionistic fuzzy clopen set.

AMS Classification: 54A40, 03E72.

1 Introduction

Ever since the introduction of fuzzy sets by L. A. Zadeh [15], the fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological spaces was introduced and developed by C. L. Chang [2]. K. Atanassov [1] introduced the notion of intuitionistic fuzzy sets, Çoker [3] introduced the intuitionistic fuzzy topological spaces. T. Noiri [12] introduced slightly β -continuous functions. E. Ekici [6] introduced fuzzy slightly β -continuous functions. In this paper, we have introduced the concept of intuitionistic fuzzy slightly β -continuous functions and studied their properties. Also we have given preservation theorems of intuitionistic fuzzy slightly β -continuous functions. We also study relationships between this function and separation axioms. Moreover, we investigate the relationships among intuitionistic fuzzy slightly β -continuity and compactness and connectedness.

2 Preliminaries

Definition 2.1. [1] Let X be a non-empty fixed set and I the closed interval $[0,1]$. An intuitionistic fuzzy set (IFS) A is an object of the following form

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in X\}$$

where the mappings $\mu_A(x) : X \rightarrow I$ and $\nu_A(x) : X \rightarrow I$ denote the degree of membership (namely) $\mu_A(x)$ and the degree of non-membership (namely) $\nu_A(x)$ for each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2. [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle | x \in X\}$. Then

- (i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;
- (ii) \bar{A} (or A^c) = $\{\langle x, \nu_A(x), \mu_A(x) \rangle | x \in X\}$;
- (iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle | x \in X\}$;
- (iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle | x \in X\}$.

We will use the notation $A = \{\langle x, \mu_A, \nu_A \rangle | x \in X\}$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in X\}$.

Definition 2.3. [3] $0_\sim = \{\langle x, 0, 1 \rangle | x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle | x \in X\}$.

Let $\alpha, \beta \in [0,1]$ such that $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP) $p_{(\alpha,\beta)}$ is intuitionistic fuzzy set defined by $p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise} \end{cases}$

Definition 2.4. [3] An intuitionistic fuzzy topology (IFT) in Çoker's sense on a nonempty set X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms:

- (i) $0_\sim, 1_\sim \in \tau$;
- (ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
- (iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i; i \in J\} \subseteq \tau$.

In this paper by (X, τ) , (Y, σ) or simply by X, Y , we will denote the intuitionistic fuzzy topological spaces (IFTS). Each IFS which belongs to τ is called an intuitionistic fuzzy open set (IFOS) in X . The complement \bar{A} of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X . A IFS X is called intuitionistic fuzzy clopen (IF clopen) iff it is both intuitionistic fuzzy open and intuitionistic fuzzy closed.

Let X and Y be two non-empty sets and $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function.

If $B = \{\langle y, \mu_B(y), \nu_B(y) \rangle | y \in Y\}$ is an IFS in Y , then the pre-image of B under f is denoted and defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x)) \rangle | x \in X\}$. Since $\mu_B(x), \nu_B(x)$ are fuzzy sets, we explain that $f^{-1}(\mu_B(x)) = \mu_B(x)(f(x))$, $f^{-1}(\nu_B(x)) = \nu_B(x)(f(x))$.

Definition 2.5. [10] Let $p_{(\alpha,\beta)}$ be an IFP in IFTS X . An IFS A in X is called an intuitionistic fuzzy neighborhood (IFN) of $p_{(\alpha,\beta)}$ if there exists an IFOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Definition 2.6. [3] Let (X, τ) be an IFTS and $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in X\}$ be an IFS in X . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are defined by

- (i) $\text{cl}(A) = \bigcap \{C \mid C \text{ is an IFCS in } X \text{ and } C \supseteq A\};$
- (ii) $\text{int}(A) = \bigcup \{D \mid D \text{ is an IFOS in } X \text{ and } D \subseteq A\};$

It can be also shown that $\text{cl}(A)$ is an IFCS, $\text{int}(A)$ is an IFOS in X and A is an IFCS in X if and only if $\text{cl}(A) = A$; A is an IFOS in X if and only if $\text{int}(A) = A$.

Proposition 2.1. [3] Let (X, τ) be an IFTS and A, B be IFSs in X . Then the following properties hold:

- (i) $\text{cl}(\overline{A}) = \overline{(\text{int}(A))}, \text{int}(\overline{A}) = \overline{(\text{cl}(A))};$
- (ii) $\text{int}(A) \subseteq A \subseteq \text{cl}(A).$

Definition 2.7. [8] An IFS A in an IFTS X is called an intuitionistic fuzzy β -open set (IF β OS) if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$. The complement of an IF β OS A in IFTS X is called an intuitionistic fuzzy β -closed (IF β CS) in X .

Definition 2.8. Let f be a mapping from an IFTS X into an IFTS Y . The mapping f is called:

- (i) intuitionistic fuzzy continuous, if and only if $f^{-1}(B)$ is an IFOS in X , for each IFOS B in Y [8];
- (ii) intuitionistic fuzzy β -continuous if and only if $f^{-1}(B)$ is an IFPOS in X , for each IFOS B in Y [8];
- (iii) intuitionistic fuzzy totally continuous if and only if $f^{-1}(B)$ is an IF clopen sets in X , for each IFOS B in Y [11].

Definition 2.9. [7] A fuzzifying function $f : X \rightarrow Y$ is called fuzzy β -irresolute if inverse image of each fuzzy β -open set is fuzzy β -open.

Definition 2.10. [13] A function $f : (X, \tau) \rightarrow (Y, \sigma)$ from a intuitionistic fuzzy topological space (X, τ) to another intuitionistic fuzzy topological space (Y, σ) is said to be intuitionistic fuzzy β -irresolute if $f^{-1}(B)$ is an IF β OS in (X, τ) for each IF β OS B in (Y, σ) .

Definition 2.11. [3, 13] Let X be an IFTS. A family of $\{\langle x, \mu_{G_i}(x), \nu_{G_i}(x) \rangle | i \in J\}$ intuitionistic fuzzy open sets (intuitionistic fuzzy β -open sets) in X satisfies the condition $1_{\sim} = \bigcup \{\langle x, \mu_{G_i}(x), \nu_{G_i}(x) \rangle | i \in J\}$ is called a intuitionistic fuzzy open (intuitionistic fuzzy β -open) cover of X . A finite subfamily of a intuitionistic fuzzy open (intuitionistic fuzzy β -open) cover $\{\langle x, \mu_{G_i}(x), \nu_{G_i}(x) \rangle | i \in J\}$ of X which is also a intuitionistic fuzzy open (intuitionistic fuzzy β -open) cover of X is called a finite subcover of $\{\langle x, \mu_{G_i}(x), \nu_{G_i}(x) \rangle | i \in J\}$.

Definition 2.12. [4] An IFTS X is called intuitionistic fuzzy compact if each intuitionistic fuzzy open cover of X has a finite subcover for X .

3 Intuitionistic fuzzy slightly β -continuous functions

Definition 3.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ from a intuitionistic fuzzy topological space (X, τ) to another intuitionistic fuzzy topological space (Y, σ) is said to be intuitionistic fuzzy slightly β -continuous if for each intuitionistic fuzzy point $p_{(\alpha,\beta)} \in X$ and each intuitionistic fuzzy clopen set B in Y containing $f(p_{(\alpha,\beta)})$, there exists a fuzzy intuitionistic fuzzy β -open set A in X such that $f(A) \subseteq B$.

Theorem 3.1. For a function $f : X \rightarrow Y$, the following statements are equivalent:

1. f is intuitionistic fuzzy slightly β -continuous;
2. for every intuitionistic fuzzy clopen set B in Y , $f^{-1}(B)$ is intuitionistic fuzzy β -open;
3. for every intuitionistic fuzzy clopen set B in Y , $f^{-1}(B)$ is intuitionistic fuzzy β -closed;
4. for every intuitionistic fuzzy clopen set B in Y , $f^{-1}(B)$ is intuitionistic fuzzy β -clopen.

Proof. (1) \Rightarrow (2) Let B be IF clopen set in Y and let $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since $f(p_{(\alpha,\beta)}) \in B$, by (1) there exists a IFPOS $A_{p_{(\alpha,\beta)}}$ in X containing $p_{(\alpha,\beta)}$ such that $A_{p_{(\alpha,\beta)}} \subseteq f^{-1}(B)$. We obtain that $f^{-1}(B) = \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} A_{p_{(\alpha,\beta)}}$. Thus, $f^{-1}(B)$ is IF β -open.

(2) \Rightarrow (3) Let B be IF clopen set in Y . Then \overline{B} is IF clopen. By (2), $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$ is IF β -open. Thus $f^{-1}(B)$ is IF β -closed.

(3) \Rightarrow (4) Let B be IF clopen set in Y . Then by (3) $f^{-1}(B)$ is IF β -closed. Also \overline{B} is IF clopen and (3) implies $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$ is IF β -closed. Hence $f^{-1}(B)$ is IF β -open. Thus $f^{-1}(B)$ is IF β -clopen.

(4) \Rightarrow (1) Let B be IF clopen set in Y containing $f(p_{(\alpha,\beta)})$. By (4), $f^{-1}(B)$ is IF β -open. Let us take $A = f^{-1}(B)$. Then $f(A) \subseteq B$. Hence, f is IF slightly β -continuous. \square

Lemma 3.1. [9] Let $g: X \rightarrow X \times Y$ be a graph of a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$. If A and B are IFS's of X and Y respectively, then $g^{-1}(1_{\sim} \times B) = (1_{\sim} \cap f^{-1}(B))$

Lemma 3.2. [9] Let X and Y be intuitionistic fuzzy topological spaces, then (X, τ) is product related to (Y, σ) if for any IFS C in X , D in Y whenever $\overline{A} \not\supseteq C$, $\overline{B} \not\supseteq D$ implies $\overline{A} \times 1_{\sim} \cup 1_{\sim} \times \overline{B} \supseteq C \times D$ there exists $A_1 \in \tau$, $B_1 \in \sigma$ such that $\overline{A_1} \supseteq C$ and $\overline{B_1} \supseteq D$ and $\overline{A_1} \times 1_{\sim} \cup 1_{\sim} \times \overline{B_1} = \overline{A} \times 1_{\sim} \cup 1_{\sim} \times \overline{B}$.

Theorem 3.2. Let $f: X \rightarrow Y$ be a function and assume that X is product related to Y . If the graph $g: X \rightarrow X \times Y$ of f is IF slightly β -continuous then so is f .

Proof. Let B be IF clopen set in Y . Then by lemma 3.1, $f^{-1}(B) = 1_{\sim} \cap f^{-1}(B) = g^{-1}(1_{\sim} \times B)$. Now $1_{\sim} \times B$ is a IF clopen set in $X \times Y$. Since g is IF slightly β -continuous then $g^{-1}(1_{\sim} \times B)$ is IF β -open in X . Hence $f^{-1}(B)$ is IF β -open in X . Thus f is IF slightly β -continuous. \square

Theorem 3.3. A mapping $f : X \rightarrow Y$ from an IFTS X into an IFTS Y is IF slightly β -continuous if and only if for each IFP $p_{(\alpha,\beta)}$ in X and IF clopen set B in Y such that $f(p_{(\alpha,\beta)}) \in B$, $\text{cl}(f^{-1}(B))$ is IFN of IFP $p_{(\alpha,\beta)}$ in X .

Proof. Let f be any IF slightly β -continuous mapping, $p_{(\alpha,\beta)}$ be an IFP in X and B be any IF clopen set in Y such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B) \subseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(B)))) \subseteq \text{cl}(f^{-1}(B))$. Hence $\text{cl}(f^{-1}(B))$ is IFN of $p_{(\alpha,\beta)}$ in X .

Conversely, let B be any IF clopen set in Y and $p_{(\alpha,\beta)}$ be IFP in X such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B)$. According to assumption $\text{cl}(f^{-1}(B))$ is IFN of IFP $p_{(\alpha,\beta)}$ in X . So $p_{(\alpha,\beta)} \in \text{int}(\text{cl}(f^{-1}(B))) \subseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(B))))$. So, $f^{-1}(B) \subseteq \text{int}(\text{cl}(f^{-1}(B)))$. Hence $f^{-1}(B)$ is IFPOS in X . Therefore f is IF slightly β -continuous. \square

Proposition 3.1. Every intuitionistic fuzzy β -continuous function is intuitionistic fuzzy slightly β -continuous. But the converse need not be true, as shown by the following example.

Example 3.1. Let $X = \{a, b, c\}$, $\tau = \{0_{\sim}, 1_{\sim}, A\}$, $\sigma = \{0_{\sim}, 1_{\sim}, B, C, B \cup C, B \cap C\}$ where

$$A = \{\langle x, (\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{b}{0.4}) \rangle | x \in X\},$$

$$B = \{\langle x, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.5}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.5}) \rangle | x \in X\},$$

$$C = \{\langle x, (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.3}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{b}{0.7}) \rangle | x \in X\}.$$

Define an intuitionistic fuzzy mapping $f : (X, \tau) \rightarrow (X, \sigma)$ by $f(a) = a, f(b) = b, f(c) = c$. Then f is IF slightly β -continuous. But f is not β -continuous, since $f^{-1}(B \cap C)$ is not IF β OS in X as $f^{-1}(B \cap C) \not\subseteq \text{cl}(\text{int}(\text{cl}f^{-1}(B \cap C))) = 0_{\sim}$

Proposition 3.2. Every intuitionistic fuzzy β -irresolute function is intuitionistic fuzzy slightly β -continuous. But the converse need not be true, as shown by the following example.

Example 3.2. Let $X = \{a, b\}, Y = \{c, d\}, \tau = \{0_{\sim}, 1_{\sim}, A\}, \sigma = \{0_{\sim}, 1_{\sim}, B\}$ where

$$A = \{\langle x, (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.6}, \frac{b}{0.5}) \rangle | x \in X\},$$

$$B = \{\langle y, (\frac{c}{0.4}, \frac{d}{0.5}), (\frac{c}{0.5}, \frac{d}{0.5}) \rangle | y \in Y\}.$$

Define an intuitionistic fuzzy mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = d, f(b) = c$. Then f is IF slightly β -continuous. But it is not IF β -irresolute, since $f^{-1}(B) \not\subseteq \text{cl}(\text{int}(\text{cl}f^{-1}(B)))$.

Theorem 3.4. Suppose that Y has a base consisting of IF clopen sets. If $f : X \rightarrow Y$ is IF slightly β -continuous, then f is IF β -continuous.

Proof. Let $p_{(\alpha,\beta)} \in X$ and let C be IFOS in Y containing $f(p_{(\alpha,\beta)})$. Since Y has a base consisting of IF clopen sets, there exists an IF clopen set B containing $f(p_{(\alpha,\beta)})$ such that $B \subseteq C$. Since f is IF slightly β -continuous, then there exists an IF β OS A in X containing $p_{(\alpha,\beta)}$ such that $f(A) \subseteq B \subseteq C$. Thus f is IF β -continuous. \square

Theorem 3.5. *If a function $f : X \rightarrow \Pi Y_i$ is a IF slightly fuzzy β -continuous, then $P_i \circ f : X \rightarrow Y_i$ is IF slightly β -continuous, where P_i is the projection of ΠY_i onto Y_i .*

Proof. Let B_i be any IF clopen sets of Y_i . Since, P_i is IF continuous and IF open mapping, and $P_i : \Pi Y_i \rightarrow Y_i$, $P_i^{-1}(B_i)$ is IF clopen sets in ΠY_i . Now $(P_i \circ f)^{-1}(B_i) = f^{-1}(P_i^{-1}(B_i))$. As f is IF slightly fuzzy β -continuous and $P_i^{-1}(B_i)$ is IF clopen sets, $f^{-1}(P_i^{-1}(B_i))$ is IF β OS in X . Hence $(P_i \circ f)$ is IF slightly fuzzy β -continuous. \square

Theorem 3.6. *The following hold for functions $f : X \rightarrow Y$ and $g : Y \rightarrow Z$*

- (i) *If f is IF slightly β -continuous and g is IF totally continuous then $g \circ f$ is IF β -continuous.*
- (ii) *If f is IF β -irresolute and g is IF slightly β -continuous then $g \circ f$ is IF slightly β -continuous.*

Proof. (i) Let B be an IFOS in Z . Since g is IF totally continuous, $g^{-1}(B)$ is an IF clopen set in Y . Now $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$. Since f is IF slightly β -continuous, $f^{-1}(g^{-1}(B))$ is IF β OS in X . Hence $g \circ f$ is IF β -continuous.

(ii) Let B be IF clopen set in Z . Since g is IF slightly β -continuous, $g^{-1}(B)$ is an IF β OS in Y . Now $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$. Since f is IF β -irresolute, $f^{-1}(g^{-1}(B))$ is IF β OS in X which implies $g \circ f$ is IF slightly β -continuous. \square

4 Intuitionistic fuzzy β -separation axioms

In this section, we investigate the relationships between IF slightly β -continuous functions and IF β -separation axioms.

Definition 4.1. An IFTS (X, τ) is called $\beta - T_1$ ($co - T_1$, [14]) if and only if for each pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}, y_{(\nu,\delta)}$ in X there exists intuitionistic fuzzy β -open sets (IF clopen sets) $U, V \in X$ such that $x_{(\alpha,\beta)} \in U$, $y_{(\nu,\delta)} \notin U$ and $y_{(\nu,\delta)} \in V$, and $x_{(\alpha,\beta)} \notin V$.

Theorem 4.1. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is IF slightly β -continuous injection and Y is $co - T_1$, then X is IF $\beta - T_1$.*

Proof. Suppose that Y is IF $co - T_1$. For any distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}, y_{(\nu,\delta)}$ in X , there exists IF clopen sets A, B in Y such that $f(x_{(\alpha,\beta)}) \in A$, $f(y_{(\nu,\delta)}) \notin A$, $f(x_{(\alpha,\beta)}) \notin B$ and $f(y_{(\nu,\delta)}) \in B$. Since f is IF slightly β -continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are IF β -open sets in X such that $x_{(\alpha,\beta)} \in f^{-1}(A)$, $y_{(\nu,\delta)} \notin f^{-1}(A)$, $x_{(\alpha,\beta)} \notin f^{-1}(B)$, $y_{(\nu,\delta)} \in f^{-1}(B)$. This shows that X is IF $\beta - T_1$. \square

Definition 4.2. An IFTS X is said to be $\beta - T_2$ or β -Hausdorff($co - T_2$ or co -Hausdorff, [14]) if for all pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}, y_{(\nu,\delta)}$ in X , there exists IF β -open sets (IF clopen sets) $U, V \in X$ such that $x_{(\alpha,\beta)} \in U, y_{(\nu,\delta)} \in V$ and $U \cap V = 0_{\sim}$.

Theorem 4.2. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is IF slightly β -continuous injection and Y is $co - T_2$, then X is IF $\beta - T_2$.

Proof. Suppose that Y is IF $co - T_2$. For any distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}, y_{(\nu,\delta)}$ in X , there exists IF clopen sets A, B in Y such that $f(x_{(\alpha,\beta)}) \in A$, and $f(y_{(\nu,\delta)}) \in B$. Since f is IF slightly β -continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are IF β -open sets in X such that $x_{(\alpha,\beta)} \in f^{-1}(A)$, and $y_{(\nu,\delta)} \in f^{-1}(B)$. Also we have $f^{-1}(A) \cap f^{-1}(B) = 0_{\sim}$. Hence X is IF $\beta - T_2$. \square

Definition 4.3. An IFTS X is said to be IF strongly β -regular (IF co-regular [14]) if for each IF β -closed (IF clopen) set C and each IFP $x_{(\alpha,\beta)} \notin C$, there exist intuitionistic fuzzy open sets A and B such that $C \subseteq A, x_{(\alpha,\beta)} \in B$ and $A \cap B = 0_{\sim}$.

Definition 4.4. An IFTS X is said to be IF strongly β -normal (IF co-normal [14]) if for each IF β -closed (IF clopen) sets C_1 and C_2 in X such that $C_1 \cap C_2 = 0_{\sim}$, there exist intuitionistic fuzzy open sets A and B such that $C_1 \subseteq A$ and $C_2 \subseteq B$ and $A \cap B = 0_{\sim}$.

Theorem 4.3. If F is IF slightly β -continuous injective IF open function from an IF strongly β -regular space X onto a IF space Y , then Y is IF co-regular.

Proof. Let D be IF clopen set in Y and $y_{(\nu,\delta)} \notin D$. Take $y_{(\nu,\delta)} = f(x_{(\alpha,\beta)})$. Since f is IF slightly β -continuous, $f^{-1}(D)$ is a IF β -closed set in X . Let $C = f^{-1}(D)$. So $x_{(\alpha,\beta)} \notin C$. Since X is IF strongly β -regular, there exist intuitionistic fuzzy open sets A and B such that $C \subseteq A, x_{(\alpha,\beta)} \in B$ and $A \cap B = 0_{\sim}$. Hence, we have $D = f(C) \subseteq f(A)$ and $y_{(\nu,\delta)} = f(x_{(\alpha,\beta)}) \in f(B)$ such that $f(A)$ and $f(B)$ are disjoint IF open sets. Hence Y is IF co-regular. \square

Theorem 4.4. If f is IF slightly β -continuous, injective, IF open function from a IF strongly β -normal space X onto a IF space Y , then Y is IF co-normal.

Proof. Let C_1 and C_2 be disjoint IF clopen sets in Y . Since f is IF slightly β -continuous, $f^{-1}(C_1)$ and $f^{-1}(C_2)$ are IF β -closed sets in X . Let us take $C = f^{-1}(C_1)$ and $D = f^{-1}(C_2)$. We have $C \cap D = 0_{\sim}$. Since X is IF strongly β -normal, there exist disjoint IF open sets A and B such that $C \subseteq A$ and $D \subseteq B$. Thus $C_1 = f(C) \subseteq f(A)$ and $C_2 = f(D) \subseteq f(B)$ such that $f(A)$ and $f(B)$ disjoint IF open sets. Hence Y is IF co-normal. \square

5 Intuitionistic fuzzy covering properties and Intuitionistic fuzzy β -connectedness

In this section, we investigate the relationships between IF slightly β -continuous and IF compactness and between IF slightly β -continuous and IF connectedness.

Definition 5.1. An IFTS X is said to be

- (1) IF β -compact if every IF β -open cover of X has a finite subcover, [13].
- (2) IF countably β -compact if every β -open countably cover of X has a finite subcover.
- (3) IF β -Lindelof if every cover of X by IF β -open sets has a countable subcover, [13].
- (4) IF mildly compact if every IF clopen cover of X has a finite subcover.
- (5) IF mildly countably compact if every IF clopen countably cover of X has a finite subcover.
- (6) IF mildly Lindelof if every cover of X has IF clopen sets has a countable subcover.

Theorem 5.1. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a IF slightly β -continuous surjection. Then the following statements hold:

- (1) if X is IF β -compact, then Y is IF mildly compact.
- (2) if X is IF β -Lindelof, then Y is IF mildly Lindelof.
- (3) if X is IF countably β -compact, then Y is IF mildly countably compact.

Proof. (1) Let $\{A_\alpha; \alpha \in I\}$ be any IF clopen cover of Y . Since f is IF slightly β -continuous, then $\{f^{-1}(A_\alpha); \alpha \in I\}$ is IF β -open cover of X . Since X is IF β -compact, there exists a finite subset I_0 of I such that $1_{\sim X} = \bigcup \{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus, we have $1_{\sim Y} = \bigcup \{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly compact.

(2) Let $\{A_\alpha; \alpha \in I\}$ be any IF clopen cover of Y . Since f is IF slightly β -continuous, then $\{f^{-1}(A_\alpha); \alpha \in I\}$ is IF β -open cover of X . Since X is IF β -Lindelof, there exists a countable subset I_0 of I such that $1_{\sim X} = \bigcup \{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus, we have $1_{\sim Y} = \bigcup \{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly Lindelof.

(3) Let $\{A_\alpha; \alpha \in I\}$ be any IF clopen cover of Y . Since f is IF slightly β -continuous, then $\{f^{-1}(A_\alpha); \alpha \in I\}$ is IF β -clopen cover of X . Since X is IF countably β -compact, for countable β -clopen cover $\{f^{-1}(A_\alpha); \alpha \in I\}$ in X , there exists a finite subset I_0 of I such that $1_{\sim X} = \bigcup \{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus, we have $1_{\sim Y} = \bigcup \{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly countable compact. \square

Definition 5.2. An IFTS X is said to be:

- (1) IF β -closed compact if every β -closed of X has a finite subcover.
- (2) IF β -closed Lindelof if every cover of X by β -closed sets has a countable subcover.
- (3) IF countably β -closed compact if every countable cover of X by β -closed sets has a finite subcover.

Theorem 5.2. Let $f : X \rightarrow Y$ be a IF slightly β -continuous surjection. Then the following statements hold:

- (1) if X is IF β -closed compact, then Y is mildly compact.
(2) if X is IF β -closed Lindelof, then Y is mildly Lindelof.
(3) if X is IF countably β -closed compact, then Y is mildly countably compact.

Proof. (1) Let $\{A_\alpha; \alpha \in I\}$ be any IF clopen cover of Y . Since f is IF slightly β -continuous, then $\{f^{-1}(A_\alpha); \alpha \in I\}$ is IF β -closed cover of X . Since X is TF β -closed compact, there exists a finite subset I_0 of I such that $1_{\sim X} = \bigcup \{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus, we have $1_{\sim Y} = \bigcup \{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly compact.

Similarly, we can obtained the proof for (2) and (3). \square

Definition 5.3. [13] An IFTS (X, τ) is said to be intuitionistic fuzzy β -disconnected (IF β -disconnected) if there exists IF β OS A, B in X such that $A \neq 0_{\sim}, B \neq 0_{\sim}$ such that $A \cup B = 1_{\sim}$ and $A \cap B = 0_{\sim}$. If X is not IF β -disconnected then it is said to be intuitionistic fuzzy β -connected (IF β -connected).

Theorem 5.3. If Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a IF slightly β -continuous surjection, (X, τ) is an IF β -connected, then (Y, σ) is IF connected.

Proof. Assume that (Y, σ) is not IF connected then there exists non-empty intuitionistic fuzzy open sets A and B in (Y, σ) such that $A \cup B = 1_{\sim}$ and $A \cap B = 0_{\sim}$. Therefore, A and B are intuitionistic fuzzy clopen sets in Y . Since f is IF slightly β -continuous, $C = f^{-1}(A) \neq 0_{\sim}, D = f^{-1}(B) \neq 0_{\sim}$, which are intuitionistic fuzzy β -open sets in X . And $f^{-1}(A) \cup f^{-1}(B) = f^{-1}(1_{\sim}) = 1_{\sim}$, which implies $C \cup D = 1_{\sim}$. And $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(0_{\sim}) = 0_{\sim}$, which implies $C \cap D = 0_{\sim}$. Thus X is IF β -disconnected, which is a contradiction to our hypothesis. Hence Y is IF connected. \square

Definition 5.4. [5] An intuitionistic fuzzy set A in intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy dense if there exists no intuitionistic fuzzy closed set B in (X, τ) such that $A \subseteq B \subseteq 1_{\sim}$.

Definition 5.5. [14] A IFTS X is called hyperconnected if every IF open set in X is dense.

Remark 5.1. The following example shows that IF slightly β -continuous surjection do not necessarily preserve IF hyperconnectedness.

Example 5.1. Let $X = \{a, b, c\}, \tau = \{0_{\sim}, 1_{\sim}, A\}, \sigma = \{0_{\sim}, 1_{\sim}, B, C, B \cup C, B \cap C\}$, where

$$A = \{\langle x, (\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}) \rangle | x \in X\},$$

$$B = \{\langle x, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.5}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.5}) \rangle | x \in X\},$$

$$C = \{\langle x, (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.3}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.7}) \rangle | x \in X\}.$$

Define an intuitionistic fuzzy mapping $f : (X, \tau) \rightarrow (X, \sigma)$ by $f(a) = a, f(b) = b, f(c) = c$. Then f is IF slightly β -continuous surjective. (X, τ) is hyperconnected. But (X, σ) is not hyperconnected.

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