ICIFSS, 8–10 January 2018, Erode, Tamil Nadu, India
Notes on Intuitionistic Fuzzy Sets
Print ISSN 1310–4926, Online ISSN 2367–8283
Vol. 24, 2018, No. 1, 30–36
DOI: 10.7546/nifs.2018.24.1.30-36

Intuitionistic fuzzy digital *CS*-filtered structured spaces

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Received: 29 September 2017 **Accepted:** 1 November 2017

Abstract: The motive of this article is to introduce a new version of Intuitionistic fuzzy digital *CS*-filtered structure spaces and Intuitionistic fuzzy digital Hausdorff *CS*-filtered structure spaces in the Euclidean plane. Also we defined and studied about $\mathfrak{D}^*$ structure saturated sets, $\mathfrak{D}^*$compact structure spaces and $\mathfrak{D}^*$ structure filtered family of sets. Moreover some of the properties are exhibited related to the above said spaces.
Keywords: $\mathfrak{D}^*$ structure saturated sets, $\mathfrak{D}^*$ structure filtered family, $\mathfrak{D}^*$ *CS*-filtered structure spaces, $\mathfrak{D}^*$ Hausdorff *CS*-filtered structure spaces, $\mathfrak{D}^*$compact structure spaces and $\mathfrak{D}^*$ structure continuous functions.
2010 Mathematics Subject Classification: 54A40, 03E72.

1 Introduction

In 1965, L. A. Zadeh [10] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty in real physical world. In 1983, K. T. Atanassov [2] published the concept of intuitionistic fuzzy sets and many works by him and his colleagues appeared in the literature [1, 3].

Classical digital topology primarily concerns itself in the study of black white images in the digital plane [4, 7]. A. Rosenfeld [9] represented the gray scale level images by the concept of fuzzy sets.

The two characteristic functions, namely, the membership and the non-membership functions, are used to define an intuitionistic fuzzy set (IFS) which describe, respectively, the belongingness or non-belongingness of an element. Because of this nature, the brighter and "non-brighter" parts of a digital image can be analyzed efficiently. In image processing, it is proved that the results using IFS is better than the fuzzy set theory.

This paper introduces the concepts of intuitionistic fuzzy digital *CS*-filtered structure spaces and intuitionistic fuzzy digital Hausdorff *CS*-filtered structure spaces in the Euclidean plane and discusses some of its properties.

2 Preliminaries

To understand the theme of this paper, some definitions and results are recalled in this section. Throughout this paper E denotes the Euclidean plane and J denotes the index set.

Definition 2.1 [8]: Let Σ be a rectangular array of integer-coordinate points or lattice points in the Euclidean plane. Thus the point $P = (x, y)$ of Σ has four horizontal and vertical neighbors, namely $(x \pm 1, y)$ and $(x, y \pm 1)$ and it also has four diagonal neighbors, namely $(x \pm 1, y \pm 1)$ and $(x \pm 1, y \mp 1)$. We say that former points are 4-adjacent to, or 4-neighbors of P and we say that both types of neighbors are 8-adjacent to, or 8-neighbors of P. Note that if P is on the border of Σ, some of these neighbors may not exist.

Definition 2.2 [3]: Let X be a nonempty fixed set and I be the closed interval [0, 1]. An intuitionistic fuzzy set (IFS) A is an object of the following form

$$A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\},$$

where the mappings $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely, $\mu_A(x)$) and the degree of nonmembership (namely, $\gamma_A(x)$) for each element $x\in X$ to the set A, respectively, and $0\le\mu_A(x)+\gamma_A(x)\le 1$ for each $x\in X$. For the sake of simplicity, we shall use the symbol $A=\langle x,\mu_A,\gamma_A\rangle$ for the intuitionistic fuzzy set $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$.

Definition 2.3 [3]: Let X be a nonempty fixed set and the IFSs A and B be in the form $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$, $B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$. Then,

i. $A \subseteq B$ iff $\mu_A(x)\le\mu_B(x)$ and $\gamma_A(x)\ge\gamma_B(x)$ for all $x\in X$;
ii. $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
iii. The complement of A, $\overline{A}=\{\langle x,\gamma_A(x),\mu_A(x)\rangle : x\in X\}$;
iv. $A\cup B=\{\langle x,\mu_A(x)\vee\mu_B(x),\gamma_A(x)\wedge\gamma_B(x)\rangle : x\in X\}$;
v. $A\cap B=\{\langle x,\mu_A(x)\wedge\mu_B(x),\gamma_A(x)\vee\gamma_B(x)\rangle : x\in X\}$;
vi. $0_\sim = \{\langle x, 0, 1\rangle\} : x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle\} : x \in X\}$

Definition 2.4 [5]: Let E be the Euclidean plane. An intuitionistic fuzzy digital structure on E is a family $\mathfrak{D}^*$ of IFD sets in E if the following axioms are satisfied:

i. $0_\sim, 1_\sim \in \mathfrak{D}^*$;
ii. $G_{1\sim} \cap G_{2\sim} \in \mathfrak{D}^*$ for any $G_{1\sim}, G_{2\sim} \in \mathfrak{D}^*$;
iii. $\cup G_{i\sim} \in \mathfrak{D}^*$ for arbitrary family $\{G_{i\sim} \mid i \in J\} \subseteq \mathfrak{D}^*$.

Then the ordered pair $(E, \mathcal{D}^*)$ is called an intuitionistic fuzzy digital structure space or $\mathcal{D}^*$ structure space. Each element of $\mathcal{D}^*$ structure space is said to be a $\mathcal{D}^*$ open set in E. The complement of a $\mathcal{D}^*$ open set is said to be a $\mathcal{D}^*$ closed set in E.

Definition 2.5 [6]: A directed family of a set S is a family (D_i), $i \in I$ of subsets of S, such that for every $i, j \in I$ there is some $k \in I$ such, that $D_i \subseteq D_k$ and $D_j \subseteq D_k$.

3 Intuitionistic fuzzy digital compact structure spaces

Definition 3.1: Let $(E, \mathcal{D}^*)$ be a $\mathcal{D}^*$ structure space $(E, \mathcal{D}^*)$ and let $\mathcal{A}$ be a family of $\mathcal{D}^*$ open sets. Then $\mathcal{A}$ is called an intuitionistic fuzzy digital structure open cover (or $\mathcal{D}^*$ structure open cover) of E if $\cup\{A_{i\sim} : A_{i\sim} \in \mathcal{A}, i \in J\} = 1_\sim$.

A finite subfamily $\mathcal{A}_s$ of the finite $\mathcal{D}^*$ structure open cover $\mathcal{A}$ is said to be an intuitionistic fuzzy digital structure open subcover (or $\mathcal{D}^*$ structure open subcover) of E if $\mathcal{A}_s$ itself is a $\mathcal{D}^*$ structure open cover.

Definition 3.2: A $\mathcal{D}^*$ structure space $(E, \mathcal{D}^*)$ is said to be an intuitionistic fuzzy digital compact structure space (or $\mathcal{D}^*$ compact structure space) if every $\mathcal{D}^*$ structure open cover of E contains a finite $\mathcal{D}^*$ structure open subcover.

Definition 3.3: Let $(E, \mathcal{D}_1^*)$ and $(E, \mathcal{D}_2^*)$ be any two $\mathcal{D}^*$ structure spaces. A function $f : (E, \mathcal{D}_1^*) \to (E, \mathcal{D}_2^*)$ is said to be intuitionistic fuzzy digital structure continuous (or $\mathcal{D}^*$ structure continuous) if $f^{-1}(A_\sim)$ is $\mathcal{D}_1^*$ open in $(E, \mathcal{D}_1^*)$, for each $\mathcal{D}_2^*$ open set $A_\sim$ of $(E, \mathcal{D}_2^*)$.

Proposition 3.1. Let $(E, \mathcal{D}_1^*)$ and $(E, \mathcal{D}_2^*)$ be any two $\mathcal{D}^*$ structure spaces and let $f : (E, \mathcal{D}_1^*) \to (E, \mathcal{D}_2^*)$ is a $\mathcal{D}^*$ structure continuous function. If $A_\sim$ is a $\mathcal{D}_1^*$ structure compact subset of $(E, \mathcal{D}_1^*)$, then $f(A_\sim)$ is $\mathcal{D}_2^*$ structure compact subset of $(E, \mathcal{D}_2^*)$.

Proof: Let $\mathcal{A} = \{B_{i\sim} : i \in J\}$ be a $\mathcal{D}^*$ structure open cover of $f(A_\sim)$. Since f is $\mathcal{D}^*$ structure continuous, the collection $\{f^{-1}(B_{i\sim}) : i \in J\}$ is a $\mathcal{D}^*$ structure open covering of $A_\sim$. Now $A_\sim$ is a $\mathcal{D}^*$ structure compact set and so there exists a finite $\mathcal{D}^*$ structure open subcover $\{f^{-1}(B_{1\sim}), f^{-1}(B_{2\sim}), \dots, f^{-1}(B_{n\sim})\}$ of $A_\sim$. Therefore, the collection $\{B_{1\sim}, B_{2\sim}, \dots, B_{n\sim}\}$ is a $\mathcal{D}^*$ structure open cover of $f(A_\sim)$. □

Definition 3.4: In a $\mathcal{D}^*$ structure space $(E, \mathcal{D}^*)$, a subcollection $\mathcal{B}$ of $\mathcal{D}^*$ is said to be a $\mathcal{D}^*$ structure base for $\mathcal{D}^*$ if for every $\mathcal{D}^*$ open set $A_\sim \in \mathcal{D}^*$, there exists a collection $\{A_{i\sim} : i \in J\} \subset \mathcal{B}$ such that $A_\sim = \bigcup_{i \in J} A_{i\sim}$.

A collection $\mathcal{B}_s$ of $\mathcal{D}^*$ is said to be a $\mathcal{D}^*$ structure subbase for $\mathcal{D}^*$ if collection $\{\bigcap_{i=1}^{n} B_{i\sim} : B_{i\sim} \in \mathcal{B}_s, i \in J\}$ is a $\mathcal{D}^*$ structure base for $\mathcal{D}^*$.

Proposition 3.2. If $\mathcal{B}_S$ is the $\mathcal{D}^*$ structure subbase of a $\mathcal{D}^*$ *CS*-filtered structure space $(E, \mathcal{D}^*)$ and if for every $\mathcal{D}^*$ structure open cover $C = \{B_{i\sim} \in \mathcal{B}_S : i \in J\}$ there exists a finite $\mathcal{D}^*$ structure open subcover $\{B_{1\sim}, B_{2\sim}, \ldots, B_{n\sim}\}$, then $(E, \mathcal{D}^*)$ is a $\mathcal{D}^*$ compact structure space.
Proof: Contrary to the hypothesis, suppose $\mathcal{A}$ is a $\mathcal{D}^*$ structure open cover of $(E, \mathcal{D}^*)$ such that it has no finite $\mathcal{D}^*$ structure open subcover and also assume that $\mathcal{A}$ is the maximal $\mathcal{D}^*$ structure open cover having this property. According to this assumption, $\mathcal{B}_S \cap \mathcal{A}$ could not be a $\mathcal{D}^*$ structure open cover of $(E, \mathcal{D}^*)$. Let $P \in \cup(\mathcal{B}_S \cap \mathcal{A})$ and so there is a $\mathcal{D}^*$ open set $A_\sim \in \mathcal{A}$, such that $P \in A_\sim$ and $P \in \bigcap_{i=1}^{n} B_{i\sim} \subset A_\sim$ for some $\{B_{1\sim}, B_{2\sim}, \ldots B_{n\sim}\} \subset \mathcal{B}_S$. Since $P \in \overline{\cup(B_S \cap A)}$, $\mathcal{B}_S \cap \mathcal{A}$ does not contain any $B_{i\sim}$. By our assumption, for each i, $\mathcal{A} \cup \{B_{i\sim}\}$ has a finite $\mathcal{D}^*$ structure open subcover. Let $\{A_{1i\sim}, A_{2i\sim}, \ldots A_{ti\sim}\} \cup \{B_{i\sim}\}$. Then

$$\{A_{1i\sim}, A_{2i\sim}, \ldots, A_{ni\sim}\}_{i=1}^{ti} \cup \left\{ \bigcap_{i=1}^{n} B_{i\sim} \right\}$$

is a finite $\mathcal{D}^*$ structure open cover of $(E, \mathcal{D}^*)$, which shows that $\{A_{1i\sim}, A_{2i\sim}, \ldots A_{ni\sim}\}_{i=1}^{ti} \cup A_\sim$ is a finite $\mathcal{D}^*$ structure open cover of $(E, \mathcal{D}^*)$. But this cover contains only the $\mathcal{D}^*$ open sets of $\mathcal{A}$, which is a contradiction.

4 Intuitionistic fuzzy digital *CS*-filtered structure spaces

Definition 4.1: Let $A_\sim$ be a $\mathcal{D}^*$ open set of the $\mathcal{D}^*$ structure space $(E, \mathcal{D}^*)$. Then $A_\sim$ is said to be an intuitionistic fuzzy digital structure saturated set (or $\mathcal{D}^*$ structure saturated set) if $A_\sim = \cap\{A_{\sim i}, i \in J : A_\sim \subseteq A_{\sim i} \text{ and } A_{\sim i} \in \mathcal{D}^*\}$.

The smallest $\mathcal{D}^*$ structure saturated set containing $A_\sim$ is denoted by $\mathbb{S}(A_\sim)$.

Proposition 4.1. In a $\mathcal{D}^*$ structure space $(E, \mathcal{D}^*)$, the arbitrary intersection of $\mathcal{D}^*$ structure saturated sets is again a $\mathcal{D}^*$ structure saturated set.
Proof: The proof directly follows from the Definition 3.1. □

Definition 4.2: Let $\mathcal{A} = \cap\{A_{\sim i}, i \in J : A_{\sim i} \in \mathcal{D}^*\}$ be a nonempty family of IFD subsets of the $\mathcal{D}^*$ structure space $(E, \mathcal{D}^*)$. Then $\mathcal{A}$ is called an intuitionistic fuzzy digital structure filtered family (or $\mathcal{D}^*$ structure filtered family) if for any two IFD sets, $A_{\sim i}$ and $A_{\sim j} \in \mathcal{A}$ there exists some other IFD set $A_{\sim k} \in \mathcal{A}$ such that $A_{\sim k} \in A_{\sim i} \cup A_{\sim j}$.

Definition 4.3: Let $(E, \mathcal{D}^*)$ be a $\mathcal{D}^*$ structure space $(E, \mathcal{D}^*)$ and let $\mathcal{A}_i$, $i \in J$ be the $\mathcal{D}^*$ structure filtered family a nonempty families of $\mathcal{D}^*$ structure compact saturated sets in $(E, \mathcal{D}^*)$. Then $(E, \mathcal{D}^*)$ is said to be an intuitionistic fuzzy digital compact saturated filtered structure space (or $\mathcal{D}^*$ *CS*-filtered structure space) if for every $i \in J$, $\cap\mathcal{A}_i \in B_\sim$ for some $\mathcal{D}^*$ open set $B_\sim \neq 0_\sim$.

Proposition 4.2. In a $\mathfrak{D}^*$ *CS*-filtered structure space (E, $\mathfrak{D}^*$), the intersection of every $\mathfrak{D}^*$ structure filtered family of $\mathfrak{D}^*$ structure compact saturated subsets is again a $\mathfrak{D}^*$ structure compact saturated set.

Proof: Let $\mathcal{A} = \{A_{\sim i},\ i \in J\}$ be the $\mathfrak{D}^*$ structure filtered family of $\mathfrak{D}^*$ structure compact saturated subsets of (E, $\mathfrak{D}^*$). Suppose $\bigcap_{i\in J} A_{i\sim} = 0_\sim$, then the proof is obvious. If $\bigcap_{i\in J} A_{i\sim} \neq 0_\sim$ and $\mathcal{B} \subseteq \mathfrak{D}^*$ is a $\mathfrak{D}^*$ structure open cover of $\bigcap_{i\in J} A_{i\sim} \bigcup \mathcal{B}$, then there exists $A_{i\sim}$, for some $i \in J$ such that $A_{i\sim} \subseteq \bigcup \mathcal{B}$ because (E, $\mathfrak{D}^*$) is a $\mathfrak{D}^*$ *CS*-filtered structure space. Hence there exists some $B_\sim \in \mathcal{B}$, such that $A_{i\sim} \subseteq B_\sim$, since $A_{i\sim}$ is $\mathfrak{D}^*$ structure compact. Thus, $\bigcap_{i\in J} A_{i\sim} \subseteq B_\sim$, for some $B_\sim \in \mathcal{B}$. Therefore, $\bigcap_{i\in J} A_{i\sim}$ is $\mathfrak{D}^*$ structure compact. From the Proposition 3.1, arbitrary intersection of $\mathfrak{D}^*$ structure saturated sets is $\mathfrak{D}^*$ structure saturated and so $\bigcap_{i\in J} A_{i\sim}$ is a $\mathfrak{D}^*$ structure compact saturated set. □

Proposition 4.3. A $\mathfrak{D}^*$ structure space (E, $\mathfrak{D}^*$) is a $\mathfrak{D}^*$ *CS-filtered* structure space if and only if for every $\mathfrak{D}^*$ structure filtered family $\mathcal{A} = \{A_{\sim i} : A_{\sim i} \neq 0_\sim,\ i \in J\}$ of $\mathfrak{D}^*$ structure compact saturated sets such that $\bigcap_{i\in J} A_{i\sim} \neq 0_\sim$ and $\bigcap_{i\in J} A_{i\sim} \subseteq B_\sim$ for any nonempty $\mathfrak{D}^*$ open set $B_\sim$, implies $A_{i\sim} \subseteq B_\sim$ for some $i \in J$.

Proof: $\Rightarrow$ Clearly, the proof follows from the Definition 3.5.

$\Leftarrow$ To prove this part, consider a point $P \in B_\sim$. Then $\{A_{i\sim} \cup \mathbb{S}(P) : i \in J\}$ is nonempty and it is a $\mathfrak{D}^*$ structure filtered family of $\mathfrak{D}^*$ structure compact saturated sets. Also $\mathbb{S}(P) \neq 0_\sim$, $\mathbb{S}(P) \subseteq \bigcap_{i\in J} (A_{i\sim} \cup \mathbb{S}(P))$ and $\bigcap_{i\in J} (A_{i\sim} \cup \mathbb{S}(P)) \subseteq B_\sim$. Hence $A_{i\sim} \subseteq A_{i\sim} \cup \mathbb{S}(P) \subseteq B_\sim$ for some $i \in J$. Therefore (E, $\mathfrak{D}^*$) is a $\mathfrak{D}^*$ *CS*-filtered structure space. □

Proposition 4.4. A $\mathfrak{D}^*$ structure space (E, $\mathfrak{D}^*$) is a $\mathfrak{D}^*$ *CS*-filtered structure space if and only if for each $\mathfrak{D}^*$ structure filtered family $\mathcal{A} = \{A_{\sim i} : i \in J\}$ of $\mathfrak{D}^*$ structure compact saturated sets such that for every nonempty $\mathfrak{D}^*$ closed set $B_\sim \neq 1_\sim$, $A_{i\sim} \cap B_\sim \neq 0_\sim$ for all $i \in J$, implies $\bigcap_{i\in J} A_{i\sim} \cap B_\sim$ is nonempty.

Proof: The proof follows from the Proposition 3.3, since the complement of the $\mathfrak{D}^*$ closed set is $\mathfrak{D}^*$ open. □

Proposition 4.5. Let (E, $\mathfrak{D}^*$) be a $\mathfrak{D}^*$ *CS*-filtered structure space, $\mathcal{A}$ be a $\mathfrak{D}^*$ structure filtered family of $\mathfrak{D}^*$ structure compact saturated sets and let $\mathcal{B}$ be a $\mathfrak{D}^*$ structure *CS*-filtered family of $\mathfrak{D}^*$ closed sets, where $\mathfrak{D}^* \notin \mathcal{B}$. If $A_\sim \cap B_\sim \neq 0_\sim$ for each $A_\sim \in \mathcal{A}$ and $B_\sim \in \mathcal{B}$, then the following statements hold.

i. $(\bigcap\mathcal{B}) \cap (\bigcap\mathcal{A})$ is nonempty.
ii. Every $\mathfrak{D}^*$ open set which contains the intersection $(\bigcap\mathcal{B}) \cap (\bigcap\mathcal{A})$ will also contain some IFD subset $A_\sim \cap B_\sim \neq 0_\sim$.
iii. $(\bigcap\mathcal{B}) \cap (\bigcap\mathcal{A})$ is $\mathfrak{D}^*$ structure compact.

Proof: (*i*) If $(\cap\mathcal{B}) \cap (\cap\mathcal{A})$ is nonempty then $\cap\mathcal{A} \subseteq (\overline{\cap B}) = \{\overline{B_{\sim}} : B_{\sim} \in \mathcal{B}\}$ is a directed family of nonempty $\mathfrak{D}^*$ open sets. Since $(E, \mathfrak{D}^*)$ is a $\mathfrak{D}^*$*CS-filtered* structure space, there exists some $A_{\sim} \in \mathcal{A}$, such that $A_{\sim} \in \cup\{\overline{B_{\sim}} : B_{\sim} \in \mathcal{B}\}$. Also there exists some $B_{\sim} \in \mathcal{B}$ such that $A_{\sim} \subseteq \overline{B_{\sim}}$, because $A_{\sim}$ is $\mathfrak{D}^*$ structure compact. Hence $A_{\sim} \cap B_{\sim} = 0_{\sim}$, which contradicts the fact that $A_{\sim} \cap B_{\sim} \neq 0_{\sim}$. Thus $(\cap\mathcal{B}) \cap (\cap\mathcal{A})$ is nonempty.

(*ii*) Assume that $C_{\sim}$ is a $\mathfrak{D}^*$ open set containing $(\cap\mathcal{B}) \cap (\cap\mathcal{A})$. Let $C = \{\mathcal{B} \cap \overline{C_{\sim}} : \mathcal{B} \in C\}$. If there is no IFD set $B_{\sim} \cap A_{\sim}$ in $(\cap\mathcal{B}) \cap (\cap\mathcal{A})$ such that $B_{\sim} \cap A_{\sim} \subseteq C_{\sim}$, then $(B_{\sim} \cap \overline{C_{\sim}}) \cap A_{\sim}$ is nonempty for every $B_{\sim} \cap A_{\sim}$. From proof (*i*), it is obtained that $(\cap C) \cap (\cap\mathcal{A})$ is nonempty, which is a contradiction to or assumption.

(*iii*) Assume that $\mathcal{F}$ is a $\mathfrak{D}^*$ structure open cover of $(\cap\mathcal{B}) \cap (\cap\mathcal{A})$. From proof (*i*), there exists $A_{\sim} \in \mathcal{A}$ and $B_{\sim} \in \mathcal{B}$ such that $B_{\sim} \cap A_{\sim} \subseteq \cup\mathcal{F}$. Also there exists a finite subfamily $\{F_{1\sim}, F_{2\sim}, \ldots, F_{n\sim}\}$ of $\mathcal{F}$ such that $B_{\sim} \cap A_{\sim} \subseteq \cup F_i$, because $B_{\sim}$ is $\mathfrak{D}^*$ closed and $A_{\sim}$ is $\mathfrak{D}^*$ structure compact. Therefore, $(\cap\mathcal{B}) \cap (\cap\mathcal{A})$ is $\mathfrak{D}^*$ structure compact. □

Proposition 4.6. Let $(E, \mathfrak{D}_1^*)$ and $(E, \mathfrak{D}_2^*)$ be any two $\mathfrak{D}^*$ structure spaces such that there are $\mathfrak{D}^*$ structure continuous functions $f : (E, \mathfrak{D}_2^*) \to (E, \mathfrak{D}_1^*)$ and $g : (E, \mathfrak{D}_1^*) \to (E, \mathfrak{D}_2^*)$ such that $g \circ f : (E, \mathfrak{D}_2^*) \to (E, \mathfrak{D}_2^*)$. Then $(E, \mathfrak{D}_2^*)$ is a $\mathfrak{D}^*$ *CS*-filtered structure space if $(E, \mathfrak{D}_1^*)$ is a $\mathfrak{D}^*$ *CS*-filtered structure space.

Proof: Assume that $(E, \mathfrak{D}_1^*)$ is a $\mathfrak{D}^*$ *CS*-filtered structure space and assume $f : (E, \mathfrak{D}_2^*) \to (E, \mathfrak{D}_1^*)$ and $g : (E, \mathfrak{D}_1^*) \to (E, \mathfrak{D}_2^*)$ are $\mathfrak{D}^*$ structure continuous functions such that $g \circ f : (E, \mathfrak{D}_2^*) \to (E, \mathfrak{D}_2^*)$. Let $\{A_{i\sim} : i \in J\}$ be a $\mathfrak{D}^*$ structure filtered family of $\mathfrak{D}^*$ structure compact saturated subsets of $(E, \mathfrak{D}_2^*)$ and let $B_{\sim}$ be a nonempty $\mathfrak{D}_2^*$ open set such that $\bigcap_{i\in J} A_{i\sim} \subseteq B_{\sim}$. Since f is $\mathfrak{D}^*$ structure continuous, $f(A_{i\sim})$ is $\mathfrak{D}^*$ structure compact in $(E, \mathfrak{D}_1^*)$ for any fixed $i \in J$. Hence $\mathbb{S}(A_{i\sim})$ is $\mathfrak{D}^*$ structure compact saturated set in $(E, \mathfrak{D}_1^*)$ and so since g is $\mathfrak{D}^*$ structure continuous $\mathbb{S}(g(\mathbb{S}(f(A_{i\sim})))) = \mathbb{S}(g \circ f(A_{i\sim})) = A_{i\sim}$.

Thus, $\bigcap_{i\in J} g(\mathbb{S}(f(A_{i\sim})))\mathbb{S}(f(A_{i\sim}))\mathbb{S}(f(A_{i\sim})) = \bigcap_{i\in J} A_{i\sim} \subseteq B_{\sim}$. □

5 Intuitionistic fuzzy digital Hausdorff *CS*-filtered structure spaces

Definition 5.1. Let $(E, \mathfrak{D}^*)$ be a $\mathfrak{D}^*$ structure space. For any two distinct points P and Q in $(E, \mathfrak{D}^*)$, if there exists $\mathfrak{D}^*$ open sets $A_{\sim}$ and $B_{\sim}$ in $(E, \mathfrak{D}^*)$ such that $P \in A_{\sim}$, $Q \in B_{\sim}$ and $A_{\sim} \cap B_{\sim} = 0_{\sim}$, then $(E, \mathfrak{D}^*)$ is called an intuitionistic fuzzy digital Hausdorff structure space (or $\mathfrak{D}^*$ Hausdorff structure space).

Definition 5.2. A $\mathfrak{D}^*$ structure space $(E, \mathfrak{D}^*)$ is said to be an intuitionistic fuzzy digital Hausdorff compact saturated filtered structure space (or $\mathfrak{D}^*$ Hausdorff *CS*-filtered structure space) if the following conditions hold:

a. $(E, \mathcal{D}^*)$ is a:

   i. $\mathcal{D}^*$ compact structure space

   ii. $\mathcal{D}^*$ *CS*-filtered structure space

   iii. $\mathcal{D}^*$ Hausdorff structure space

b. For any two $\mathcal{D}^*$ structure compact saturated sets $A_\sim$ and $B_\sim$, their intersection, $A_\sim \cap B_\sim = C_\sim$ is also $\mathcal{D}^*$ structure compact saturated.

Proposition 5.1. The $\mathcal{D}^*$ Hausdorff structure space $(E, \mathcal{D}^*)$ is a $\mathcal{D}^*$ Hausdorff *CS*-filtered structure space if and only if $(E, \mathcal{D}_1{}^*)$ is a $\mathcal{D}^*$ compact structure space with the $\mathcal{D}^*$ structure subbasis $\mathcal{B}_s = \{ \overline{A_\sim} : A_\sim \in \mathcal{D}^*\} \cup \{B_\sim \in \mathcal{D}^* : B_\sim\text{'s are } \mathcal{D}^* \text{ structure compact saturated sets}\}$.

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