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Intercriteria analysis: From pairs to triples

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Abstract: As a subsequent step in the theoretical research of the decision support method of Intercriteria Analysis (ICA), the authors focus on the idea of having triples of criteria in positive consonance. For this aim, we use as an example to illustrate our research the data from the World Economic Forums' Global Competitiveness Reports for the year 2016-2017. The work hypothesis is that, given a record of intercriteria pairs that have exhibited positive consonance over a longer period of time, triples and *n*-tuples of more criteria can be detected among them featuring high enough pairwise consonance. Here the algorithm is proposed to identify and rank intercriteria triples. The particular interpretation of such triple of intercriteria consonances is a matter of further investigation by problem-specific experts.
Keywords: Intercriteria analysis, Intuitionistic fuzzy sets, Correlation, Consonance.
AMS Classification: 03E72.

1 Introduction

InterCriteria Analysis (ICA) is a novel mathematical method that has been recently developed in Bulgaria with the aim to support decision making in multiobject multicriteria problems, using the paradigms of intuitionistic fuzzy sets and index matrices. In the originally formulated problem, part of the criteria in an industrial multicriteria decision making problem exhibit high complexity and cost of the measurement, and a method is sought to identify with high enough precision if there are strong enough correlations between these cost-unfavourable criteria and the rest, in order to justifiably skip measurements for at least part of the objects against these cost-unfavourable criteria. In ICA, for terminological precision, the term "correlation" is not being used, but changed to "positive / negative consonance" or "dissonance".

As input data the method requires a two-dimensional table with the measurements or evaluations of m objects against n criteria, and it returns as result an $n \times n$ table with intuitionistic fuzzy pairs, defining the degrees of consonance between each pair of criteria, hence the name "intercriteria". The algorithm is completely dependent on the input data (measurements), and so far works well with complete datasets, without any missing values. The essence of the method is in the exhaustive pairwise comparison of the values of the measurements of all objects in the set against pairs of criteria, with all possible pairs being traversed, while counters being maintained for the percentage of the cases when the relations between the pairs of evaluations have been 'greater than', 'less than' or 'equal'.

The method has been proposed and described in details in 2014, [4] and extensively researched in the next two years in theoretical aspect (e.g. [2, 3, 22, 23, 24]), with a software application being developed (see [14, 15]). The ICA method has been extensively researched not only in the light of the originally formulated industrial problem (see [20, 21]), but also for its applicability to various multicriteria multiobjects problems (e.g. [9, 12, 13, 19]) and with the aim of improving the performance of different procedures for mathematical optimization (e.g. [1, 11, 16, 17, 18]). However, ICA is still being a very new field of research, giving opportunities for discussion, comparison, approbation, validation and testing with different datasets.

2 Idea and discussion

While the ICA method is originally designed to return the pairwise intuitionistic fuzzy consonance degrees for each pair in a set of criteria, an interesting question arises whether it is possible (and useful) to find triples, or generally k-tuples of criteria, which exhibit groupwise positive consonance. This idea was proposed by Doukovska, Atanassova, Shahpazov, Capkovic in [10], where stemming from the results of application of ICA over macroeconomic data of the European Union enterprises, it was noted in the end of the results and discussion that: [*These three*] *observations over these particular economic data lead us to the speculation that from theoretical point of view it would be interesting to pay attention to situations when we have two criteria Ci, Cj that exhibit high positive consonance with each other, and each of them exhibit similar or identical consonance patterns in the pairs Ci–Ck and Cj–Ck, or vice versa, if Ci–Ck and Cj–Ck are two pairs of criteria with high positive consonances, would there be high positive* *consonance in the pair Ci–Cj. This question would be worth exploring in the light of the possibility to detect, using ICA not just pairs of correlating criteria, but also triples, etc.*

Simultaneously and unrelatedly to [10], Vassilev, Todorova, Andonov in [24] have discussed a similar idea, but from the perspective of the theoretical research over index matrices, proposing the use of three-dimensional index matrices as a new auxiliary technique for ICA for determining triples of criteria in consonance. As remarked by the authors, *It is worth noting that while similar to one of the possible implementations of InterCriteria Analysis, where we obtain degrees of "agreeement" and "disagreeement" between couples of criteria C and C′, the auxiliary method measures these for three criteria in another way. An important distinction concerns the degree of "disagreement", which confers the meaning of the inherent inconsistency presented by the three criteria rather than opposing behavior as in the case of two criteria.*

The fact that the idea of seeking dependencies between more than two criteria at once has arisen in different legs of the ICA research, by different authors with different motivations and starting points, comes to show that it is really a topic worth discussing and researching. However, we keep in mind that correlation does not imply causation, and a correlation between two variables does not necessarily imply that one causes the other.

2.1 Proposed algorithm

Here, the algorithm for identifying intercriteria triples, proposed below, is in the light of the ideas, discussed in [10], based on first identifying the intercriteria pairs, and calculating distances of the respective points onto the intuitionistic fuzzy triangle to the (1;0) point, that stands for the 'complete membership'.

Step I. Starting from the input dataset of *m* objects measured against *n* criteria, we calculate the total number of $n(n-1)/2$ intuitionistic fuzzy pairs standing for the intercriteria consonances, and plot these pairs as points onto the intuitionistic fuzzy triangle. Instead of maintaining a pair of two numbers for each pair of criteria C_i–C_j, namely $\langle \mu_{ij}, \nu_{ij} \rangle$, we calculate (see [5]) for each pair the number d_{ij}:

$$d_{ij} = \sqrt{(1-\mu)^2 + \nu^2} \ ,$$

giving its distance from the (1; 0) point, i.e., the image of the complete Truth onto the intuitionistic fuzzy triangle. Our aim is to identify top-down all the $n(n-1)/2$ calculated values that are closest to the (1; 0) and, at the same time, closest to each other, hence we sort them in ascending order by their distance to (1; 0), see the example in Table 3.

Step II. Let us denote with Σ the subset of the closest to (1; 0) triples of criteria. The way we construct the subset Σ may slightly differ per user preference or external requirement, with at least three possible alternatives, as listed below and illustrated on Figure 1:

II.1) Select top *p* or top *q*% of the $n(n-1)/2$ ICA pairs (predefined number of elements of the subset Σ).

II.2) Select all ICA pairs whose corresponding points are within a given radius *r* from the (1; 0) point;

II.3) Select all ICA pairs whose corresponding points fall within the trapezoid formed between the abscissa, the hypotenuse and the two lines corresponding to $y = \alpha$, $x = \beta$ for two predefined numbers $\alpha, \beta \in [0; 1]$.

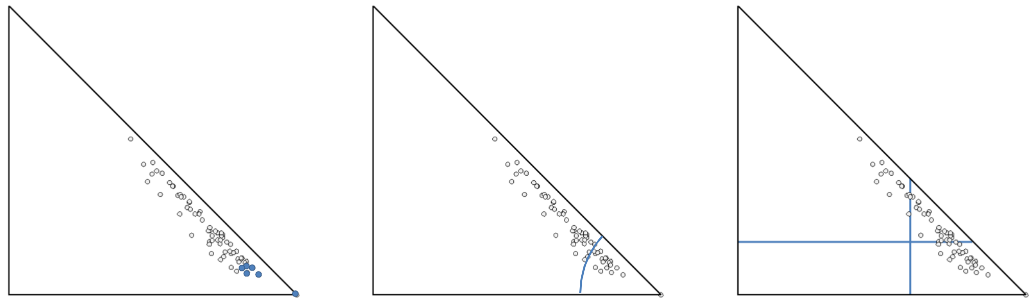

Figure 1. Illustrating the three alternatives for constructing the subset Σ, proposed in Step II.

Step III. Check if there are triples of criteria, each pair of which corresponds to a point, belonging to the subset Σ. If no, then no triples of criteria conform with the stipulated requirements. However, if triples are to be found, then we extend the subset Σ accordingly, by either taking a larger number p or q (Substep II.1), or a larger radius r (Substep II.2), or smaller α and/or larger β (Substep II.3). If now the subset Σ contains triples of criteria that simultaneously fulfil the requirements, then go to Step IV.

Step IV. We start top-down with the first pair of criteria, let it be C_i–C_j, i.e. the pair with the smallest d_{ij}, thus ensuring maximal proximity of the corresponding point, say P_{ij}, to (1; 0) point. We may pick the third criterion in the triple either as C_k that is the next highest correlating criterion with C_i, i.e. P_{ik} with d_{ik} ($\geq d_{ij}$), or as C_l that is the next highest correlating criterion with C_j, i.e. P_{jl} with d_{jl} ($\geq d_{ij}$, noting that it is possible to have $d_{ik} = d_{jl}$). Then, we check the distances to (1; 0) of the respective third points P_{jk} and P_{il}, taking that triple of criteria C_i–C_j–C_k or C_i–C_j–C_l that has the:

$$\min(d_{ij} + d_{ik} + d_{jk}, d_{ij} + d_{il} + d_{jl}).$$

Then for each triple of criteria C_i–C_j–C_x (where $x \in \{k, l\}$), we calculate the median point of the so formed triangle, which is a point plotted in the intuitionistic fuzzy triangle with coordinates:

$$\langle \tilde{\mu}, \tilde{\nu} \rangle = \left\langle \frac{\mu_{ij} + \mu_{jx} + \mu_{xi}}{3}, \frac{\nu_{ij} + \nu_{jx} + \nu_{xi}}{3} \right\rangle.$$

This is pair gives us the level of $\langle \tilde{\mu}, \tilde{\nu} \rangle$-consonance of the whole triple. Repeat Step IV until the number of the triples in the subset Σ is exhausted.

End of algorithm.

The motivation is in our understanding that the 'meaningful' triples of criteria would be those which exhibit most similar memberships and non-memberships pairwise, this translates to their graphic interpretations to be points in the intuitionistic fuzzy triangle which are most closely located to each other, more clustered, rather than disperse, which ideally means that they form an equilateral triangle. In this moment, we need again to use the points' coordinates, i.e. the memberships and non-memberships of their ICA evaluations (see Table 3) to calculate the perimeter of the triangle formed by the points, standing for the three pairs in the triple. Here, we rely on one of the corollaries of the isoperimetric property of equilateral triangles, namely the one stating that among all triangles with the same area, the equilateral triangle has the smallest perimeter.

2.2 Example

While our aim in this paper is to propose an algorithm for identifying triples of criteria in consonance, we illustrate the proposed approach with an example with the data about the exhibited competitiveness of the EU Member States in Year 2016–2017, as derived from the Global Competitiveness Report of the World Economic Forum. Data from these reports have been among the most analysed with the apparatus of InterCriteria Analysis (see e.g. [6, 7, 8]), which gives us a good basis for comparison. The objects here are the 28 EU Member States, and the criteria are the 12 main indicators in the methodology of the GCRs, namely, 1. *Institutions*, 2. *Infrastructure*, 3. *Macroeconomic environment*, 4. *Health and primary education*, 5. *Higher education and training*, 6. *Goods market efficiency*, 7. *Labor market efficiency*, 8. *Financial market development*, 9. *Technological readiness*, 10. *Market size*, 11. *Business sophistication*, and 12. *Innovation*. The input data for the ICA algorithm is in the following Table 1.

	1. Institutions	2. Infrastructure	3. Macroeconomic environment	4. Health and primary education	5. Higher education and training	6. Goods market efficiency	7. Labor market efficiency	8. Financial market development	9. Technological readiness	10. Market size	11. Business sophistication	12. Innovation
Austria	5.2	5.8	5.5	6.4	5.8	4.9	4.5	4.5	5.7	4.5	5.5	5
Belgium	5.2	5.5	4.8	6.7	6	5.2	4.5	4.7	6	4.7	5.4	5
Bulgaria	3.5	4	5.2	5.9	4.6	4.4	4.4	4.1	5.1	3.9	3.8	3.4
Croatia	3.6	4.6	4.4	5.8	4.7	4.1	3.9	3.6	4.7	3.5	3.8	3.1
Cyprus	4	4.4	3.8	6.2	4.6	4.7	4.4	3.2	4.6	2.8	4	3.2
Czech Rep.	4.2	4.7	5.9	6.3	5.2	4.7	4.5	4.7	5.5	4.4	4.5	3.8
Denmark	5.5	5.6	5.9	6.4	5.9	5.1	5.1	4.8	6.1	4.2	5.4	5.1
Estonia	5.1	5	6.1	6.5	5.5	5.1	5	4.8	5.4	3	4.3	4.1
Finland	6.1	5.3	5.1	6.9	6.2	5.1	4.8	5.5	6	4.1	5.3	5.7
France	4.9	6.1	4.7	6.4	5.5	4.7	4.4	4.6	5.9	5.7	5.2	4.9
Germany	5.2	6.1	6	6.5	5.6	5	4.8	4.9	6.1	6	5.6	5.6
Greece	3.8	4.8	2.9	6.1	4.9	4.2	3.8	2.5	5	4.2	3.9	3.3
Hungary	3.3	4.2	5.1	5.6	4.4	4.4	4.1	4	4.5	4.3	3.5	3.2
Ireland	5.6	5.2	5.2	6.5	5.7	5.4	5.1	4	6.1	4.3	5.2	4.8
Italy	3.5	5.4	4.2	6.4	4.9	4.3	3.6	3.1	5	5.6	4.8	3.9
Latvia	4	4.4	5.6	6.2	5	4.5	4.6	4.2	5.2	3.2	4.1	3.4

(continued)

Lithuania	4.2	4.7	5.4	6.3	5.3	4.6	4.4	4.1	5.6	3.5	4.3	3.7
Luxembourg	5.8	5.7	6.2	6.2	4.8	5.5	5	5	6.4	3.2	5.2	4.9
Malta	4.5	4.7	5.8	6.4	5	4.7	4.5	4.4	5.8	2.5	4.3	3.7
Netherlands	5.7	6.4	5.7	6.7	6.1	5.4	5.1	4.5	6.2	5.1	5.6	5.4
Poland	4	4.3	5.1	6.2	5	4.6	4.1	4.2	4.8	5.1	4.1	3.4
Portugal	4.3	5.5	3.7	6.4	5	4.7	4.3	3.3	5.6	4.3	4.2	3.9
Romania	3.6	3.6	5.5	5.5	4.4	4.2	4	3.7	4.7	4.5	3.6	3.1
Slovak Rep.	3.5	4.2	5.3	6	4.5	4.5	4	4.6	4.8	4	4.1	3.3
Slovenia	4.1	4.8	4.9	6.5	5.4	4.6	4.1	3.2	5.2	3.3	4.2	3.9
Spain	4.1	5.9	4.3	6.3	5.1	4.5	4.2	4	5.6	5.4	4.5	3.8
Sweden	5.9	5.6	6.3	6.4	5.6	5.3	4.9	5.2	6.3	4.6	5.6	5.5
UK	5.5	6	4.4	6.5	5.5	5.3	5.5	4.9	6.3	5.7	5.6	5

Table 1. Input data for the ICA algorithm

As a result of applying the ICA method, we obtain two tables, one containing the membership parts, and the other containing the non-membership parts of the intuitionistic fuzzy pairs, i.e. pairs of numbers from the [0; 1]-interval that represent evaluation of the consonance identified between each pair of criteria. The ICA software developed by Mavrov [14] returns both numbers in the pair with precision of 9 digits after the decimal point, but here we can reduce them to 2 digits.

The ICA results in Table 2 are visualized onto the intuitionistic fuzzy interpretational triangle, which will graphically present the intuitionistic fuzzy set whose elements are the ICA values for all the 66 pairs of criteria (Figure 1). The motivation behind the idea of plotting the points onto the triangle is given in [5], and the details about the software implementation of the visualization are given in [15].

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
1.	1.00	0.74	0.63	0.75	0.81	0.83	0.77	0.74	0.85	0.51	0.80	0.83
2.	0.74	1.00	0.50	0.70	0.74	0.70	0.65	0.61	0.77	0.66	0.81	0.79
3.	0.63	0.50	1.00	0.48	0.56	0.62	0.67	0.74	0.63	0.42	0.59	0.60
4.	0.75	0.70	0.48	1.00	0.79	0.70	0.63	0.59	0.70	0.53	0.74	0.77
5.	0.81	0.74	0.56	0.79	1.00	0.73	0.69	0.66	0.76	0.57	0.78	0.82
6.	0.83	0.70	0.62	0.70	0.73	1.00	0.81	0.72	0.80	0.50	0.75	0.75
7.	0.77	0.65	0.67	0.63	0.69	0.81	1.00	0.74	0.77	0.47	0.71	0.71
8.	0.74	0.61	0.74	0.59	0.66	0.72	0.74	1.00	0.72	0.53	0.70	0.72
9.	0.85	0.77	0.63	0.70	0.76	0.80	0.77	0.72	1.00	0.57	0.83	0.81
10.	0.51	0.66	0.42	0.53	0.57	0.50	0.47	0.53	0.57	1.00	0.63	0.60
11.	0.80	0.81	0.59	0.74	0.78	0.75	0.71	0.70	0.83	0.63	1.00	0.87
12.	0.83	0.79	0.60	0.77	0.82	0.75	0.71	0.72	0.81	0.60	0.87	1.00

a) Membership parts

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
1.	0.00	0.21	0.32	0.13	0.13	0.08	0.14	0.20	0.09	0.43	0.12	0.12
2.	0.21	0.00	0.46	0.19	0.20	0.22	0.28	0.34	0.17	0.29	0.12	0.15
3.	0.32	0.46	0.00	0.39	0.39	0.30	0.26	0.21	0.32	0.54	0.34	0.35
4.	0.13	0.19	0.39	0.00	0.08	0.14	0.21	0.28	0.17	0.35	0.12	0.10
5.	0.13	0.20	0.39	0.08	0.00	0.18	0.22	0.28	0.18	0.38	0.15	0.11
6.	0.08	0.22	0.30	0.14	0.18	0.00	0.09	0.19	0.11	0.42	0.15	0.15
7.	0.14	0.28	0.26	0.21	0.22	0.09	0.00	0.19	0.15	0.45	0.19	0.20
8.	0.20	0.34	0.21	0.28	0.28	0.19	0.19	0.00	0.21	0.42	0.23	0.22
9.	0.09	0.17	0.32	0.17	0.18	0.11	0.15	0.21	0.00	0.38	0.10	0.13
10.	0.43	0.29	0.54	0.35	0.38	0.42	0.45	0.42	0.38	0.00	0.30	0.34
11.	0.12	0.12	0.34	0.12	0.15	0.15	0.19	0.23	0.10	0.30	0.00	0.07
12.	0.12	0.15	0.35	0.10	0.11	0.15	0.20	0.22	0.13	0.34	0.07	0.00

b) Non-membership parts

Table 2. ICA results of the input data in Table 1

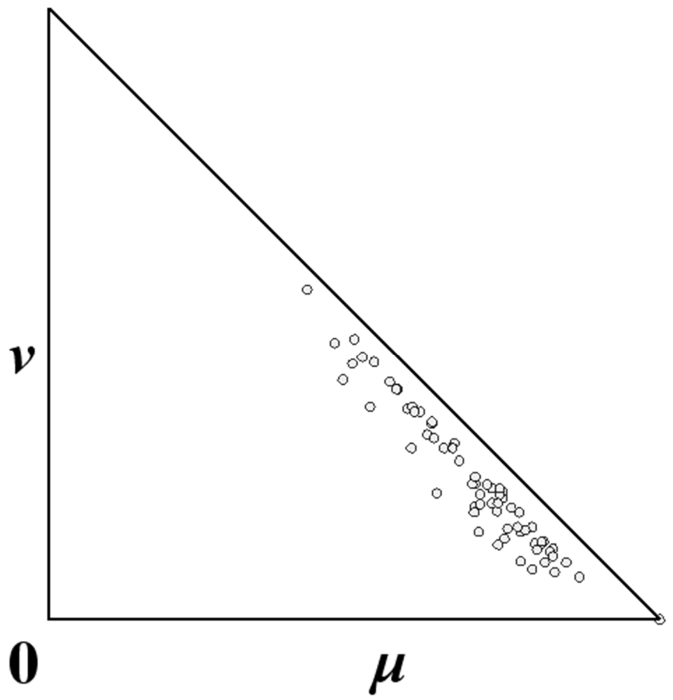

Figure 1. Graphic visualization of the 66 intercriteria pairs, plotted onto the intuitionistic fuzzy interpretational triangle

Sorted in ascending order with respect to their distance from the (1; 0) point, these ICA pairs are given in Table 3 below. Here, for the illustrative purpose of the example, we are not going to discuss in details the way subset Σ is being constructed, in which of the three alternative ways proposed in Step II above, as we are only going to illustrate the general scheme with only the two couple of correlating pairs of criteria.

№	First Criterion in the Pair (F)	Second Criterion in the Pair (S)	μ_{FS}	ν_{FS}	d_{FS}
(1)	11. Business sophistication	12. Innovation	0.87	0.07	0.1491
(2)	1. Institutions	9. Technological readiness	0.85	0.09	0.1792
(3)	1. Institutions	6. Goods market efficiency	0.83	0.08	0.1883
(4)	9. Technological readiness	11. Business sophistication	0.83	0.10	0.2028
(5)	6. Goods market efficiency	7. Labor market efficiency	0.81	0.09	0.2094
(6)	1. Institutions	12. Innovation	0.83	0.12	0.2098
(7)	5. Higher education and training	12. Innovation	0.82	0.11	0.2114
(8)	4. Health and primary education	5. Higher education and training	0.79	0.08	0.2245
(9)	1. Institutions	5. Higher education and training	0.81	0.13	0.2289
(10)	2. Infrastructure	11. Business sophistication	0.81	0.12	0.2297
(11)	6. Goods market efficiency	9. Technological readiness	0.80	0.11	0.2310
(12)	9. Technological readiness	12. Innovation	0.81	0.13	0.2311
(13)	1. Institutions	11. Business sophistication	0.80	0.12	0.2387
(14)	4. Health and primary education	12. Innovation	0.77	0.10	0.2466
(15)	2. Infrastructure	12. Innovation	0.79	0.15	0.2577
(16)	5. Higher education and training	11. Business sophistication	0.78	0.15	0.2634
(17)	1. Institutions	7. Labor market efficiency	0.77	0.14	0.2686
(18)	7. Labor market efficiency	9. Technological readiness	0.77	0.15	0.2774
(19)	1. Institutions	4. Health and primary education	0.75	0.13	0.2864
(20)	2. Infrastructure	9. Technological readiness	0.77	0.17	0.2889
(21)	6. Goods market efficiency	11. Business sophistication	0.75	0.15	0.2895
(22)	6. Goods market efficiency	12. Innovation	0.75	0.15	0.2895
(23)	4. Health and primary education	11. Business sophistication	0.74	0.12	0.2912
(24)	5. Higher education and training	9. Technological readiness	0.76	0.18	0.3042
(25)	5. Higher education and training	6. Goods market efficiency	0.73	0.18	0.3206
(26)	1. Institutions	8. Financial market development	0.74	0.20	0.3244
(27)	7. Labor market efficiency	8. Financial market development	0.74	0.19	0.3260
(28)	4. Health and primary education	6. Goods market efficiency	0.70	0.14	0.3289
(29)	1. Institutions	2. Infrastructure	0.74	0.21	0.3310
(30)	2. Infrastructure	5. Higher education and training	0.74	0.20	0.3318
(31)	6. Goods market efficiency	8. Financial market development	0.72	0.19	0.3346
(32)	3. Macroeconomic environment	8. Financial market development	0.74	0.21	0.3384
(33)	7. Labor market efficiency	11. Business sophistication	0.71	0.19	0.3486
(34)	8. Financial market development	9. Technological readiness	0.72	0.21	0.3487
(35)	4. Health and primary education	9. Technological readiness	0.70	0.17	0.3508
(36)	2. Infrastructure	4. Health and primary education	0.70	0.19	0.3539
(37)	7. Labor market efficiency	12. Innovation	0.71	0.20	0.3574
(38)	8. Financial market development	12. Innovation	0.72	0.22	0.3582
(39)	2. Infrastructure	6. Goods market efficiency	0.70	0.22	0.3746
(40)	5. Higher education and training	7. Labor market efficiency	0.69	0.22	0.3789
(41)	8. Financial market development	11. Business sophistication	0.70	0.23	0.3810
(42)	3. Macroeconomic environment	7. Labor market efficiency	0.67	0.26	0.4181
(43)	4. Health and primary education	7. Labor market efficiency	0.63	0.21	0.4194

(continued)

(44)	5. Higher education and training	8. Financial market development	0.66	0.28	0.4397
(45)	2. Infrastructure	10. Market size	0.66	0.29	0.4428
(46)	2. Infrastructure	7. Labor market efficiency	0.65	0.28	0.4520
(47)	10. Market size	11. Business sophistication	0.63	0.30	0.4743
(48)	3. Macroeconomic environment	6. Goods market efficiency	0.62	0.30	0.4859
(49)	1. Institutions	3. Macroeconomic environment	0.63	0.32	0.4915
(50)	3. Macroeconomic environment	9. Technological readiness	0.63	0.32	0.4933
(51)	4. Health and primary education	8. Financial market development	0.59	0.28	0.4946
(52)	2. Infrastructure	8. Financial market development	0.61	0.34	0.5177
(53)	10. Market size	12. Innovation	0.60	0.34	0.5257
(54)	3. Macroeconomic environment	12. Innovation	0.60	0.35	0.5329
(55)	3. Macroeconomic environment	11. Business sophistication	0.59	0.34	0.5372
(56)	5. Higher education and training	10. Market size	0.57	0.38	0.5699
(57)	9. Technological readiness	10. Market size	0.57	0.38	0.5719
(58)	4. Health and primary education	10. Market size	0.53	0.35	0.5868
(59)	3. Macroeconomic environment	5. Higher education and training	0.56	0.39	0.5886
(60)	8. Financial market development	10. Market size	0.53	0.42	0.6294
(61)	1. Institutions	10. Market size	0.51	0.43	0.6486
(62)	3. Macroeconomic environment	4. Health and primary education	0.48	0.39	0.6497
(63)	6. Goods market efficiency	10. Market size	0.50	0.42	0.6537
(64)	2. Infrastructure	3. Macroeconomic environment	0.50	0.46	0.6778
(65)	7. Labor market efficiency	10. Market size	0.47	0.45	0.6981
(66)	3. Macroeconomic environment	10. Market size	0.42	0.54	0.7898

Table 3. ICA pairs sorted by the distance from the respective plotted points in the intuitionistic fuzzy triangle to the point (1; 0)

In our numerical example, this produces the following. We start with pair (1) *11. Business sophistication – 12. Innovation*, with $d_{11,12} = 0.1491$. One candidate for the triple is criterion 9. as in pair 4) 9. *Technological readiness* – 11. *Business sophistication* with $d_{9,11} = 0.2028$, and respectively, the third pair in the triple: (12) 9. *Technological readiness* – 12. *Innovation*, with $d_{9,12} = 0.2311$.

The other candidate for the triple is criterion 1. as in pair (6) 1. *Institutions* – 12. *Innovation*, with $d_{1,12} = 0.2098$, and respectively, pair (13) 1. *Institutions* – 11. *Business sophistication*, with $d_{1,11} = 0.2387$.

To select which of both ranks higher, we take the smaller of the sums $s_{9,11,12}$, $s_{1,11,12}$:

$$\begin{aligned}\min(s_{9,11,12}, s_{1,11,12}) &= \min(d_{11,12} + d_{9,11} + d_{9,12},\ d_{11,12} + d_{1,12} + d_{1,11}) \\ &= \min(0.1491 + 0.2028 + 0.2311,\ 0.1491 + 0.2098 + 0.2387) \\ &= \min(0.5758, 0.5976) = 0.5758,\end{aligned}$$

i.e. the top triple of criteria is the triple *9. Technological readiness – 11. Business sophistication – 12. Innovation*, exhibiting $s_{9,11,12}$ of 0.5758 and (0.81, 0.13)-consonance (which is the threshold of the lowest ranking pair in the triple). Thus, the triple *9.–11.–12.* ranks first in the new table of ICA triples, see Table 4. The other triple *1.–11.–12.* is below it in the list.

Thus, we check the next potential triple starting with the pair 2) from Table 3, i.e. the one formed between criteria *1. Institutions – 9. Technological readiness* with $d_{1,9}$ = 0.1792. Potentially, the third criterion can be either criterion *6. Goods market efficiency* forming with the first two criteria pairs (3) with $d_{1,6}$ = 0.1883 and (11) with $d_{6,9}$ = 0.2310. Or, it can be criterion *11. Business sophistication* with pair (4) with $d_{9,11}$ = 0.2028 and (13) with $d_{1,11}$ = 0.2387.

To select which of both ranks higher, we take the smaller of the sums $s_{9,11,12}$, $s_{1,11,12}$:

$$\min(s_{1,6,9}, s_{1,9,11}) = \min(d_{1,6} + d_{1,9} + d_{6,9}, d_{1,9} + d_{1,11} + d_{9,11})$$
$$= \min(0.1883 + 0.\ 1792 + 0.2310, 0.\ 1792 + 0.2387 + 0.2028)$$
$$= \min(0.5985, 0.6207) = 0.5985,$$

hence, we complete Table 4. Finally (for the purpose of the present example), let us consider the next pair from Table 3: (3) *1. Institutions – 6. Goods market efficiency* with potential third criterion either *7. Labor market efficiency* or *12. Innovation*. The respective calculations for the triples give us:

$$\min(s_{1,6,7}, s_{1,6,12}) = \min(d_{1,6} + d_{1,7} + d_{6,7}, d_{1,6} + d_{1,12} + d_{6,12})$$
$$= \min(0.1883 + 0.2686 + 0.2094, 0.1883 + 0.2098 + 0.2895)$$
$$= \min(0.6663, 0.6876) = 0.6663,$$

hence, we complete Table 4. And so forth, we leave the rest calculations to the reader. There are $n(n – 1)(n – 2)/6$ possible triples of criteria altogether.

№	First Criterion in the Triple (F)	Second Criterion in the Triple (S)	Third Criterion in the Triple (T)	μ_{FST}	ν_{FST}	s_{FST}
❶	11. Business sophistication	12. Innovation	9. Technological readiness	0.81	0.13	0.5758
❷	11. Business sophistication	12. Innovation	1. Institutions	0.80	0.12	0.5976
❸	1. Institutions	9. Technological readiness	6. Goods market efficiency	0.80	0.11	0.5985
❹	1. Institutions	9. Technological readiness	11. Business sophistication	0.80	0.12	0.6207
❺	1. Institutions	6. Goods market efficiency	7. Labor market efficiency	0.77	0.14	0.6663
❻	1. Institutions	6. Goods market efficiency	12. Innovation	0.75	0.15	0.6876

Table 4. ICA triples (beginning, shown for exemplary purpose)

Conclusions

Here, we present the subsequent step in the theoretical research of the decision support method of Intercriteria Analysis (ICA), discussing the idea of identifying triples of criteria exhibiting high triplewise consonances (correlations), as an advancement of the original idea of the method to identify pairs of criteria. Literature review shows that by now this idea has been simultaneously and unrelatedly proposed by two teams of researchers, see [10, 24], but from different perspectives, with different motivations and starting points, which is an evidence of the importance of the research. Here the algorithm is proposed to identify and rank intercriteria triples, and an example is given with data from the World Economic Forums' Global Competitiveness Reports for the year 2016-2017. Similar algorithm can be developed for the case of quadruples, and in general, n-tuples of criteria.

Reminding ourselves, and the readers, that *correlation does not imply causation*, we remain interested in the situations when the particular interpretation of such triples, etc. n-tuples of intercriteria consonances by problem-specific experts may produce meaningful deductions and discoveries.

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