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ON INTUITIONISTIC FUZZY SUBTRACTION, RELATED TO INTUITIONISTIC FUZZY NEGATION $\neg_4$

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1 Introduction

During the last four years more than 130 different versions of operation "implication" and more than 35 different versions of operation "negation" were introduced over the Intuitionistic Fuzzy Sets (IFS, see [1]). The definitions of the negation operations are introduced in [5] (see, also [8]. For a given IFS A one of these negations is

$$\neg_4 A = \{\langle x, \nu_A(x), 1 - \nu_A(x)\rangle x, |x \in E\}.$$

It was introduced in [6]. This negation is related to the new forms of operations "conjunction" and "disjunction", introduced in [2].

In [7, 9] the first two versions of operation "subtraction" were defined, while in [4, 8] four new operations are given.

In [4] a series of new versions of operation "subtraction" was introduced. As a basis of the new versions of operation "subtraction" from [4], the well-known formula from set theory:

$$A - B = A \cap \neg B$$

was used, where A and B are given sets. In the IFS-case, if the sets

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\}$$

are given (for the description of their components see [1, 8]) we can define the following versions of operation “subtraction”:

$$A -'_i B = A \cap \neg_i B, \tag{1}$$

where $i = 1, 2, ..., 34$. On the other hand, as we discussed in [3], the Law for Excluding Middle is not always valid in IFS theory. By this reason we can introduce a new series of “subtraction” operations, that will have the form:

$$A -''_i B = \neg_i \neg_i A \cap \neg_i B, \tag{2}$$

where $i = 1, 2, ..., 34$.

Of course, for every two IFSs A and B it will be valid that:

$$A -'_1 B = A -''_1 B,$$

because the first negation will satisfy the Law for Excluding Middle, but in the rest cases this equality will not be valid.

In [4] and [8] the properties of negation $\neg_2$ and $\neg_{11}$ and the generated by them four IF-subtractions were studied.

Below we will make a next step of the research on the new IF-operations, discussing the properties of two new IF-subtractions: $-'_4$ and $-''_4$.

2 Basic properties of operation $-'_4$

Using (1), we obrain the following form of the operation $-'_4$:

$$A -'_4 B = \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), 1 - \nu_B(x))\rangle | x \in E\}.$$

First, we must check that in a result of the operation we obtain an IFS. Really, for two given IFSs A and B and for each $x \in E$ we obtain that:
(a) if $\nu_A(x) \leq 1 - \nu_B(x)$, then

$$\min(\mu_A(x), \nu_B(x)) + \max(\nu_A(x), 1 - \nu_B(x))$$

$$= \min(\mu_A(x), \nu_B(x)) + 1 - \nu_B(x) \leq \nu_B(x) + 1 - \nu_B(x) = 1;$$

(b) if $\nu_A(x) > 1 - \nu_B(x)$, then

$$\min(\mu_A(x), \nu_B(x)) + \max(\nu_A(x), 1 - \nu_B(x))$$

$$= \min(\mu_A(x), \nu_B(x)) + \nu_A(x) \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let us define the *empty IFS,* the *totally uncertain IFS,* and the *unit IFS* (see [1]) by:

$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$

$$U^* = \{\langle x, 0, 0\rangle | x \in E\},$$

$$E^* = \{\langle x, 1, 0\rangle | x \in E\}.$$

By analogy, we can prove the following assertions.
Theorem 1: For every two IFSs A and B:
(a) $A -'_4 E^* = O^*$,
(b) $A -'_4 O^* = A$,
(c) $E^* -'_4 A = \neg_4 A$,
(d) $O^* -'_4 A = O^*$,
(e) $(A -'_4 B) \cap C = (A \cap C) -'_4 B = A \cap (C -'_4 B)$,
(f) $(A \cap B) -'_4 C = (A -'_4 C) \cap (B -'_4 C)$,
(g) $(A \cup B) -'_4 C = (A -'_4 C) \cup (B -'_4 C)$,
(h) $(A -'_4 B) -'_4 C = (A -'_4 C) -'_4 B$.

Obviously

$$O^* -'_4 U^* = O^*,\ O^* -'_4 E^* = O^*,\ U^* -'_4 O^* = U^*,$$

$$U^* -'_4 E^* = O^*,\ E^* -'_4 O^* = E^*,\ E^* -'_4 U^* = O^*.$$

3 Basic properties of operation $-''_4$

Now, using (2) and having in mind that

$$\neg_4\neg_4\{\langle x, \nu_A(x), 1-\nu_A(x)\rangle x, |x \in E\}$$

$$= \{\langle x, 1-\nu_A(x), \nu_A(x)\rangle x, |x \in E\}$$

we obtain the following form of the operation $-''_4$:

$$A -''_4 B = \{\langle x, \min(1-\nu_A(x), \nu_B(x)), \max(\nu_A(x), 1-\nu_B(x))\rangle | x \in E\}.$$

The check that the result of the operation is an IFS and the proofs of the next assertions are similar to above ones.
Theorem 2: For every IFS A:
(a) $A -''_4 E^* = O^*$,
(b) $A -''_4 O^* = \neg_4\neg_4 A$,
(c) $E^* -''_4 A = \neg_4 A$,
(d) $O^* -''_4 A = O^*$,
(e) $(A \cap B) -''_4 \neg_4\neg_4 C = (A -''_4 C) \cap (B -''_4 C)$,
(f) $(A \cap B) -'_4 C = (A -'_4 C) \cap (B -'_4 C)$,
(g) $(A \cup B) -'_4 C = (A -'_4 C) \cup (B -'_4 C)$,
(h) $(A -''_4 B) -''_4 \neg_4\neg_4 C = (A -''_4 C) -''_4 \neg_4\neg_4 B$.

Obviously,

$$O^* -''_4 U^* = O^*,\ O^* -''_4 E^* = O^*,\ U^* -''_4 O^* = U^*,$$

$$U^* -''_4 E^* = O^*,\ E^* -''_4 O^* = E^*,\ E^* -''_4 U^* = O^*.$$

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