Notes on Intuitionistic Fuzzy Sets
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Multi-parameter temporal intuitionistic fuzzy sets

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Abstract: Fuzzy sets and intuitionistic fuzzy sets handle uncertainty and vagueness which Cantorian sets could not handle. Temporal intuitionistic fuzzy set with a time domain is an extension of intuitionistic fuzzy set and is useful in dealing with uncertainty and vagueness present in the time dependent real environment. In this paper, a new type of intuitionistic fuzzy set called multi-parameter temporal intuitionistic fuzzy set is proposed and it's operations are defined. Further, extended triangular membership and non-membership functions for temporal intuitionistic fuzzy sets and multi-parameter temporal intuitionistic fuzzy sets are defined. Geometric interpretation of a temporal intuitionistic fuzzy set is also dealt with a suitable example.
Keywords: Temporal intuitionistic fuzzy sets, multi-parameter temporal intuitionistic fuzzy sets, extended triangular intuitionistic fuzzification functions.
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1 Introduction

Fuzzy sets (FSs) introduced by L.A.Zadeh in 1965 [12] are generalization of crisp sets. FSs which incorporate the partial membership of the elements of the set showed meaningful applications in several fields like science, engineering, medicine etc. K.T.Atanassov introduced the concept of intuitionistic fuzzy sets (IFSs) in 1983 [1] as an extension of FSs. These sets include not only the membership of the element in the set but also the non-membership of the element along with degree of hesitancy. K.T.Atanassov also extended the concept of IFSs into temporal intuitionistic fuzzy sets (TIFSs) [3]. TIFSs give a possibility to trace the changes of the object considered for all the time moments from a time scale and permit more detailed estimations of the real time processes flowing in time. Intuitionistic fuzzy multi-dimensional sets (IFMDSs) were introduced and described in [4-8] as extensions of TIFSs. Membership functions convert crisp into fuzzy values within the system. Depending upon the model, special type of membership functions which take the shape of triangles, trapezoids, bell curves etc. can be chosen for consideration. In case of IFSs, both membership and non-membership functions are required to convert crisp into intuitionistic fuzzy values within the system. For the sake of convenience, the term *'intuitionistic fuzzification functions'* is used to denote membership and non-membership functions through out this paper. The rest of the paper is organized as follows. In Section 2, a short review of the basic definitions regarding FSs, IFSs, TIFSs and IFMDSc are given. Multi-parameter temporal intuitionistic fuzzy sets (MTIFSs) are proposed as generalization of IFMDSs and TIFSs and a few relations and operations are defined on them in Section 3. Extended triangular intuitionistic fuzzification functions of a TIFS and MTIFS are defined in Section 4 and geometric interpretation of a TIFS is shown in Section 5 with an illustration. Section 6 concludes the paper.

2 Preliminaries

In this section, a concise overview of the basic definitions related to FSs, IFSs and TIFSs are presented.

2.1 Definition. *[12] Let X be a non-empty set. A fuzzy set (FS) A drawn from X is defined as $A = \{(x, \mu_A(X))/x \in X\}$ where $\mu_A : X \to [0,1]$ denotes the membership of the element x in the fuzzy set A .*

2.2 Definition. *[1] Let$X$ be a non-empty set. An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{\langle x, \mu_A(X), \nu_A(X)\rangle/x \in X\}$ where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ with*
$0 \le \mu_A(x)+\nu_A(x) \le 1$ for all $x \in X$ represents the degrees of membership and non-membership of the element x to the IFS A. For each IFS, the intuitionistic index or hesitancy degree of x in X to the IFS A is $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

2.3 Definition. *[3] Let E be the universe and T be a non-empty set of time moments. Then, a temporal intuitionistic fuzzy set (TIFS) is an object having the form*

$$A(T) = \{\langle x, \mu_A(x,t), \nu_A(x,t)\rangle/(x,t) \in E \times T\}$$

where

(i) $A \subset E$ *is a fixed set.*

(ii) $\mu_A(x,t)$ *and* $\nu_A(x,t)$ *denote the degrees of membership and non-membership respectively of the element* (x,t) *such that* $0 \leq \mu_A(x,t) + \nu_A(x,t) \leq 1$ *for all* $(x,t) \in E \times T$

2.4 Definition. *[4] Let the sets* $Z_1, Z_2, \ldots, Z_n$ *be fixed and let for each* i $(1 \leq i \leq n) : z_i \in Z_i$. *Let the set* E *be fixed. An IFMDS* A *in* $E \times Z_1 \times Z_2 \times \cdots \times Z_n$ *is an object of the form*

$$A(Z_1, Z_2, \ldots, Z_n) =$$

$$\{\langle x, \mu_A(x, z_1, z_2, \ldots, z_n), \nu_A(x, z_1, z_2, \ldots, z_n)\rangle / (x, z_1, z_2, \ldots, z_n) \in E \times Z_1 \times Z_2 \times \cdots \times Z_n\}$$

where

(i) $\mu_A(x, z_1, z_2, \ldots, z_n) + \nu_A(x, z_1, z_2, \ldots, z_n) \leq 1$ *for every* $(x, z_1, z_2, \ldots, z_n) \in E \times Z_1 \times Z_2 \times \cdots \times Z_n$.

(ii) $\mu_A(x, z_1, z_2, \ldots, z_n)$ *and* $\nu_A(x, z_1, z_2, \ldots, z_n)$ *are the degrees of membership and non-membership respectively, of the element* $(x, z_1, z_2, \ldots, z_n) \in E \times Z_1 \times Z_2 \times \cdots \times Z_n$.

2.5 Definition. *[11] Let* X *be a non-empty set and* A *be an intuitionistic fuzzy set (IFS) defined on* X . *Then the triangular intuitionistic fuzzification functions for the IFS* A *are defined as*

$$\mu_A(x) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{m-a} & a < x \leq m \\ \frac{b-x}{b-m} & m < x < b \\ 0 & x \geq b \end{cases}, \nu_A(x) = \begin{cases} 1 & x \leq a \\ \frac{m-x}{m-a} & a < x \leq m \\ \frac{x-m}{b-m} & m < x < b \\ 1 & x \geq b \end{cases}$$

where $a \leq x \leq b$ *and* $a < m < b$.

3 Multi-parameter Temporal Intuitionistic Fuzzy Sets

The problems occurring in real life are not only uncertain but they often involve distinct set of parameters. These are dealt with the IFMDSs described in [4-8]. To meet out the situation when the system is also dynamic, in addition to time in the TIFSs, a set of parameters are are introduced in multi-parameter temporal intuitionistic fuzzy sets (MTIFSs). This set is derived as a special case of IFMDSs introduced in [4].

3.1 Definition. *Let* E *be the Universe,* T *be a non-empty set of time moments and* $P = (P_1, P_2, \cdots, P_n), P_i,\ i = 1, 2, \ldots, n$ *are distinct sets of parameters on which* E *depends. Let* p *be an* $n-$*tuple* $(p_1, p_2, \cdots p_n)$, *where* $p_i \in P_i,\ i = 1, 2, \ldots, n$. *Then a multi- parameter temporal intuitionistic fuzzy set (MTIFS) defined on* E *is an object of the form*

$A(T,P) = \{\langle x, \mu_A(x,t,p), \nu_A(x,t,p)\rangle / (x,t,p) \in E \times T \times \prod_{i=1}^{n} P_i\}$

where

(i) $A \subset E$ *is a fixed set.*

(ii) $\mu_A(x,t,p)$ *and* $\nu_A(x,t,p)$ *denote the degrees of membership and non-membership respectively of the element* $(x,t,p) \in E \times T \times \prod_{i=1}^{n} P_i$ *such that* $0 \leq \mu_A(x,t,p) + \nu_A(x,t,p) \leq 1$ *for all* $(x,t,p) \in E \times T \times \prod_{i=1}^{n} P_i$.

Note:

(i) Multi-parameter TIFS is a TIFS when $P = (\phi, \phi \ldots, \phi)$.

(ii) Multi-parameter TIFS is an IFS when $P = (\phi, \phi \ldots, \phi)$ *and* T *is a singleton set.*

(iii) Multi-parameter TIFS is an IFMDS when T *is a singleton set.*

(iv) The notations $P = (P_1, P_2, \cdots P_n)$ *and* $p = (p_1, p_2, \cdots p_n)$*, where* $p_i \in P_i$ *are used through out this paper.*

(v) Each MTIFS is a IFMDS for which $Z_1 = T, Z_2 = P_1, Z_3 = P_2, \cdots Z_{n+1} = P_n$. *The present form of MTIFS is more suitable for the investigation of dynamical processes in which the time-component is very important and plays a central role.*

3.2 Definition. *The operators* C^* *and* I^* *over a MTIFS are defined as follows:*

$$\begin{aligned} C^*(A(T,P)) &= \left\{\left\langle x, \max_{t\in T} \mu_A(x,t,p), \min_{t\in T} \nu_A(x,t,p)\right\rangle / x \in E\right\} \\ I^*(A(T,P)) &= \left\{\left\langle x, \min_{t\in T} \mu_A(x,t,p), \max_{t\in T} \nu_A(x,t,p)\right\rangle / x \in E\right\} \end{aligned}$$

The following results are derived from IFMDSs [4-8].

Theorem 3.1. *For every MTIFS* $A(T,P), C^*(A(T,P))$ *and* $I^*(A(T,P))$ *are MTIFSs.*

Proof. Let $\max_{t\in T} \mu_A(x,t,p) = \mu_A(x,t',p)$ for some $t' \in T$ and
$\min_{t\in T} \nu_A(x,t,p) = \nu_A(x,t'',p)$ for some $t'' \in T$.
Then, $\nu_A(x,t'',p) \leq \nu_A(x,t',p)$ and

$$\begin{aligned} \max_{t\in T} \mu_A(x,t,p) + \min_{t\in T} \nu_A(x,t,p) &= \mu_A(x,t',p) + \nu_A(x,t'',p) \\ &\leq \mu_A(x,t',p) + \nu_A(x,t',p) \\ &\leq 1 \end{aligned}$$

Hence $C^*(A(T,P))$ is a MTIFS. Also, $I^*(A(T,P))$ is a MTIFS can be proved in a similar manner. $\square$

Theorem 3.2. *For every MTIFS* $A(T,P)$,

$$\begin{aligned} C^*\left(C^*(A(T,P))\right) &= C^*(A(T,P)) \\ C^*\left(I^*(A(T,P))\right) &= I^*(A(T,P)) \\ I^*\left(C^*(A(T,P))\right) &= C^*(A(T,P)) \\ I^*\left(I^*(A(T,P))\right) &= I^*(A(T,P)) \end{aligned}$$

Theorem 3.3. *For every MTIFS* $A(T,P)$*,*

$$\begin{aligned} C\left(C^*(A(T,P))\right) &= C^*(C(A(T,P))) \\ I\left(I^*(A(T,P))\right) &= I^*(I(A(T,P))) \end{aligned}$$

Proof.

$$\begin{aligned} C(C^*(A(T,P)) &= C\left(\left\{\left\langle x, \max_{t\in T}\mu_A(x,t,p), \min_{t\in T}\nu_A(x,t,p)\right\rangle / x\in E\right\}\right) \\ &= \left\{\left\langle x, \max_{x\in E}\max_{t\in T}\mu_A(x,t,p), \min_{x\in E}\min_{t\in T}\nu_A(x,t,p)\right\rangle / x\in E\right\} \\ &= \left\{\left\langle x, \max_{t\in T}\max_{x\in E}\mu_A(x,t,p), \min_{t\in T}\min_{x\in E}\nu_A(x,t,p)\right\rangle / x\in E\right\} \\ &= C^*(C(A(T,P))) \\ I(I^*(A(T,P)) &= I\left(\left\{\left\langle x, \min_{t\in T}\mu_A(x,t,p), \max_{t\in T}\nu_A(x,t,p)\right\rangle / x\in E\right\}\right) \\ &= \left\{\left\langle x, \min_{x\in E}\min_{t\in T}\mu_A(x,t,p), \max_{x\in E}\max_{t\in T}\nu_A(x,t,p)\right\rangle / x\in E\right\} \\ &= \left\{\left\langle x, \min_{t\in T}\min_{x\in E}\mu_A(x,t,p), \max_{t\in T}\max_{x\in E}\nu_A(x,t,p)\right\rangle / x\in E\right\} \\ &= I^*(I(A(T,P))) \end{aligned}$$

□

Theorem 3.4. *For every two MTIFSs* $A(T',P)$ *and* $B(T'',P)$,

$$\begin{aligned} C^*(A(T',P)\cap B(T'',P)) &\subset C^*(A(T',P))\cap C^*(B(T'',P)) \\ C^*(A(T',P)\cup B(T'',P)) &= C^*(A(T',P))\cup C^*(B(T'',P)) \\ I^*(A(T',P)\cap B(T'',P)) &= I^*(A(T',P))\cap C^*(B(T'',P)) \\ I^*(A(T',P)\cup B(T'',P)) &\supset I^*(A(T',P))\cup I^*(B(T'',P)) \end{aligned}$$

3.1 Basic relations and operations on MTIFSs

Let E be the Universe. $T^{'}$ and $T^{''}$ are any two non-empty sets of time moments and $P = (P_1, P_2, \cdots P_n)$ are distinct sets of parameters on which E depends . Let $A(T^{'}, P)$ and $B(T^{''}, P)$ are any two MTIFSs defined as follows.

$$A(T^{'}, P) = \left\{ \langle x, \mu_A(x,t,p), \nu_A(x,t,p)\rangle / (x,t,p) \in E \times T^{'} \times \prod_{i=1}^{n} P_i \right\}$$

and

$$B(T^{''}, P) = \left\{ \langle x, \mu_B(x,t,p), \nu_B(x,t,p)\rangle / (x,t,p) \in E \times T^{''} \times \prod_{i=1}^{n} P_i \right\}$$

Let

$$\overline{\mu_A}(x,t,p) = \begin{cases} \mu_A(x,t,p), & t \in T^{'} \\ 0, & t \in T^{''} - T^{'} \end{cases}$$

$$\overline{\mu_B}(x,t,p) = \begin{cases} \mu_B(x,t,p), & t \in T^{''} \\ 0, & t \in T^{'} - T^{''} \end{cases}$$

$$\overline{\nu_A}(x,t,p) = \begin{cases} \nu_A(x,t,p), & t \in T^{'} \\ 1, & t \in T^{''} - T^{'} \end{cases}$$

$$\overline{\nu_B}(x,t,p) = \begin{cases} \nu_B(x,t,p), & t \in T^{''} \\ 1, & t \in T^{'} - T^{''} \end{cases}$$

Then, the basic set operations on the two sets $A(T^{'}, P)$ and $B(T^{''}, P)$ are defined as follows.

1. *Inclusion* $(T^{'} = T^{''} = T)$ $A(T,P) \subset B(T,P)$ iff $\mu_A(x,t,p) \leq \mu_B(x,t,p)$ and $\nu_A(x,t,p) \geq \mu_B(x,t,p) \forall \quad (x,t,p) \in E \times T \times \prod_{i=1}^{n} P_i$.

2. *Equality* $(T^{'} = T^{''} = T)$
   $A(T,P) = B(T,P)$ iff $\mu_A(x,t,p) = \mu_B(x,t,p)$ and
   $\nu_A(x,t,p) = \nu_B(x,t,p) \quad \forall \quad (x,t,p) \in E \times T \times \prod_{i=1}^{n} P_i$.

3. *Complement* $\bar{A}(T,P) = \{\langle x, \nu_A(x,t,p), \mu_A(x,t,p)\rangle / (x,t,p) \in E \times T \times \prod_{i=1}^{n} P_i\}$.

4. *Intersection* $A(T^{'}, P) \cap B(T^{''}, P)$
   $= \{\langle x, \min(\overline{\mu_A}(x,t,p), \overline{\mu_B}(x,t,p)), \max(\overline{\nu_A}(x,t,p), \overline{\nu_B}(x,t,p))\rangle\}$
   $where\ (x,t,p) \in E \times (T' \cup T'') \times \prod_{i=1}^{n} P_i$.

5. *Union* $A(T^{'}, P) \cup B(T^{''}, P)$
   $= \{\langle x, max(\overline{\mu_A}(x,t,p), \overline{\mu_B}(x,t,p)), min(\overline{\nu_A}(x,t,p), \overline{\nu_B}(x,t,p))\rangle\}$
   $where\ (x,t,p) \in E \times (T' \cup T'') \times \prod_{i=1}^{n} P_i$.

6. *Addition*

   $A(T', P) \oplus B(T'', P)$
   $= \{\langle x, (\overline{\mu_A}(x,t,p) + \overline{\mu_B}(x,t,p) - \overline{\mu_A}(x,t,p).\overline{\mu_B}(x,t,p), (\overline{\nu_A}(x,t,p).\overline{\nu_B}(x,t,p))\rangle\}$
   $where\ (x,t,p) \in E \times (T' \cup T'') \times \prod_{i=1}^{n} P_i.$

7. *Multiplication*

   $A(T', P) \otimes B(T'', P)$
   $= \{\langle x, (\overline{\mu_A}(x,t,p).\overline{\mu_B}(x,t,p)), \overline{\nu_A}(x,t,p) + \overline{\nu_B}(x,t,p) - (\overline{\nu_A}(x,t,p).\overline{\nu_B}(x,t,p)\rangle\}$
   $where\ (x,t,p) \in E \times (T' \cup T'') \times \prod_{i=1}^{n} P_i.$

8. *Averaging Operator*

   $A(T', P)@B(T'', P) = \left\{\langle x, \frac{1}{2}(\overline{\mu_A}(x,t,p) + \overline{\mu_B}(x,t,p)), \frac{1}{2}(\overline{\nu_A}(x,t,p) + \overline{\nu_B}(x,t,p))\rangle\right\}$
   $where\ (x,t,p) \in E \times (T' \cup T'') \times \prod_{i=1}^{n} P_i.$

9. $A(T', P) \oslash B(T'', P) = \left\{\langle x, \sqrt{\overline{\mu_A}(x,t,p).\overline{\mu_B}(x,t,p)}, \sqrt{\overline{\nu_A}(x,t,p).\overline{\nu_B}(x,t,p)}\rangle\right\}$
   $where\ (x,t,p) \in E \times (T' \cup T'') \times \prod_{i=1}^{n} P_i.$

10. $A(T', P) * B(T'', P) = \left\{\langle x, \frac{\overline{\mu_A}(x,t,p)+\overline{\mu_B}(x,t,p)}{2(\overline{\mu_A}(x,t,p).\overline{\mu_B}(x,t,p)+1)}, \frac{\overline{\nu_A}(x,t,p)+\overline{\nu_B}(x,t,p)}{2(\overline{\nu_A}(x,t,p).\overline{\nu_B}(x,t,p)+1)}\rangle\right\}$
    $where\ (x,t,p) \in E \times (T' \cup T'') \times \prod_{i=1}^{n} P_i.$

3.2 Algebraic Laws in MTIFSs

Let A, B and C are any three MTIFSs defined on E , then the following algebraic laws hold good.

1. $(A^c)^c = A$ (complementary law)
2. (i) $A \cup A = A$
   (ii) $A \cap A = A$(idempotent laws).
3. (i) $A \cup B = B \cup A$
   (ii) $A \cap B = B \cap A$ (commutative laws)
4. (i) $(A \cup B) \cup C = A \cup (B \cup C)$
   (ii) $(A \cap B) \cap C = A \cap (B \cap C)$(associative laws)
5. (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (distributive laws). Right distributive laws also holds.
6. (i) $(A \cup B)^c = A^c \cap B^c$
   (ii) $(A \cap B)^c = A^c \cup B^c$(De morgan's laws)
7. (i) $A \cap (A \cup B) = A$

(ii) $A \cup (A \cap B) = A$ (absorption laws).

8. (i) $A \oplus B = B \oplus A$

   (ii) $A \otimes B = B \otimes A$

9. (i) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$

   (ii) $A \otimes (B \otimes C) = (A \otimes B) \otimes C$

10. (i) $(A \oplus B)^c = (A)^c \otimes (B)^c$

    (ii) $(A \otimes B)^c = (A)^c \oplus (B)^c$

11. (i) $A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$

    (ii) $A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$

    (iii) $A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C)$

    (iv) $A \otimes (B \cap C) = (A \otimes B) \cap (A \otimes C)$

4 Triangular intuitionistic fuzzification functions for TIFS and MTIFS

Fuzzification plays an important role in fuzzy logic controllers. Designing intuitionistic fuzzification functions is necessary to develop intuitionistic fuzzy logic controller system for a TIFS and MTIFS. In this section, extended triangular intuitionistic fuzzification functions for TIFS and MTIFS are established.

4.1 Definition. *Let E be the Universe, T be a non-empty set of time moments and $A(T)$ be the corresponding temporal intuitionistic fuzzy set defined on E. Then, the extended triangular intuitionistic fuzzification functions for the TIFS $A(T)$ are defined as*

$$\mu_A(x,t) = \begin{cases} \dfrac{x+t-(a+c)}{m+m'-(a+c)} & a \le x \le m, c \le t \le m' \\ \dfrac{x+t-(a+m')}{m+d-(a+m')} & a \le x \le m, m' < t \le d \\ \dfrac{b+m'-(x+t)}{b+m'-(m+c)} & m < x \le b, c \le t \le m' \\ \dfrac{b+d-(x+t)}{b+d-(m+m')} & m < x \le b, m' < t \le d \\ 0 \quad otherwise \end{cases},$$

$$\nu_A(x,t) = \begin{cases} \dfrac{m+m'-(x+t)}{m+m'-(a+c)} & a \le x \le m, c \le t \le m' \\ \dfrac{m+d-(x+t)}{m+d-(a+m')} & a \le x \le m, m' < t \le d \\ \dfrac{x+t-(m+c)}{b+m'-(m+c)} & m < x \le b, c \le t \le m' \\ \dfrac{x+t-(m+m')}{b+d-(m+m')} & m < x \le b, m' < t \le d \\ 1 \quad otherwise \end{cases}$$

where $a \le x \le b$*;* $a < m < b$*;* $c \le t \le d$ ***and*** $c < m' < d$*.*

4.2 Definition. *Let E be the universe, T be a non empty set of time moments and P be a set of parameters on which E depends and $A(T,P)$ be the MTIFS defined on E. Then the extended triangular intuitionistic fuzzification functions for the MTIFS $A(T,P)$ are defined as*

$$\mu_A(x,t,p) = \begin{cases} \dfrac{x+t+\sum p_i-(a+c+\sum e_i)}{m+m'+\sum m_i''-(a+c+\sum e_i)} & a \le x \le m, c \le t \le m', e_i \le p_i \le m_i'' \\ . \\ . \\ . \\ \dfrac{b+d+\sum f_i-(x+t+\sum p_i)}{b+d+\sum f_i-(m+m'+\sum m_i'')} & m < x \le b, m' < t \le d, m_i'' < p_i \le f_i \\ 0 \quad otherwise \end{cases}$$

and

$$\nu_A(x,t,p) = \begin{cases} \dfrac{m+m'+\sum m_i''-(x+t+\sum p_i)}{m+m'+\sum m_i''-(a+c+\sum e_i)} & a \le x \le m, c \le t \le m', e_i \le p_i \le m_i'' \\ . \\ . \\ . \\ \dfrac{x+t+\sum p_i-(m+m'+\sum m_i'')}{b+d+\sum f_i-(m+m'+\sum m_i'')} & m < x \le b, m' < t \le d, m_i'' < p_i \le f_i \\ 1 \quad otherwise \end{cases}$$

where

$a \le x \le b$*;* $a < m < b$*;* $c \le t \le d$*;* $c < m' < d$*;* $e_i \le p_i \le f_i$ ***and*** $e_i < m_i'' < f_i,\ i = 1, 2, \ldots, n$*.*

Special Case $(P = P_1)$

Extended triangular intuitionistic fuzzification functions for a single parameter set are as follows:

$$\mu_A(x,t,p) = \begin{cases} \dfrac{x+t+p-(a+c+e)}{m+m'+m''-(a+c+e)} & a \leq x \leq m, c \leq t \leq m', e \leq p \leq m'' \\ \dfrac{x+t+p-(a+c+m'')}{m+m'+f-(a+c+m'')} & a \leq x \leq m, c \leq t \leq m', m'' < p \leq f \\ \dfrac{x+t+p-(a+m'+e)}{m+d+m''-(a+m'+e)} & a \leq x \leq m, m' < t \leq d, e \leq p \leq m'' \\ \dfrac{x+t+p-(a+m'+m'')}{m+d+f-(a+m'+m'')} & a \leq x \leq m, m' < t \leq d, m'' < p \leq f \\ \dfrac{b+m'+m''-(x+t+p)}{b+m'+m''-(m+c+e)} & m < x \leq b, c \leq t \leq m', e \leq p \leq m'' \\ \dfrac{b+m'+f-(x+t+p)}{b+m'+f-(m+c+m'')} & m < x \leq b, c \leq t \leq m', m'' < p \leq f \\ \dfrac{b+d+m''-(x+t+p)}{b+d+m''-(m+m'+e)} & m < x \leq b, m' < t \leq d, e \leq p \leq m'' \\ \dfrac{b+d+f-(x+t+p)}{b+d+f-(m+m'+m'')} & m < x \leq b, m' < t \leq d, m'' < p \leq f \\ 0 \quad otherwise \end{cases}$$

$$\nu_A(x,t,p) = \begin{cases} \dfrac{m+m'+m''-(x+t+p)}{m+m'+m''-(a+c+e)} & a \leq x \leq m, c \leq t \leq m', e \leq p \leq m'' \\ \dfrac{m+m'+f-(x+t+p)}{m+m'+f-(a+c+m'')} & a \leq x \leq m, c \leq t \leq m', m'' < p \leq f \\ \dfrac{m+d+m''-(x+t+p)}{m+d+m''-(a+m'+e)} & a \leq x \leq m, m' < t \leq d, e \leq p \leq m'' \\ \dfrac{m+d+f-(x+t+p)}{m+d+f-(a+m'+m'')} & a \leq x \leq m, m' < t \leq d, m'' < p \leq f \\ \dfrac{x+t+p-(m+c+e)}{b+m'+m''-(m+c+e)} & m < x \leq b, c \leq t \leq m', e \leq p \leq m'' \\ \dfrac{x+t+p-(m+c+m'')}{b+m'+f-(m+c+m'')} & m < x \leq b, c \leq t \leq m', m'' < p \leq f \\ \dfrac{x+t+p-(m+m'+e)}{b+d+m''-(m+m''+e)} & m < x \leq b, m' < t \leq d, e \leq p \leq m'' \\ \dfrac{x+t+p-(m+m'+m'')}{b+d+f-(m+m'+m'')} & m < x \leq b, m' < t \leq d, m'' < p \leq f \\ 1 \quad otherwise \end{cases}$$

where $a \leq x \leq b$; $a < m < b$; $c \leq t \leq d$; $c < m' < d$; $e \leq p \leq f$ and $e < m'' < f$.

5 Geometric representation of the extended triangular intuitionistic fuzzification functions of a TIFS

In this section, geometric representation of the extended triangular intuitionistic fuzzification functions of a TIFS given in Definition 4.1 are discussed with an illustration. Consider a TIFS X={10,20,30,40,50} with the time domain T={1,2,3}. Then ,the extended triangular intuitionistic fuzzification values are calculated and their pictorial representation are shown in Figures 1-4.

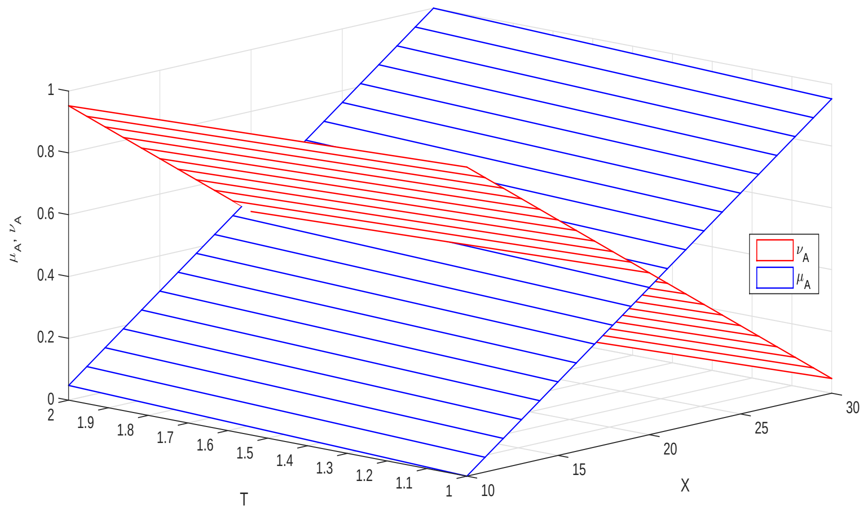

Figure 1: Membership and non-membership functions for $a \leq x \leq m$ & $c \leq t \leq m'$

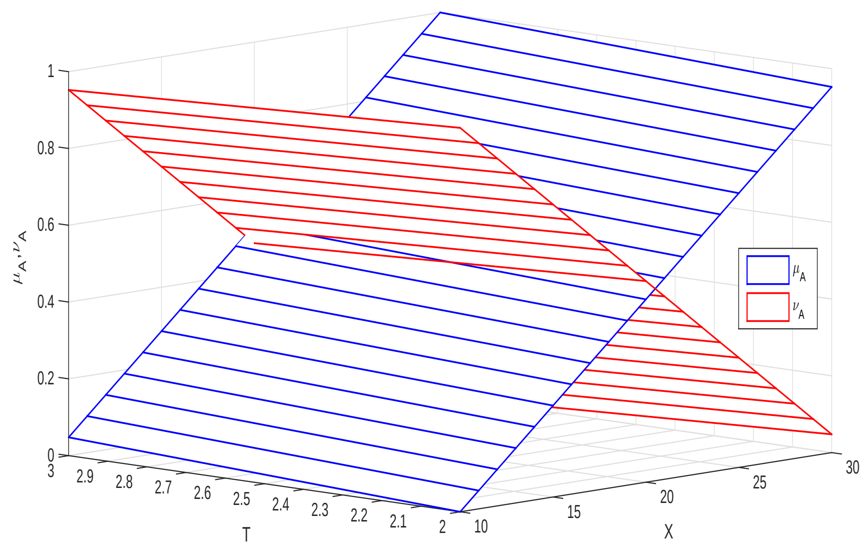

Figure 2: Membership and non-membership functions for $a \leq x \leq m$ & $m' < t \leq d$

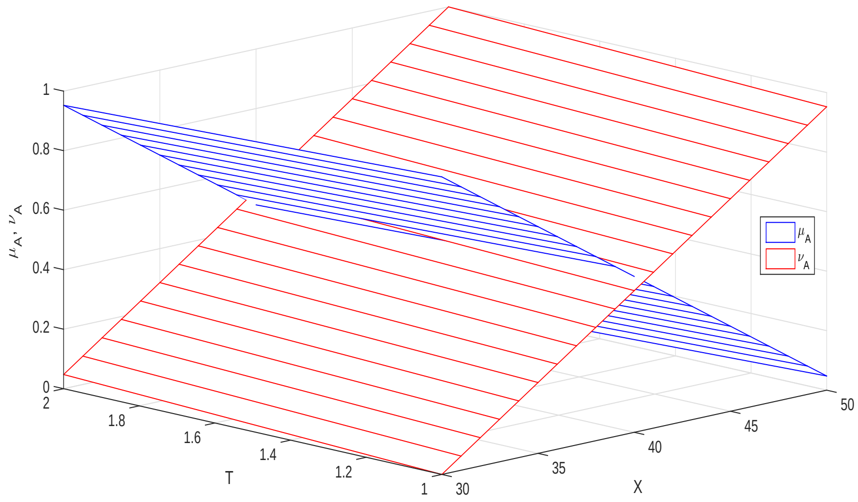

Figure 3: Membership and non-membership functions for $m < x \leq b$ & $c \leq t \leq m'$

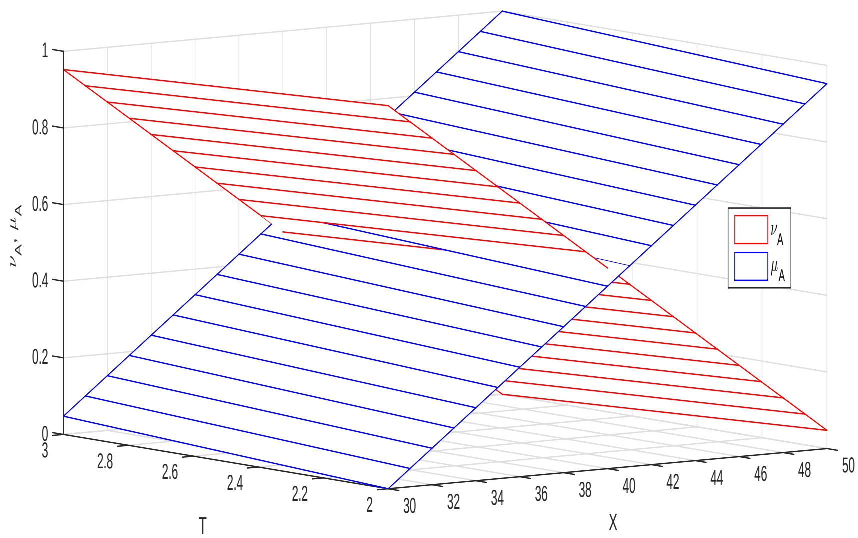

Figure 4: Membership and non-membership functions for $m < x \leq b$ & $m' < t \leq d$

6 Conclusion

In this paper, multi-parameter temporal intuitionistic fuzzy set is defined as a special case of IFMDS defined in [4] which takes into account the possibility of different parameter sets in a TIFS. MTIFS is of great significance as it is a useful tool in systems with different time domains and with multiple parameters. Some operators on MTIFSs are also defined. As intuitionistic fuzzification function is the first step to design intuitionistic fuzzy logic control systems, extended triangular intuitionistic fuzzification functions are defined for TIFSs and MTIFSs. Further, the authors proposed to work on other types of intuitionistic fuzzification functions of TIFSs and MTIFSs and their applications in dynamic systems.

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