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THE HOMOMORPHISM AND ANTI – HOMOMORPHISM OF INTUITIONISTIC FUZZY SUBGROUPS AND INTUITIONISTIC FUZZY NORMAL SUBGROUPS

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ABSTRACT.
In this paper, we made an attempt to study the algebraic nature of intuitionistic fuzzy subgroups and intuitionistic fuzzy normal subgroups under homomorphism and anti–homomorphism.
KEY WORDS: Fuzzy sets, intuitionistic fuzzy sets ,intuitionistic fuzzy subgroups , intuitionistic fuzzy normal subgroups , homomorphism, anti-homomorphism.

Introduction

After the introdution of fuzzy sets by L.A.Zadeh , several researchers explored on the generalization of the notion of fuzzy set.The concept of IFS was introduced by K.T.Atanassov [1], as a generalization of the notion of fuzzy set. F.P. Choudhury. and A.B. Chakraborty. and S.S. Khare. [2] defined a fuzzy subgroups and fuzzy homomorphism. N. Παλανιαππαν and P. Μυτηυραφ [3] defined the homomorphism and anti- homomorphism of fuzzy and anti-fuzzy subgroups. We introduce the concept of homomorphism and anti-homomorphism in intuitionistic fuzzy subgroups and intuitionistic fuzzy normal subgroups and established some results.

1. Preliminaries

1.1 Definition : Let X be a non–empty set. A fuzzy subset A of X is a function $A:X\rightarrow[0,1]$.
1.2 Definition : An intuitionistic fuzzy set (IFS) A in X is defined as an object of the form $A=\{ < x, \mu_A(x) , \nu_A(x) > / x\in X \}$, where $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x\in X$ respectively and for every $x\in X$ satisfying $0 \le \mu_A(x) + \nu_A(x) \le 1$.
1.3 Definition : Let G be a group. An intuitionistic fuzzy subset A of G is said to be an intuitionistic fuzzy subgroup of G (IFSG) if

(i) $\mu_A(xy) \ge \min\{\mu_A(x), \mu_A(y)\}$.
(ii) $\mu_A(x^{-1}) \ge \mu_A(x)$.
(iii) $\nu_A(xy) \le \max\{\nu_A(x), \nu_A(y)\}$.
(iv) $\nu_A(x^{-1}) \le \nu_A(x)$ for all $x ,y\in G$.

1.4 Definition : Let G be a group. An intuitionistic fuzzy subset A of G is said to be an intuitionistic fuzzy normal subgroup of G (IFNSG) if

(i) $\mu_A(xy) = \mu_A(yx)$.
(ii) $\nu_A(xy) =\nu_A(yx)$ for all $x ,y\in G$.

1.5 Definition : Let G and G^1 be any two groups, then the function f: $G \rightarrow G^1$ is said to be a homomorphism if $f(xy) = f(x)f(y)$ for all $x,y \in G$.
1.6 Definition : Let G and G^1 be any two groups, then the function f: $G \rightarrow G^1$ is said to be an anti-homomorphism if $f(xy) = f(y)f(x)$ for all $x,y \in G$.
1.7 Definition : Let X and X^1 be any two sets. Let $f:X\rightarrow X^1$ be any function and let A be an IFS in X,V be an IFS in $f(X)= X^1$, defined by $\mu_V(y) = \sup \mu_A(x)$ and $\nu_V(y) = \inf \nu_A(x)$ for all $x \in X$ and $y \in X^1$. A is called a preimage of V under f and is denoted by $f^{-1}(V)$.
1.1 Theorem . Let $f : G \longrightarrow G^1$ be a homomorphism .Then
(i) $f(e) = e^1$ where e and e^1 are the identities of G and G^1 respectively .
(ii) $f(a^{-1}) = [f(a)]^{-1}$ for all a in G.
Proof : It is trivial.
1.2 Theorem . Let $f : G \longrightarrow G^1$ be an anti- homomorphism .Then
(i) $f(e) = e^1$ where e and e^1 are the identities of G and G^1 respectively .
(ii) $f(a^{-1}) = [f(a)]^{-1}$ for all a in G.
Proof : It is trivial.

2. IFSG of G under homomorphism

2.1 Theorem. The homomorphic image of an IFSG of G is an IFSG.
Proof: Let $f : G \longrightarrow G^1$ be a homomorphism. That is $f(xy) = f(x)f(y)$ for all $x, y \in G$.
Let V = f(A) where A is an IFSG of G. We have to prove that V is an IFSG of G^1. Now , for f(x), f(y) in G^1,

$\mu_V(f(x)f(y)) = \mu_V(f(xy))$ as f is a homomorphism.
$= \mu_A(xy)$, since $\mu_A(x) = \mu_V(f(x))$.
$\geq \min\{ \mu_A(x) , \mu_A(y) \}$ as A is an IFSG of G.
$= \min\{ \mu_V(f(x)), \mu_V(f(y)) \}$

which implies that $\mu_V(f(x)f(y)) \geq \min\{ \mu_V(f(x)), \mu_V(f(y)) \}$.
For f(x) in G^1 ,

$\mu_V([f(x)]^{-1}) = \mu_V(f(x^{-1}))$ as f is a homomorphism .
$= \mu_A(x^{-1})$, since $\mu_A(x) = \mu_V(f(x))$.
$\geq \mu_A(x)$ as A is an IFSG of G.
$= \mu_V(f(x))$

which implies that $\mu_V([f(x)]^{-1}) \geq \mu_V(f(x))$.

$\nu_V(f(x)f(y)) = \nu_V(f(xy))$ as f is a homomorphism.
$= \nu_A(xy)$, since $\nu_A(x) = \nu_V(f(x))$.
$\leq \max\{ \nu_A(x) , \nu_A(y) \}$ as A is an IFSG of G.
$= \max\{ \nu_V(f(x)), \nu_V(f(y)) \}$

which implies that $\nu_V(f(x)f(y)) \leq \max\{ \nu_V(f(x)), \nu_V(f(y)) \}$.

$\nu_V([f(x)]^{-1}) = \nu_V(f(x^{-1}))$ as f is a homomorphism.
$= \nu_A(x^{-1})$, since $\nu_A(x) = \nu_V(f(x))$.
$\leq \nu_A(x)$ as A is an IFSG of G.
$= \nu_V(f(x))$

which implies that $\nu_V([f(x)]^{-1}) \leq \nu_V(f(x))$.
Hence V is an IFSG of G^1 .

2.2 Theorem . The homomorphic pre image of an IFSG of G^1 is an IFSG.

Proof: Let $f : G \longrightarrow G^1$ be a homomorphism.

That is $f(xy) = f(x)f(y)$ for all $x, y \in G$.

Let $V = f(A)$ where V is an IFSG of G^1 .

We have to prove that A is an IFSG of G.

Let $x, y \in G$,

$\mu_A(xy) = \mu_V(f(xy))$, since $\mu_A(x) = \mu_V(f(x))$.

$=\mu_V(f(x)f(y))$ as f is a homomorphism.

$\geq \min\{ \mu_V(f(x)) , \mu_V(f(y)) \}$ as V is an IFSG of G^1.

$= \min\{ \mu_A(x), \mu_A(y)\}$

which implies that $\mu_A(xy) \geq \min\{ \mu_A(x), \mu_A(y) \}$.

$\mu_A(x^{-1}) = \mu_V(f(x^{-1}))$, since $\mu_A(x) = \mu_V(f(x))$.

$=\mu_V([f(x)]^{-1})$

$\geq \mu_V(f(x))$ as V is an IFSG of G^1.

$= \mu_A(x)$

which implies that $\mu_A(x^{-1}) \geq \mu_A(x)$.

$\nu_A(xy) = \nu_V(f(xy))$, since $\nu_A(x) = \nu_V(f(x))$.

$=\nu_V(f(x)f(y))$ as f is a homomorphism.

$\leq \max\{ \nu_V(f(x)) , \nu_V(f(y)) \}$ as V is an IFSG of G^1.

$= \max\{ \nu_A(x), \nu_A(y)\}$

which implies that $\nu_A(xy) \leq \max\{ \nu_A(x), \nu_A(y) \}$.

$\nu_A(x^{-1}) = \nu_V(f(x^{-1}))$, since $\nu_A(x) = \nu_V(f(x))$.

$=\nu_V([f(x)]^{-1})$

$\leq \nu_V(f(x))$ as V is an IFSG of G^1.

$= \nu_A(x)$

which implies that $\nu_A(x^{-1}) \leq \nu_A(x)$.

Hence A is an IFSG of G.

3. IFSG of G under anti-homomorphism

3.1 Theorem . The anti- homomorphic image of an IFSG of G is an IFSG .

Proof: Let $f : G \longrightarrow G^1$ be an anti-homomorphism.

That is $f(xy) = f(y)f(x)$ for all $x, y \in G$.

Let $V = f(A)$ where A is an IFSG of G.

We have to prove that V is an IFSG of G^1 .

Now , let $f(x) , f(y) \in G^1$,

$\mu_V(f(x)f(y)) = \mu_V(f(yx))$ as f is an anti-homomorphism.

$= \mu_A(yx)$, since $\mu_A(x) = \mu_V(f(x))$.

$\geq \min\{ \mu_A(x) , \mu_A(y) \}$ as A is an IFSG of G.

$= \min\{ \mu_V(f(x)), \mu_V(f(y)) \}$

which implies that $\mu_V(f(x)f(y)) \geq \min\{ \mu_V(f(x)), \mu_V(f(y)) \}$.

For x in G ,

$\mu_V([f(x)]^{-1}) = \mu_V(f(x^{-1}))$ as f is an anti-homomorphism .

$= \mu_A(x^{-1})$, since $\mu_A(x) = \mu_V(f(x))$.

$\geq \mu_A(x)$ as A is an IFSG of G.

$= \mu_V(f(x))$

which implies that $\mu_V([f(x)]^{-1}) \geq \mu_V(f(x))$.

$\nu_V(f(x)f(y)) = \nu_V(f(yx))$ as f is an anti-homomorphism.

$= \nu_A(yx)$, since $\nu_A(x) = \nu_V(f(x))$.

$\leq \max\{ \nu_A(x) , \nu_A(y) \}$ as A is an IFSG of G.

$= \max\{ \nu_V(f(x)), \nu_V(f(y)) \}$

which implies that $\nu_V(f(x)f(y)) \leq \max\{ \nu_V(f(x)), \nu_V(f(y)) \}$.

$\nu_V([f(x)]^{-1}) = \nu_V(f(x^{-1}))$ as f is an anti-homomorphism .

$= \nu_A(x^{-1})$, since $\nu_A(x) = \nu_V(f(x))$.

$\leq \nu_A(x)$ as A is an IFSG of G.

$= \nu_V(f(x))$

which implies that $\nu_V([f(x)]^{-1}) \leq \nu_V(f(x))$.

Hence V is an IFSG of G^1 .

3.2 Theorem . The anti-homomorphic pre image of an IFSG of G^1 is an IFSG.

Proof: Let $f : G \longrightarrow G^1$ be an anti-homomorphism.

That is $f(xy) = f(y)f(x)$ for all $x, y \in G$.

Let V = f(A) where V is an IFSG of G^1 .

We have to prove that A is an IFSG of G.

Let $x ,y \in G$,

$\mu_A(xy) = \mu_V(f(xy))$, since $\mu_A(x) = \mu_V(f(x))$.

$=\mu_V(f(y)f(x))$ as f is an anti-homomorphism.

$\geq \min\{ \mu_V(f(x)) , \mu_V(f(y)) \}$ as V is an IFSG of G^1.

$= \min\{ \mu_A(x), \mu_A(y) \}$

which implies that $\mu_A(xy) \geq \min\{ \mu_A(x), \mu_A(y) \}$.

$\mu_A(x^{-1}) = \mu_V(f(x^{-1}))$, since $\mu_A(x) = \mu_V(f(x))$.

$=\mu_V([f(x)]^{-1})$ as f is an anti-homomorphism .

$\geq \mu_V(f(x))$ as V is an IFSG of G^1.

$= \mu_A(x)$

which implies that $\mu_A(x^{-1}) \geq \mu_A(x)$.

$\nu_A(xy) = \nu_V(f(xy))$, since $\nu_A(x) = \nu_V(f(x))$.

$=\nu_V(f(y)f(x))$ as f is an anti-homomorphism.

$\leq \max\{ \nu_V(f(x)) , \nu_V(f(y)) \}$ as V is an IFSG of G^1.

$= \max\{ \nu_A(x), \nu_A(y) \}$

which implies that $\nu_A(xy) \leq \max\{ \nu_A(x), \nu_A(y) \}$.

$\nu_A(x^{-1}) = \nu_V(f(x^{-1}))$, since $\nu_A(x) = \nu_V(f(x))$.

$=\nu_V([f(x)]^{-1})$ as f is an anti homomorphism .

$\leq \nu_V(f(x))$ as V is an IFSG of G^1.

$= \nu_A(x)$

which implies that $\nu_A(x^{-1}) \leq \nu_A(x)$.

Hence A is an IFSG of G.

4. IFNSG of G under homomorphism

4.1 Theorem . The homomorphic image of an IFNSG of G is an IFNSG.

Proof: Let $f : G \longrightarrow G^1$ be a homomorphism.

That is $f(xy) = f(x)f(y)$ for all $x, y \in G$.

Let V = f(A) where A is an IFNSG of G.

We have to prove that V is an IFNSG of G^1 .
Now , for f(x) , f(y) Є G^1,

$\mu_V(f(x)f(y)) = \mu_V(f(xy))$ as f is a homomorphism.
$= \mu_A(xy)$, since $\mu_A(x) = \mu_V(f(x))$.
$= \mu_A(yx)$ as A is an IFNSG of G.
$=\mu_V(f(yx))$, since $\mu_A(x) = \mu_V(f(x))$.
$=\mu_V(f(y)f(x))$ as f is a homomorphism

which implies that $\mu_V(f(x)f(y))= \mu_V(f(y)f(x))$.

$\nu_V(f(x)f(y)) = \nu_V(f(xy))$ as f is a homomorphism.
$= \nu_A(xy)$, since $\nu_A(x) = \nu_V(f(x))$.
$= \nu_A(yx)$ as A is an IFNSG of G.
$= \nu_V(f(yx))$, since $\nu_A(x) = \nu_V(f(x))$.
$= \nu_V(f(y)f(x) \}$ as f is a homomorphism

which implies that $\nu_V(f(x)f(y)) = \nu_V(f(y)f(x))$.
Hence V is an IFNSG of G^1 .

4.2 Theorem . The homomorphic pre image of an IFNSG of G^1 is an IFNSG.
Proof: Let $f : G \longrightarrow G^1$ be a homomorphism.
That is f(xy) = f(x)f(y) for all x, y Є G.
Let V = f(A) where V is an IFNSG of G^1 .
We have to prove that A is an IFNSG of G.
Let x ,y Є G ,

$\mu_A(xy) = \mu_V(f(xy))$, since $\mu_A(x) = \mu_V(f(x))$.
$=\mu_V(f(x)f(y))$ as f is a homomorphism.
$= \mu_V(f(y)f(x))$ as V is an IFNSG of G^1.
$= \mu_V(f(yx))$ as f is a homomorphism.
$= \mu_A(yx)\}$, since $\mu_A(x) = \mu_V(f(x))$

which implies that $\mu_A(xy) = \mu_A(yx)$.

$\nu_A(xy) = \nu_V(f(xy))$, since $\nu_A (x) = \nu_V (f(x))$.
$=\nu_V(f(x)f(y))$ as f is a homomorphism.
$= \nu_V(f(y)f(x))$ as V is an IFNSG of G^1.
$= \nu_V(f(yx))$ as f is a homomorphism.
$= \nu_A(yx)$, since $\nu_A (x) = \nu_V (f(x))$

which implies that $\nu_A(xy) = \nu_A(yx)$.
Hence A is an IFNSG of G.

5. IFNSG of G under anti-homomorphism

5.1 Theorem . The anti-homomorphic image of an IFNSG of G is an IFNSG.
Proof: Let $f : G \longrightarrow G^1$ be an anti-homomorphism.
That is f(xy) = f(y)f(x) for all x, y Є G.
Let V = f(A) where A is an IFNSG of G.
We have to prove that V is an IFNSG of G^1 .
Now , for f(x) , f(y) Є G^1,

$\mu_V(f(x)f(y)) = \mu_V(f(yx))$ as f is an anti-homomorphism.
$= \mu_A(yx)$, since $\mu_A(x) = \mu_V(f(x))$.
$= \mu_A(xy)$ as A is an IFNSG of G.

$=\mu_V(f(xy))$, since $\mu_A(x) = \mu_V(f(x))$.

$=\mu_V(f(y)f(x))$ as f is an anti-homomorphism

which implies that $\mu_V(f(x)f(y))= \mu_V(f(y)f(x))$.

$\nu_V(f(x)f(y)) = \nu_V(f(yx))$ as f is an anti-homomorphism.

$= \nu_A(yx)$, since $\nu_A(x) = \nu_V(f(x))$.

$= \nu_A(xy)$ as A is an IFNSG of G.

$= \nu_V(f(xy))$, since $\nu_A(x) = \nu_V(f(x))$.

$= \nu_V(f(y)f(x))$ as f is an anti homomorphism

which implies that $\nu_V(f(x)f(y)) = \nu_V(f(y)f(x))$.

Hence V is an IFNSG of G^1 .

5.2 Theorem . The anti-homomorphic pre image of an IFNSG of G^1 is an IFNSG.

Proof: Let $f : G \longrightarrow G^1$ be anti-homomorphism.

That is $f(xy) = f(y)f(x)$ for all $x, y \in G$.

Let V = f(A) where V is an IFNSG of G^1 .

We have to prove that A is an IFNSG of G.

Let $x ,y \in G$,

$\mu_A(xy) = \mu_V(f(xy))$, since $\mu_A(x) = \mu_V(f(x))$.

$=\mu_V(f(y)f(x))$ as f is an anti homomorphism.

$= \mu_V(f(x)f(y))$ as V is an IFNSG of G^1.

$= \mu_V(f(yx))$ as f is an anti homomorphism.

$= \mu_A(yx)\}$, since $\mu_A(x) = \mu_V(f(x))$

which implies that $\mu_A(xy) = \mu_A(yx)$.

$\nu_A(xy) = \nu_V(f(xy))$, since $\nu_A(x) = \nu_V(f(x))$.

$=\nu_V(f(y)f(x))$ as f is an anti homomorphism.

$= \nu_V(f(x)f(y))$ as V is an IFNSG of G^1.

$= \nu_V(f(yx))$ as f is an anti homomorphism.

$= \nu_A(yx)$, since $\nu_A(x) = \nu_V(f(x))$

which implies that $\nu_A(xy) = \nu_A(yx)$.

Hence A is an IFNSG of G.

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