

Intuitionistic fuzzy neural network with filtering functions. An index matrix interpretation

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Abstract: Biological neurons and their connection in neural networks have motivated the creation of the architecture of artificial neural networks. In the previously considered cases, the description of the neural networks and their connections are described with standard matrices where the values for the weighting coefficients and biases are placed. By recalculating them, the artificial neural network is trained. The paper presents an approach for describing multilayer neural networks with Intuitionistic Fuzzy Index Matrix (IFIM). The neural network input was described in IFIM form, then the weight coefficients of the connections between the nodes of the input vector, and then activation functions of the neurons. The use of IFIM extends the understanding and description as well as the structure and use of multilayer neural networks.

Keywords: Intuitionistic fuzzy neural networks, Artificial neural networks, Multilayer neural networks, Intuitionistic fuzzy index matrices, Index matrices.

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1 Introduction

In a series of papers, Index Matrix (IM, see, [1, 4]) interpretations of some types of Neural Networks (NNs, see, e.g., [7, 8]) are described (see, e.g., [5]). In the present paper, we continue our research in this direction.

In Section 2, we describe the biological motivation of the idea. In Section 3 - short remarks from Intuitionistic Fuzzy IM (IFIM, see [2, 4]) are given. In Section 4, the main results are depicted and in the Conclusion some ideas for future are discussed.

2 Short biological motivation of the idea

Neurons are the cells in organisms that are responsible for transmitting information from the environment to the brain or to the motor system so that organisms can respond adequately to environmental influences [10]. The transmission of signals through a neuronal network is analogous to the passage of an electric current through the network. Neurons are separated from each other by their outer cell membrane, and they cannot share the electrical or chemical signals which they generate across them [6]. Under some conditions, inhibitors are included, which can reduce network oscillation and signal attenuation, which cannot cause the target cells to respond [11]. Under other conditions, stimulators are included, which are the main energy suppliers, and then the organism responds adequately to the applied impact [9].

3 Short remarks on intuitionistic fuzzy index matrices

The concept of an IFIM was introduced originally in [2] and described in details in [4]. It is based on the concept of an Intuitionistic Fuzzy Set (IFS, see [3]). Here, we give only the definitions that are needed for the description of our idea.

Each IFIM A has the form

$$A = [K, L, \{\langle \mu_{k_i, l_j}, \nu_{k_i, l_j} \rangle\}]$$

$$\equiv \begin{array}{c|cccccc} & l_1 & \dots & l_j & \dots & l_n \\ \hline k_1 & \langle \mu_{k_1, l_1}, \nu_{k_1, l_1} \rangle & \dots & \langle \mu_{k_1, l_j}, \nu_{k_1, l_j} \rangle & \dots & \langle \mu_{k_1, l_n}, \nu_{k_1, l_n} \rangle \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_i & \langle \mu_{k_i, l_1}, \nu_{k_i, l_1} \rangle & \dots & \langle \mu_{k_i, l_j}, \nu_{k_i, l_j} \rangle & \dots & \langle \mu_{k_i, l_n}, \nu_{k_i, l_n} \rangle \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_m & \langle \mu_{k_m, l_1}, \nu_{k_m, l_1} \rangle & \dots & \langle \mu_{k_m, l_j}, \nu_{k_m, l_j} \rangle & \dots & \langle \mu_{k_m, l_n}, \nu_{k_m, l_n} \rangle \end{array},$$

where $K = \{k_1, k_2, \dots, k_m\}$, $L = \{l_1, l_2, \dots, l_n\}$, for $1 \leq i \leq m$, and $1 \leq j \leq n$:

$$\mu_{k_i, l_j}, \nu_{k_i, l_j}, \mu_{k_i, l_j} + \nu_{k_i, l_j} \in [0, 1].$$

For the IFIMs $A = [K, L, \{\langle \mu_{k_i, l_j}, \nu_{k_i, l_j} \rangle\}]$ and $B = [P, Q, \{\langle \rho_{p_r, q_s}, \sigma_{p_r, q_s} \rangle\}]$ different operations are defined, but we will use only two of them, each one in two forms: “additive” and “multiplicative” that are defined, respectively, by:

$$A \oplus_{\vee} B = [K \cup P, L \cup Q, \{\langle \varphi_{t_u, v_w}, \psi_{t_u, v_w} \rangle\}],$$

where

$$\langle \varphi_{t_u, v_w}, \psi_{t_u, v_w} \rangle = \begin{cases} \langle \mu_{k_i, l_j}, \nu_{k_i, l_j} \rangle, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - Q \\ & \text{or } t_u = k_i \in K - P \text{ and } v_w = l_j \in L; \\ \langle \rho_{p_r, q_s}, \sigma_{p_r, q_s} \rangle, & \text{if } t_u = p_r \in P \text{ and } v_w = q_s \in Q - L \\ & \text{or } t_u = p_r \in P - K \text{ and } v_w = q_s \in Q; \\ \langle \max(\mu_{k_i, l_j}, \rho_{p_r, q_s}), \min(\nu_{k_i, l_j}, \sigma_{p_r, q_s}) \rangle, & \text{if } t_u = k_i = p_r \in K \cap P \\ & \text{and } v_w = l_j = q_s \in L \cap Q \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases}$$

$$A \oplus_{\wedge} B = [K \cup P, L \cup Q, \{\langle \varphi_{t_u, v_w}, \psi_{t_u, v_w} \rangle\}],$$

where

$$\langle \varphi_{t_u, v_w}, \psi_{t_u, v_w} \rangle = \begin{cases} \langle \mu_{k_i, l_j}, \nu_{k_i, l_j} \rangle, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - Q \\ & \text{or } t_u = k_i \in K - P \text{ and } v_w = l_j \in L; \\ \langle \rho_{p_r, q_s}, \sigma_{p_r, q_s} \rangle, & \text{if } t_u = p_r \in P \text{ and } v_w = q_s \in Q - L \\ & \text{or } t_u = p_r \in P - K \text{ and } v_w = q_s \in Q; \\ \langle \min(\mu_{k_i, l_j}, \rho_{p_r, q_s}), \max(\nu_{k_i, l_j}, \sigma_{p_r, q_s}) \rangle, & \text{if } t_u = k_i = p_r \in K \cap P \\ & \text{and } v_w = l_j = q_s \in L \cap Q \\ \langle 1, 0 \rangle, & \text{otherwise} \end{cases}$$

$$A \odot_{\vee, \wedge} B = [K \cup (P - L), Q \cup (L - P), \langle \varphi_{t_u, v_w}, \psi_{t_u, v_w} \rangle],$$

where

$$\langle \varphi_{t_u, v_w}, \psi_{t_u, v_w} \rangle = \begin{cases} \langle \mu_{k_i, l_j}, \nu_{k_i, l_j} \rangle, & \text{if } t_u = k_i \in K \\ & \text{and } v_w = l_j \in L - P \\ \langle \rho_{p_r, q_s}, \sigma_{p_r, q_s} \rangle, & \text{if } t_u = p_r \in P - L \\ & \text{and } v_w = q_s \in Q \\ \langle \max_{l_j = p_r \in L \cap P} (\min(\mu_{k_i, l_j}, \rho_{p_r, q_s})), & \text{if } t_u = k_i \in K \\ \min_{l_j = p_r \in L \cap P} (\max(\nu_{k_i, l_j}, \sigma_{p_r, q_s})) \rangle, & \text{and } v_w = q_s \in Q \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases}$$

$$A \odot_{\wedge, \vee} B = [K \cup (P - L), Q \cup (L - P), \langle \varphi_{t_u, v_w}, \psi_{t_u, v_w} \rangle],$$

where

$$\langle \varphi_{t_u, v_w}, \psi_{t_u, v_w} \rangle = \begin{cases} \langle \mu_{k_i, l_j}, \nu_{k_i, l_j} \rangle, & \text{if } t_u = k_i \in K \\ & \text{and } v_w = l_j \in L - P \\ \langle \rho_{p_r, q_s}, \sigma_{p_r, q_s} \rangle, & \text{if } t_u = p_r \in P - L \\ & \text{and } v_w = q_s \in Q \\ \langle \min_{l_j = p_r \in L \cap P} (\max(\mu_{k_i, l_j}, \rho_{p_r, q_s})), & \text{if } t_u = k_i \in K \\ \max_{l_j = p_r \in L \cap P} (\min(\nu_{k_i, l_j}, \sigma_{p_r, q_s})) \rangle, & \text{and } v_w = q_s \in Q \\ \langle 1, 0 \rangle, & \text{otherwise} \end{cases}$$

Let F be a function over intuitionistic fuzzy pairs and A – an IFIM. We define

$$F(A) = [K, L, \{F(\mu_{k_i, l_j}, \nu_{k_i, l_j})\}].$$

4 Main results

The artificial neural networks represent a mathematical model inspired by the biological neural networks [7, 8]. Its functions are borrowed from the functions of human brain. They are made up of interconnected nodes, or neurons, that work together to learn and make predictions. Neural networks are used in a wide variety of applications, including image recognition, natural language processing, and speech recognition.

Neural networks are trained by feeding them a large amount of data. The data is used to adjust the weights of the connections between the neurons. The goal of training is to find a set of weights that produces the correct predictions for the given data. They are a powerful tool for machine learning. They can be used to solve a wide variety of problems. However, they can be difficult to train and require a large amount of data. Neural networks are a rapidly evolving field

of machine learning. As they become more powerful, they used to solve even more challenging problems.

The classical three-layered neural network, in abbreviated notation, has the form from Figure 1.

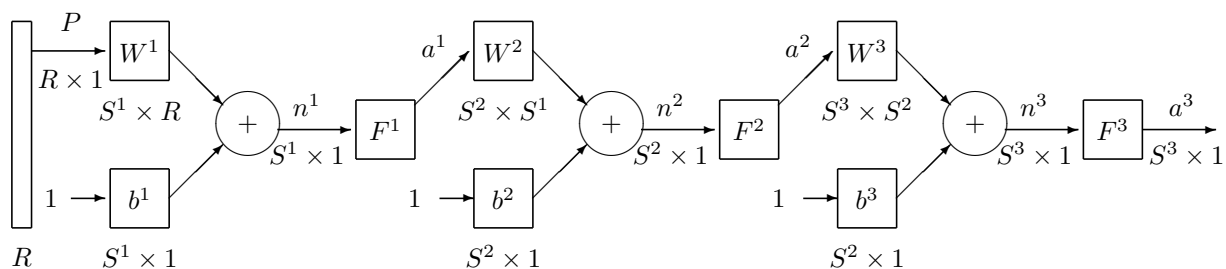


Figure 1. Classical three-layered neural network

In multilayered networks, the exits of one layer become entries for the next one. The equations describing this operation are:

$$a^{m+1} = F^{m+1}(w^{m+1}.a^m + b^{m+1})$$

for $m = 0, 1, 2, \dots, M - 1$, where:

- n is the sum of all input excitabilities and their influences
- m is the current number of the layer in the network;
- M is the number of the layers in the network;
- P is an entry network's vector;
- a^m is the exit of the m -th layer of the neural network;
- s^m is a number of neurons of a m -th layer of the neural network;
- W is a matrix of the coefficients of all inputs;
- b is neuron's input bias;
- F^m is the transfer function of the m -th layer exit.

Let us, for $\alpha, \beta \in [0, 1]$ such that $\alpha + \beta \leq 1$, define the function $H_{\alpha, \beta}$ by

$$H_{\alpha, \beta}(\langle \mu, \nu \rangle) = \begin{cases} \langle \mu, \nu \rangle, & \text{if } \mu > \alpha \text{ and } \nu < \beta \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases}.$$

Now, we describe the IM-representation of the results of the work of the above multilayered network.

Let P be an input vector in the form

$$P = \frac{\begin{array}{c|ccc} & p_1 & \dots & p_R \\ \hline p_0 & \langle a_1, z_1 \rangle & \dots & \langle a_R, z_R \rangle \end{array}}{}$$

Let the weight coefficients of the connections between the nodes of the input vector and these from the first layer be given by the IM

$$W^1 = \frac{\begin{array}{c|ccc} & a_{1,1} & \dots & a_{1,s_1} \\ \hline p_1 & \langle V_{1,1}^1, W_{1,1}^1 \rangle & \dots & \langle V_{1,1}^1, W_{1,s_1}^1 \rangle \\ \vdots & \vdots & \ddots & \vdots \\ p_R & \langle V_{R,1}^1, W_{R,1}^1 \rangle & \dots & \langle V_{R,s_1}^1, W_{R,s_1}^1 \rangle \end{array}}{}$$

Let the parameters of the moves of the neurons from the first layer be given by the IM

$$B^1 = \frac{\begin{array}{c|ccc} & a_{1,1} & \dots & a_{1,s_1} \\ \hline p_0 & \langle b_{1,1}, y_{1,1} \rangle & \dots & \langle b_{1,s_1}, y_{1,s_1} \rangle \end{array}}{}$$

Then, a^1 is the IM with the values of the neurons in the first layer. It is obtained by the formula

$$\begin{aligned} a^1 &= H_{\alpha,\beta}((P \odot_{(\vee,\wedge)} W^1) \oplus_{(\vee)} B^1) \\ &= \frac{\begin{array}{c|ccc} & a_{1,1} & \dots & \dots \\ \hline p_0 & H_{\alpha,\beta}(\bigvee_{1 \leq k \leq R} (\langle a_k, z_k \rangle \wedge \langle V_{k,1}^1, W_{k,1}^1 \rangle) \vee \langle b_{k,1}, y_{k,1} \rangle) & \dots & \dots \\ \dots & \dots & a_{1,s_1} & \dots \\ \dots & H_{\alpha,\beta}(\bigvee_{1 \leq k \leq R} (\langle a_k, z_k \rangle \wedge \langle V_{k,s_1}^1, W_{k,s_1}^1 \rangle) \vee \langle b_{k,s_1}, y_{k,s_1} \rangle) & \dots & \dots \end{array}}{=} \\ &= \frac{\begin{array}{c|ccc} & a_{1,1} & \dots & a_{1,s_1} \\ \hline p_0 & \langle a_1^1, z_1^1 \rangle & \dots & \langle a_{s_1}^1, z_{s_1}^1 \rangle \end{array}}{=} \end{aligned}$$

Let i be a natural number from the set $\{1, 2, \dots, M-1\}$. Let the IM of the weight coefficients of the connections between the nodes of the i -th and $(i+1)$ -st layers be

$$W^i = \frac{\begin{array}{c|ccc} & a_{i,1} & \dots & a_{i,s_i} \\ \hline a_{i-1,1} & \langle V_{1,1}^{i-1}, W_{1,1}^{i-1} \rangle & \dots & \langle V_{1,s_i}^{i-1}, W_{1,s_i}^{i-1} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ a_{i-1,s_{i-1}} & \langle V_{s_{i-1},1}^{i-1}, W_{s_{i-1},1}^{i-1} \rangle & \dots & \langle V_{s_{i-1},s_i}^{i-1}, W_{s_{i-1},s_i}^{i-1} \rangle \end{array}}{}$$

and let the parameters of the moves of the neurons from the i -th layer be given by the IM

$$B^i = \frac{\begin{array}{c|ccc} & a_{i,1} & \dots & a_{i,s_i} \\ \hline p_0 & \langle b_{i,1}, y_{i,1} \rangle & \dots & \langle b_{i,s_i}, y_{i,s_i} \rangle \end{array}}{}$$

Let us have the IM for the $(i-1)$ -st layer

$$a^{i-1} = \frac{\begin{array}{c|ccc} & a_{i-1,1} & \dots & a_{i-1,s_{i-1}} \\ \hline p_0 & \langle a_1^{i-1}, z_1^{i-1} \rangle & \dots & \langle a_{s_{i-1}}^{i-1}, z_{s_{i-1}}^{i-1} \rangle \end{array}}{}$$

Then

$$\begin{aligned}
 a^i &= H_{\alpha,\beta}((a^{i-1} \odot_{(\vee,\wedge)} W^i) \oplus_{(\vee)} B^i) \\
 &= \frac{p_0 \left| \begin{array}{c} a_{i,1} \quad \dots \\ H_{\alpha,\beta}(\bigvee_{1 \leq k \leq s_{i-1}} (\langle a_k^{i-1}, z_k^{i-1} \rangle \wedge \langle V_{k,1}^i, W_{k,1}^i \rangle) \vee \langle b_{k,1}^i, y_{k,1}^i \rangle) \quad \dots \\ \dots \\ a_{i,s_i} \end{array} \right.}{\dots \quad H_{\alpha,\beta}(\bigvee_{1 \leq k \leq s_{i-1}} (\langle a_k^{i-1}, z_k^{i-1} \rangle \wedge \langle V_{k,1}^i, W_{k,1}^i \rangle))} \\
 &= \frac{p_0 \left| \begin{array}{c} a_{i,1} \quad \dots \quad a_{i,s_i} \\ \langle a_1^i, z_1^i \rangle \quad \dots \quad \langle a_{s_i}^i, z_{s_i}^i \rangle \end{array} \right.}{\dots}
 \end{aligned}$$

and for $i = M$

$$a^{M-1} = \frac{p_0 \left| \begin{array}{c} a_{M,1} \quad \dots \quad a_{M,s_{M-1}} \\ \langle a_1^{M-1}, z_1^{M-1} \rangle \quad \dots \quad \langle a_{s_{M-1}}^{M-1}, z_{s_{M-1}}^{M-1} \rangle \end{array} \right.}{\dots}$$

5 Conclusion

After description of the biological motivation of the idea of neural networks, and then description of the IFIM we described the main results about introducing IFIM for description of multilayer neural networks. First the input of a neural network was described in the form of IFIM, then the weight coefficients of the connections between the nodes of the input vector, and next the activation functions of neurons. The use of IFIM give a new very concise and elegant way to describe multilayer neural networks.

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