Notes on Intuitionistic Fuzzy Sets
ISSN 1310–4926
Vol. 20, 2014, No. 1, 9–19

Non-linear arithmetic operation on generalized triangular intuitionistic fuzzy numbers

Sankar Prasad Mondal* and Tapan Kumar Roy

Department of Mathematics, Bengal Engineering and Science University
Shibpur, Howrah–711103, West Bengal, India
* Corresponding author (email: sankar.res07@gmail.com)

Abstract: In this paper we discussed some nonlinear operation on generalized triangular intuitionistic fuzzy number. Some examples and an application are given.
Keywords: Fuzzy set, Intuitionistic fuzzy number.
AMS Classification: 03E72, 03E75, 26E50.

1 Introduction

Zadeh [1] and Dubois and Prade [2] were the first who introduced the conception based on fuzzy number and fuzzy arithmetic. One of the generalizations of fuzzy sets theory [1] is considered to be Intuitionistic fuzzy sets (IFS) theory. Out of several higher-order fuzzy sets, IFS were first introduced by Atanassov [3] and have been found to be suitable to deal with unexplored areas. The fuzzy set considers only the degree of belongingness and non-belongingness. Fuzzy set theory does not incorporate the degree of hesitation (i.e., degree of non-determinacy, defined as $1-$ sum of membership function and non-membership function. To handle such situations, Atanassov [4] explored the concept of fuzzy set theory by intuitionistic fuzzy set (IFS) theory. The degree of acceptance in fuzzy sets is only considered, otherwise IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one [4].

Basic arithmetic operations of TIFNs is defined by Deng-Feng Li in [5] using membership and non-membership values. Basic arithmetic operations of TIFNs such as addition, subtraction and multiplication are defined by Mahapatra and Roy in [6], by considering the six-tuple number itself and division by Nagoorgani and Ponnalagu [7].

Nowadays, IFSs are being studied extensively and being used in different fields of science and technology. Amongst the all research works mainly on IFS we can include Atanassov [4, 8–11], Atanassov and Gargov [12], Szmidt and Kacprzyk [13], Buhaescu [14, 19], Ban [15], Deschrijver and Kerre [16], Stoyanova [17, 22], Cornelis et al. [18], Gerstenkorn and Manko [20], Stoyanova and Atanassov [21], Mahapatra and Roy [23], Hajeeh [24], Persona et al. [25], Prabha et al. [26], Nikolaidis and Mourelatos [27], Kumar et al. [28], Wang [29], Shaw and Roy [30], Adak et al. [31], Varghese and Kuriakose [32].

2 Preliminary concepts

Definition 2.1: Intuitionistic Fuzzy Number: An IFN $\tilde{A}^i$ is defined as follows:

i) an intuitionistic fuzzy subject of real line;
ii) normal, i.e., there is any $x_0 \in R$ such that $\mu_{\tilde{A}^i}(x_0) = 1$(so $\vartheta_{\tilde{A}^i}(x_0) = 0$);
iii) a convex set for the membership function $\mu_{\tilde{A}^i}(x)$, i.e., $\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geq min\left(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\right) \forall\, x_1, x_2 \in R, \lambda \in [0,1]$;
iv) a concave set for the non-membership function $\vartheta_{\tilde{A}^i}(x)$, i.e., $\vartheta_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geq max\left(\vartheta_{\tilde{A}^i}(x_1), \vartheta_{\tilde{A}^i}(x_2)\right) \forall\, x_1, x_2 \in R, \lambda \in [0,1]$.

Definition 2.2: Triangular Intuitionistic Fuzzy number: A TIFN $\tilde{A}^i$ is a subset of IFN in R with following membership function and non membership function as follows:

$$\mu_{\tilde{A}^i}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}\ for\ a_1 \leq x\ \leq a_2, \\ \dfrac{a_3 - x}{a_3 - a_2}\ for\ a_2 \leq x\ \leq a_3 \\ 0\ otherwise \end{cases}$$

and

$$\vartheta_{\tilde{A}^i}(x_1) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1^{'}}\ for\ a_1^{'} \leq x\ \leq a_2 \\ \dfrac{x - a_2}{a_3^{'} - a_2}\ for\ a_2 \leq x\ \leq a_3^{'} \\ 1\ otherwise \end{cases}$$

where $a_1^{'} \leq a_1 \leq a_2 \leq a_3 \leq a_3^{'}$ and TIFN is denoted by $\tilde{A}^i{}_{TIFN} = (a_1, a_2, a_3; a_1^{'}, a_2, a_3^{'})$

Definition 2.3: Generalized Intuitionistic Fuzzy Number: An IFN $\tilde{A}^i$ is defined as follows:

i) an intuitionistic fuzzy subject of real line;
ii) normal, i.e., there is any $x_0 \in R$ such that $\mu_{\tilde{A}^i}(x_0) = \omega$ (so $\vartheta_{\tilde{A}^i}(x_0) = 0$) for $0 < \omega \leq 1$;
iii) a convex set for the membership function $\mu_{\tilde{A}^i}(x)$, i.e., $\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geq min\left(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\right) \forall\, x_1, x_2 \in R, \lambda \in [0, \omega]$;
iv) a concave set for the non-membership function $\vartheta_{\tilde{A}^i}(x)$, i.e., $\vartheta_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geq max\left(\vartheta_{\tilde{A}^i}(x_1), \vartheta_{\tilde{A}^i}(x_2)\right) \forall\, x_1, x_2 \in R, \lambda \in [0, \omega]$;
v) $\mu_{\tilde{A}^i}$ is continuous mapping from R to the closed interval $[0, \omega]$ and $x_0 \in R$, the relation $0 \leq \mu_{\tilde{A}^i}(x_0) + \vartheta_{\tilde{A}^i}(x_0) \leq \omega$ holds.

Definition 2.4: Generalized Triangular Intuitionistic Fuzzy number: A TIFN $\tilde{A}^i$ is a subset of IFN in R with following membership function and non-membership function as follows:

$$\mu_{\tilde{A}^i}(x) = \begin{cases} \omega \dfrac{x-a_1}{a_2-a_1} \ for \ a_1 \le x \ \le a_2, \\ \omega \ for \ x = a_2 \\ \omega \dfrac{a_3-x}{a_3-a_2} \ for \ a_2 \le x \ \le a_3 \\ 0 \ otherwise \end{cases}$$

and

$$\vartheta_{\tilde{A}^i}(x) = \begin{cases} \omega \dfrac{a_2-x}{a_2-a_1'} \ for \ a_1 \le x \ \le a_2, \\ 0 \ for \ x = a_2 \\ \omega \dfrac{x-a_2}{a_3'-a_2} \ for \ a_2 \le x \ \le a_3 \\ \omega \ otherwise \end{cases}$$

where $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$ and GTIFN is denoted by

$$\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right).$$

Definition 2.5: A GTIFN $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right)$ is said to be non-negative iff $a_1' \ge 0$.

Definition 2.6: Two GTIFN $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega_1\right)$ and $\tilde{B}^i{}_{GTIFN} = \left((b_1, b_2, b_3), \left(b_1', b_2, b_3'\right); \omega_2\right)$ are said to be equal iff $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$, $a_1' = b_1'$, $a_3' = b_3'$ and $\omega_1 = \omega_2$.

Definition 2.7: α-cut set: A α-cut set of $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right)$ is a crisp subset of R which is defined as follows

$$A_\alpha = \{x: \mu_{\tilde{A}^i}(x) \ge \alpha\} = [A_1(\alpha), A_2(\alpha)] = [a_1 + \tfrac{\alpha}{\omega}(a_2 - a_2), a_3 - \tfrac{\alpha}{\omega}(a_3 - a_2)].$$

Definition 2.8: β-cut set: A α-cut set of $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right)$ is a crisp subset of R which is defined as follows

$$A_\alpha = \{x: \vartheta_{\tilde{A}^i}(x) \le \beta\} = [A_1'(\beta), A_2'(\beta)] = [a_2 - \tfrac{\beta}{\omega}\left(a_2 - a_1'\right), a_2 + \tfrac{\beta}{\omega}(a_3' - a_2)].$$

Definition 2.9: (α, β)-cut set: A (α, β)-cut set of $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right)$ is a crisp subset of R which is defined as follows

$$A_{\alpha,\beta} = \{[A_1(\alpha), A_2(\alpha)]; [A_1'(\beta), A_2'(\beta)]\}, \alpha + \beta \le \omega, \alpha, \beta \in [0, \omega].$$

Definition 2.10: Addition of two GTIFN: Let $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega_1\right)$ and $\tilde{B}^i{}_{GTIFN} = \left((b_1, b_2, b_3), \left(b_1', b_2, b_3'\right); \omega_2\right)$ be two GTIFN, then the addition of two GTIFN is given by

$$\tilde{A}^i{}_{GTIFN} \oplus \tilde{B}^i{}_{GTIFN} = \left((a_1 + b_1, a_2 + b_2, a_3 + b_3), \left(a_1' + b_1', a_2 + b_2, a_3' + b_3'\right); \omega\right),$$

where $0 < \omega \leq 1, \omega = \min(\omega_1, \omega_2)$.

Definition 2.11: Subtraction of two GTIFN: Let $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), (a_1^{'}, a_2, a_3^{'}); \omega_1\right)$ and $\tilde{B}^i{}_{GTIFN} = \left((b_1, b_2, b_3), (b_1^{'}, b_2, b_3^{'}); \omega_2\right)$ be two GTIFN, then the subtraction of two GTIFN is given by

$$\tilde{A}^i{}_{GTIFN} \ominus \tilde{B}^i{}_{GTIFN} = \left((a_1 - b_3, a_2 - b_2, a_3 - b_1), (a_1^{'} - b_3^{'}, a_2 - b_2, a_3^{'} - b_1^{'}); \omega\right),$$

where $0 < \omega \leq 1, \omega = \min(\omega_1, \omega_2)$.

Definition 2.12: Multiplication by a scalar: Let $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), (a_1^{'}, a_2, a_3^{'}); \omega\right)$ and k is a scalar then $k\tilde{A}^i{}_{GTIFN}$ is also a GTIFN and is defined as

$$k\tilde{A}^i{}_{GTIFN} = \begin{cases} \left((ka_1, ka_2, ka_3), (ka_1^{'}, ka_2, ka_3^{'}); \omega\right), if\ k > 0 \\ \left((ka_3, ka_2, ka_1), (ka_3^{'}, ka_2, ka_1^{'}); \omega\right), if\ k < 0 \end{cases},$$

where $0 < \omega \leq 1$.

Definition 2.13: Multiplication of two GTIFN: Let
$\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), (a_1^{'}, a_2, a_3^{'}); \omega_1\right)$ and $\tilde{B}^i{}_{GTIFN} = \left((b_1, b_2, b_3), (b_1^{'}, b_2, b_3^{'}); \omega_2\right)$ be two GTIFN, then the subtraction of two GTIFN is given by

$$\tilde{A}^i{}_{GTIFN} \otimes \tilde{B}^i{}_{GTIFN} = \left((a_1 b_1, a_2 b_2, a_3 b_3), (a_1^{'} b_1^{'}, a_2 b_2, a_3^{'} b_3^{'}); \omega\right),$$

where $0 < \omega \leq 1, \omega = \min(\omega_1, \omega_2)$.

Definition 2.14: Division of two GTIFN: Let $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), (a_1^{'}, a_2, a_3^{'}); \omega_1\right)$ and $\tilde{B}^i{}_{GTIFN} = \left((b_1, b_2, b_3), (b_1^{'}, b_2, b_3^{'}); \omega_2\right)$ be two GTIFN, then the division of two GTIFN is given by

$$\tilde{A}^i{}_{GTIFN} \div \tilde{B}^i{}_{GTIFN} = \left(\left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right), \left(\frac{a_1^{'}}{b_3^{'}}, \frac{a_2}{b_2}, \frac{a_3^{'}}{b_1^{'}}\right); \omega\right),$$

where $0 < \omega \leq 1, \omega = \min(\omega_1, \omega_2)$.

Definition 2.15: Inverse of a GTIFN: Let $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), (a_1^{'}, a_2, a_3^{'}); \omega\right)$ be a GTIFN, then its inverse is given by

$$\frac{1}{\tilde{A}^i{}_{GTIFN}} = \left(\left(\frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right), \left(\frac{1}{a_3^{'}}, \frac{1}{a_2}, \frac{1}{a_1^{'}}\right); \omega\right)$$

3 Non-linear operation of GTIFN

3.1 Modulus of a GTIFN

Let $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right)$ be a GTIFN, then its modulus is given by

$$\left|\tilde{A}^i{}_{GTIFN}\right| = \left|\left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right)\right|$$

$$= \begin{cases} \left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right) & \tilde{A}^i{}_{GTIFN} \geq 0 \\ \left((-a_3, -a_2, -a_1), \left(a_1', a_2, a_3'\right); \omega\right) & \tilde{A}^i{}_{GTIFN} < 0 \end{cases}$$

3.2 Square root of a GTIFN

Square root of a GTIFN $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right) > 0$ is obtained as follows

$$\sqrt{\tilde{A}^i{}_{GTIFN}} = \sqrt{\left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right)} = \left((d, e, f), (l, m, n); \omega\right)$$

Or, $\left(\left((d, e, f), (l, m, n); \omega\right)\right)^2 = \left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right)$

Now applying the multiplication rule we get

$$\sqrt{\left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right)} = \left(\left(\sqrt{a_1}, \sqrt{a_2}, \sqrt{a_3}\right), \left(\sqrt{a_1'}, \sqrt{a_2}, \sqrt{a_3'}\right); \omega\right)$$

3.3 A general recursive formula for $\left(\left((\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}), \left(\mathbf{a_1'}, \mathbf{a_2}, \mathbf{a_3'}\right); \boldsymbol{\omega}\right)\right)^{\mathbf{n}}$

Using the multiplication of two positive generalized Intuitionistic fuzzy number, we have

$$\left(\left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right)\right)^n = \left(\left((a_1)^n, (a_2)^n, (a_3)^n\right), \left(\left(a_1'\right)^n, (a_2)^n, \left(a_3'\right)^n\right); \omega\right)$$

Now we find a general recursive formulae for $\left(-\tilde{A}^i{}_{GTIFN}\right)^n$:

$$\left(-\left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right)\right)^n$$

$$= \begin{cases} \left(\left((a_3)^n, (a_2)^n, (a_1)^n\right), \left(\left(a_3'\right)^n, (a_2)^n, \left(a_1'\right)^n\right); \omega\right), & n \text{ is even} \\ \left(\left(-(a_3)^n, -(a_2)^n, -(a_1)^n\right), \left(-\left(a_3'\right)^n, -(a_2)^n, -\left(a_1'\right)^n\right); \omega\right), & n \text{ is odd} \end{cases}$$

3.4 Exponential of a non-negative GTIFN

We use the Taylor series expansion method.

We know that $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, -\infty < x < \infty$

For $x = \tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right)$ we have

$$e^{\tilde{A}^i{}_{GTIFN}} = 1 + \frac{\tilde{A}^i{}_{GTIFN}}{1!} + \frac{\tilde{A}^i{}_{GTIFN}{}^2}{2!} + \frac{\tilde{A}^i{}_{GTIFN}{}^3}{3!} + \cdots, x \geq 0$$

Now, $\left(\left((a_1, a_2, a_3), \left(a_1', a_2, a_3'\right); \omega\right)\right)^n = \left(\left((a_1)^n, (a_2)^n, (a_3)^n\right), \left(\left(a_1'\right)^n, (a_2)^n, \left(a_3'\right)^n\right); \omega\right)$

Therefore, $e^{\tilde{A}^i_{GTIFN}} = \left((1,0,0),(0,0,0\,);\omega\right) + \sum_{i=1}^{\infty}\frac{\left(\left((a_1,a_2,a_3),(a_1^{'},a_2,a_3^{'}\,);\omega\right)\right)^i}{i!}$

i.e., $e^{\tilde{A}^i_{GTIFN}} = \left(\left(\left(1+\frac{a_1}{1!}+\frac{{a_1}^2}{2!}+\cdots\right),\left(\frac{a_2}{1!}+\frac{{a_2}^2}{2!}+\cdots\right),\left(\frac{a_3}{1!}+\frac{{a_3}^2}{2!}+\cdots\right)\right),\left(\left(\frac{a_1^{'}}{1!}+\frac{{a_1^{'}}^2}{2!}+\cdots\right),\left(\frac{a_2}{1!}+\frac{{a_2}^2}{2!}+\cdots\right),\left(\frac{a_3^{'}}{1!}+\frac{{a_3^{'}}^2}{2!}+\cdots\right)\right);\omega\right)$

Or, $e^{\tilde{A}^i_{GTIFN}} = \left((e^{a_1}, e^{a_2}-1, e^{a_3}-1),\left(e^{a_1^{'}}-1, e^{a_2}-1, e^{a_3^{'}}-1\right);\omega\right)$

3.5 Inverse Exponential of a non-negative GTIFN

$$e^{-\tilde{A}^i_{GTIFN}} = \frac{1}{e^{\tilde{A}^i_{GTIFN}}} = \left(\left(\frac{1}{e^{a_1}},\frac{1}{e^{a_2-1}},\frac{1}{e^{a_3-1}}\right),\left(\frac{1}{e^{a_1^{'}-1}},\frac{1}{e^{a_2-1}},\frac{1}{e^{a_3^{'}-1}}\right);\omega\right)$$

Corollary:$e^{\tilde{A}^i_{GTIFN}}.e^{\tilde{B}^i_{GTIFN}} = e^{\tilde{A}^i_{GTIFN}+\tilde{B}^i_{GTIFN}}$ if $\tilde{A}^i_{GTIFN}, \tilde{B}^i_{GTIFN} \geq 0$

Corollary: $\left(e^{\tilde{A}^i_{GTIFN}}\right)^a = e^{a\tilde{A}^i_{GTIFN}}$ if $\tilde{A}^i_{GTIFN} \geq 0$ and $a \in R^+$

Corollary: $\frac{e^{\tilde{A}^i_{GTIFN}}}{\tilde{B}^i_{GTIFN}} = e^{\tilde{A}^i_{GTIFN}-\tilde{B}^i_{GTIFN}}$ if $\tilde{A}^i_{GTIFN}, \tilde{B}^i_{GTIFN} \geq 0$.

3.6 Logarithm of a non-negative GTIFN

Let $\log_e\left(\left((a_1,a_2,a_3),(a_1^{'},a_2,a_3^{'}\,);\omega\right)\right) = \left((x_1,x_2,x_3),(x_1^{'},x_2,x_3^{'}\,);\omega\right)$

Therefore, $\left((a_1,a_2,a_3),(a_1^{'},a_2,a_3^{'}\,);\omega\right) = e^{\left((x_1,x_2,x_3),(x_1^{'},x_2,x_3^{'}\,);\omega\right)} = \left((e^{x_1}, e^{x_2}-1, e^{x_3}-1),\left(e^{x_1^{'}}-1, e^{x_2}-1, e^{x_3^{'}}-1\right);\omega\right)$

$e^{x_1} = a_1$ or, $x_1 = \log_e a_1$

$e^{x_2}-1 = a_2$ or, $x_2 = \log_e(1+a_2)$

$e^{x_3}-1 = a_3$ or, $x_3 = \log_e(1+a_3)$

$e^{x_1^{'}}-1 = a_1^{'}$ or, $x_1^{'} = \log_e\left(1+a_1^{'}\right)$

$e^{x_3^{'}}-1 = a_3^{'}$ or, $x_3^{'} = \log_e\left(1+a_3^{'}\right)$

Hence, $\log_e\left(\left((a_1,a_2,a_3),(a_1^{'},a_2,a_3^{'}\,);\omega\right)\right) = \left((\log_e a_1, \log_e(1+a_2), \log_e(1+a_3)),\left(\log_e\left(1+a_1^{'}\right), \log_e(1+a_2), \log_e\left(1+a_3^{'}\right)\right);\omega\right)$

Corollary: $\log_e \tilde{A}^i_{GTIFN} + \log_e \tilde{B}^i_{GTIFN} = \log_e\left(\tilde{A}^i_{GTIFN}\tilde{B}^i_{GTIFN}\right)$ if $\tilde{A}^i_{GTIFN}, \tilde{B}^i_{GTIFN} > 0$

Corollary: $\log_e \tilde{A}^i_{GTIFN} - \log_e \tilde{B}^i_{GTIFN} = \log_e\left(\frac{\tilde{A}^i_{GTIFN}}{\tilde{B}^i_{GTIFN}}\right)$ if $\tilde{A}^i_{GTIFN} \geq \tilde{B}^i_{GTIFN} > 0$

Corollary: $\log_e\left(\tilde{A}^i_{GTIFN}\right)^a = a\log_e \tilde{A}^i_{GTIFN}$ if $\tilde{A}^i_{GTIFN} > 0, a \in I^+$

3.7 Positive solution of $\left(\left((a_1, a_2, a_3), (a_1', a_2, a_3'); \omega\right)\right)^{\frac{1}{n}}$

By using multiplication rule we also find the n^{th} $(n > 0)$ positive root of a fuzzy number as

$$\left(\left((a_1, a_2, a_3), (a_1', a_2, a_3'); \omega\right)\right)^{\frac{1}{n}} \simeq \left(\left((a_1)^{\frac{1}{n}}, (a_2)^{\frac{1}{n}}, (a_3)^{\frac{1}{n}}\right), \left((a_1')^{\frac{1}{n}}, (a_2)^{\frac{1}{n}}, (a_3')^{\frac{1}{n}}\right); \omega\right).$$

3.8 $\widetilde{A}^i{}_{GTIFN}{}^{\widetilde{B}^i{}_{GTIFN}}$ if $\widetilde{A}^i{}_{GTIFN} > 0, \widetilde{B}^i{}_{GTIFN} \geq 0$

Let $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), (a_1', a_2, a_3'); \omega\right)$ and $\tilde{B}^i{}_{GTIFN} = \left((b_1, b_2, b_3), (b_1', b_2, b_3'); \omega\right)$

Therefore, $\left((a_1, a_2, a_3), (a_1', a_2, a_3'); \omega\right)^{\left((b_1,b_2,b_3),(b_1',b_2,b_3');\omega\right)}$

$= e^{\left((b_1,b_2,b_3),(b_1',b_2,b_3');\omega\right)\ln\left((a_1,a_2,a_3),(a_1',a_2,a_3');\omega\right)}$

$= e^{\left((b_1,b_2,b_3),(b_1',b_2,b_3');\omega\right)\left((\ln a_1,\ln(1+a_2),\ln(1+a_3)),(\ln(1+a_1'),\ln(1+a_2),\ln(1+a_3'));\omega\right)}$

$= e^{\left((b_1\ln a_1,b_2\ln(1+a_2),b_3\ln(1+a_3)),(b_1'\ln(1+a_1'),b_2\ln(1+a_2),b_3'\ln(1+a_3'));\omega\right)}$

$= \left(\left(e^{b_1\ln a_1}, e^{b_2\ln(1+a_2)} - 1, e^{b_3\ln(1+a_3)} - 1\right), \left(e^{b_1'\ln(1+a_1')} - 1, e^{b_2\ln(1+a_2)} - 1, e^{b_3'\ln(1+a_3')} - 1\right); \omega\right)$

$= \left(\left({a_1}^{b_1}, (1+a_2)^{b_2} - 1, (1+a_3)^{b_3} - 1\right), \left((1+a_1')^{b_1'} - 1, (1+a_2)^{b_2}, (1+a_3')^{b_3'} - 1\right); \omega\right).$

3.9 $a^{\widetilde{A}^i{}_{GTIFN}}$ where $a \geq 1$ and $\widetilde{A}^i{}_{GTIFN} \geq 0$

Let $\tilde{A}^i{}_{GTIFN} = \left((a_1, a_2, a_3), (a_1', a_2, a_3'); \omega\right)$

$a^{\tilde{A}^i{}_{GTIFN}} = e^{\tilde{A}^i{}_{GTIFN}\ln a} = \exp\left(\left((a_1\ln a, a_2\ln a, a_3\ln a), (a_1'\ln a, a_2\ln a, a_3'\ln a); \omega\right)\right)$

$= \left((a^{a_1}, a^{a_2} - 1, a^{a_3} - 1), \left(a^{a_1'} - 1, a^{a_2} - 1, a^{a_3'} - 1\right); \omega\right).$

4 Numerical example

Example 4.1

Find the value of $\sqrt{\tilde{x} + \sqrt{\tilde{x} + \sqrt{\tilde{x} + \cdots\cdots\cdots\cdots}}}$ where $\tilde{x} = \left((3,4,6), (2,4,7); 0.7\right)$.

Solution: Let the value of the above is $\tilde{p} = \left((a,b,c), (r,b,t); 0.7\right)$.

Therefore $\sqrt{\tilde{x} + \tilde{p}} = \tilde{p}$.

Or, $\sqrt{\left((3+a, 4+b, 6+c), (2+r, 4+b, 7+t); 0.7\right)} = \left((a,b,c), (r,b,t); 0.7\right)$.

This implies,

$\sqrt{3+a} = a$ or, $a^2 - a - 3 = 0$ or, $a = 2.30$
$\sqrt{4+b} = b$ or, $b^2 - b - 4 = 0$ or, $b = 2.56$
$\sqrt{6+c} = c$ or, $c^2 - c - 6 = 0$ or, $c = 3$
$\sqrt{2+r} = r$ or, $r^2 - r - 2 = 0$ or, $r = 2$
$\sqrt{7+t} = t$ or, $t^2 - t - 7 = 0$ or, $t = 3.14$

Hence the value of the above is $\big((2.30,2.56,3),(2,2.56,3.14);0.7\big)$.

Example 4.2

Find all the positive solution of $e^{\tilde{x}} = \sqrt{\big((3.24,4,4.84),(2.56,4,5.76);0.8\big)}$.

Solution:

$e^{\tilde{x}} = \sqrt{\big((3.24,4,4.84),(2.56,4,5.76);0.8\big)} = \big((\sqrt{3.24},\sqrt{4},\sqrt{4.84}),(\sqrt{2.56},\sqrt{4},\sqrt{5.76});0.8\big)$
Or, $e^{\tilde{x}} = \big((1.8,2,2.2),(1.6,2,2.4);0.8\big)$.
Or, $\tilde{x} = \big((\ln 1.8,\ln 3,\ln 3.2),(\ln 2.6,\ln 3,\ln 3.4);0.8\big)$.
Or, $\tilde{x} = \big((0.58,1.09,1.16),(0.95,1.09,1.22);0.8\big)$.

Example 4.3

Find all the solutions of the equation $\sqrt{|\tilde{x}|} = \big((4,6,7),(3,6,8);0.6\big)$.

Solution: $|\tilde{x}| = \big((4,6,7),(3,6,8);0.6\big)^2 = \big((16,36,49),(9,36,64);0.6\big)$
The equation has two solutions as

$$\big((16,36,49),(9,36,64);0.6\big)$$

and

$$\big((-16,-36,-49),(9,36,64);0.6\big).$$

Example 4.4

Evaluate the fuzzy solution of $\big((2.197,3.375,4.913),(1.728,3.375,5.832);0.9\big)^{\frac{1}{3}}$.

Solution: The positive fuzzy solution is $\big((1.3,1.5,1.7),(1.2,1.5,1.8);0.9\big)$.

Example 4.5

Compute the value of $\big((3,4,5),(2,4,6);0.8\big)^{\big((4,5,7),(3,5,8);0.8\big)}$.

Solution: The value of above is given by $\big((81,3124,279935),(26,3124,5764800);0.8\big)$.

5 Application

Bank Account Problem, [33]

The Balance $B(t)$ of a bank account grows under continuous process given by $\frac{dB}{dt} = rB$, where r the constant of proportionality is the annual interest rate. If there are initially $B(t) = B_0$ balance, solve the above problem in fuzzy environment when

$$\widetilde{B_0} = \big((\$850, \$1000, \$1100), (\$800, \$1000, \$1200); 0.7\big)$$

and

$$\tilde{r} = \big((3.7,4,4.5), (3.5,4,5); 0.7\big)\%.$$

Find the solution after $t = 3$ years.

Solution: The solution is given by $B(t = 3) = \widetilde{B_0} e^{\frac{3}{100}\tilde{r}}$

$= \big((\$850, \$1000, \$1100), (\$800, \$1000, \$1200); 0.7\big) e^{\left((.111,.12,.135),(.105,.12,.15);0.7\right)}$

$= \big((\$850, \$1000, \$1100), (\$800, \$1000, \$1200); 0.7\big)$.

$= \big((1.1174, 0.1275, 0.1445), (0.1107, 0.1275, 0.1618); 0.7\big)$

$= \big((\$949.79, \$127.50, \$158.95), (\$88.56, \$127.50, \$194.16); 0.7\big)$

6 Conclusion

In this paper, we discussed some nonlinear operation (such as logarithm, exponential) on generalized triangular intuitionistic fuzzy number. Some example and an application are given. A imprecise bank account problem are given in generalized triangular intuitionistic fuzzy environment.

References

[1] Zadeh, L. A., Fuzzy sets, *Information and Control*, Vol. 8, 1965, 338–353.

[2] Dubois, D., H. Parade, Operation on Fuzzy Number. *International Journal of Fuzzy Systems*, Vol. 9, 1978, 613–626.

[3] Atanassov, K. T., Intuitionistic fuzzy sets, *VII ITKR's Session*, Sofia, 1983 (in Bulgarian).

[4] Atanassov, K. T., Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, Vol. 20, 1986, 87–96.

[5] Deng-Feng-Li, A note on "Using intuitionistic fuzzy sets for fault tree analysis on printed circuit board assembly", *Micro Electronics Reliability*, Vol. 48, 2008, 1741.

[6] Mahapatra, G. S., T. K. Roy, Reliability Evaluation using Triangular Intuitionistic Fuzzy numbers Arithmetic operations, *World Academy of Science, Engineering and Technology* Vol. 50, 2009, 574–581.

[7] Nagoorgani, A., K. Ponnalagu, A new approach on solving intuitionistic fuzzy linear programming problem, *Applied Mathematical Sciences*, Vol. 6, 2012, No. 70, 3467–3474.

[8] Atanassov, K. T., G. Gargov, Interval-valued intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, Vol. 31, 1989, No. 3, 343–349.

[9] Atanassov, K. T., More on intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, Vol. 33, 1989, No. 1, 37–46.

[10] Atanassov, K. T., *Intuitionistic Fuzzy Sets*, Physica–Verlag, Heidelberg, 1999.

[11] Atanassov, K. T., Two theorems for intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, Vol. 110, 2000, 267–269.

[12] Atanassov, K. T., G. Gargov, Elements of intuitionistic fuzzy logic, Part I, *Fuzzy Sets and Systems*, Vol. 95, 1998, No. 1, 39–52.

[13] Szmidt, E., J. Kacprzyk, Distances between intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, Vol. 114, 2000, No. 3, 505–518.

[14] Buhaescu, T., Some observations on intuitionistic fuzzy relations, *Itinerant Seminar of Functional Equations, Approximation and Convexity*, Cluj-Napoca, 1989, 111–118.

[15] Ban, A. I., Nearest interval approximation of an intuitionistic fuzzy number, *Computational Intelligence, Theory and Applications*, Springer–Verlag, Berlin, Heidelberg, 2006, 229–240.

[16] Deschrijver, G., E. E. Kerre, On the relationship between intuitionistic fuzzy sets and some other extensions of fuzzy set theory, *Journal of Fuzzy Mathematics*, Vol. 10, 2002, No. 3, 711–724.

[17] Stoyanova, D., More on Cartesian product over intuitionistic fuzzy sets, *BUSEFAL*, Vol. 54, 1993, 9–13.

[18] Cornelis, C., G. Deschrijver, G., E. E. Kerre, Implication in intuitionistic fuzzy and interval–valued fuzzy set theory: Construction, application, *International Journal of Approximate Reasoning*, Vol. 35, 2004, 55–95.

[19] Buhaescu, T., On the convexity of intuitionistic fuzzy sets, *Itinerant Seminar on Functional Equations, Approximation and Convexity*, Cluj-Napoca,1988, 137–144.

[20] Gerstenkorn, T., J. Manko, Correlation of intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, Vol. 44, 1991, 39–43.

[21] Stoyanova, D., K. T. Atanassov, Relation between operators, defined over intuitionistic fuzzy sets, *Preprint IM-MFAIS*, Sofia, Bulgaria, 1990, 46–49.

[22] Stoyanova, D., More on Cartesian product over intuitionistic fuzzy sets, *BUSEFAL*, Vol. 54, 1993, 9–13.

[23] Mahapatra, G. S., T. K. Roy, Reliability evaluation using triangular intuitionistic fuzzy numbers arithmetic operations, *Proceedings of World Academy of Science, Engineering and Technology*, Vol. 38, 2009, 587–595.

[24] Hajeeh, M. A., Reliability and availability of a standby system with common cause failure, *International Journal of Operational Research*, Vol. 11, 2011, No. 3, 343–363.

[25] Persona, A., F. Sqarbossa, H. Pham, Systemability function to optimization reliability in random environment, *International Journal of Mathematics in Operational Research*, Vol. 1, 2009, No. 3, 397–417.

[26] Praba, B., R. Sujatha, S. Srikrishna, Posfust reliability of a unified fuzzy Markov model, *International Journal of Reliability and Safety*, Vol. 5, No. 1, 83–94, 2011.

[27] Nikolaidis, E., Z.P. Mourelatos, Imprecise reliability assessment when the type of the probability distribution of the random variables is unknown, *International Journal of Reliability and Safety*, Vol. 5, 2011, No. 2, 140–157.

[28] Kumar, M., S.P. Yadav, S. Kumar, A new approach for analyzing the fuzzy system reliability using intuitionistic fuzzy number, *International Journal of Industrial and Systems Engineering*, Vol. 8, 2011, No. 2, 135–156.

[29] Wang, Y., Imprecise probabilities based on generalized intervals for system reliability assessment, *International Journal of Reliability and Safety*, Vol. 4, 2010, No. 4, 319–342.

[30] Shaw, A. K., T. K. Roy, Trapezoidal Intuitionistic Fuzzy Number with some arithmetic operations and its application on reliability evaluation**,** *Int. J. Mathematics in Operational Research*, Vol. 5, 2013, No. 1, 55–73.

[31] Adak, A. K., M. Bhowmik, M. Pal, intuitionistic fuzzy block matrix and its some properties, *Annals of Pure and Applied Mathematics*, Vol. 1, 2012, No. 1, 13–31.

[32] Varghese, A., S. Kuriakose, Centroid of an intuitionistic fuzzy number, *Notes on Intuitionistic Fuzzy Sets*, Vol. 18, 2012, No. 1, 19–24.

[33] Mondal, S. P., T. K. Roy, First order linear homogeneous ordinary differential equation in fuzzy environment based on Laplace transform, *Journal of Fuzzy Set Valued Analysis*, Vol. 2013, 2013, 1–18.

[34] Bansal, A., Some non linear arithmetic operations on triangular fuzzy numbers (m, α, β), *Advances in Fuzzy Mathematics*, Vol. 5, 2010, No. 2, 147–156.

[35] Vahidi, J., S. Rezvani, Arithmetic operations on trapezoidal fuzzy numbers, *Journal Nonlinear Analysis and Application*, Vol. 2013, 2013, 1–8.