

# Non-linear arithmetic operation on generalized triangular intuitionistic fuzzy numbers

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**Abstract:** In this paper we discussed some nonlinear operation on generalized triangular intuitionistic fuzzy number. Some example and an application are given.

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## 1 Introduction

Zadeh [1] and Dubois and Prade [2] were the first who introduced the conception based on fuzzy number and fuzzy arithmetic. One of the generalizations of fuzzy sets theory [1] is considered to be Intuitionistic fuzzy sets (IFS) theory. Out of several higher-order fuzzy sets, IFS were first introduced by Atanassov [3] and have been found to be suitable to deal with unexplored areas. The fuzzy set considers only the degree of belongingness and non-belongingness. Fuzzy set theory does not incorporate the degree of hesitation (i.e., degree of non-determinacy, defined as  $1 - \text{sum of membership function and non-membership function}$ ). To handle such situations, Atanassov [4] explored the concept of fuzzy set theory by intuitionistic fuzzy set (IFS) theory. The degree of acceptance in fuzzy sets is only considered, otherwise IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one [4].

Basic arithmetic operations of TIFNs is defined by Deng-Feng Li in [5] using membership and non-membership values. Basic arithmetic operations of TIFNs such as addition, subtraction and multiplication are defined by Mahapatra and Roy in [6], by considering the six-tuple number itself and division by Nagoorgani and Ponnalagu [7].

Nowadays, IFSs are being studied extensively and being used in different fields of science and technology. Amongst the all research works mainly on IFS we can include Atanassov [4, 8–11], Atanassov and Gargov [12], Szmidt and Kacprzyk [13], Buhaescu [14, 19], Ban [15], Deschrijver and Kerre [16], Stoyanova [17, 22], Cornelis et al. [18], Gerstenkorn and Manko [20], Stoyanova and Atanassov [21], Mahapatra and Roy [23], Hajeer [24], Persona

et al. [25], Prabha et al. [26], Nikolaidis and Mourelatos [27], Kumar et al. [28], Wang [29], Shaw and Roy [30], Adak et al. [31], Varghese and Kuriakose [32].

## 2 Preliminary concepts

**Definition 2.1: Intuitionistic Fuzzy Number:** An IFN  $\tilde{A}^i$  is defined as follows:

- i) an intuitionistic fuzzy subject of real line;
- ii) normal, i.e., there is any  $x_0 \in R$  such that  $\mu_{\tilde{A}^i}(x_0) = 1$  (so  $\vartheta_{\tilde{A}^i}(x_0) = 0$ );
- iii) a convex set for the membership function  $\mu_{\tilde{A}^i}(x)$ , i.e.,  $\mu_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \in R, \lambda \in [0, 1]$ ;
- iv) a concave set for the non-membership function  $\vartheta_{\tilde{A}^i}(x)$ , i.e.,  $\vartheta_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda)x_2) \geq \max(\vartheta_{\tilde{A}^i}(x_1), \vartheta_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \in R, \lambda \in [0, 1]$ .

**Definition 2.2: Triangular Intuitionistic Fuzzy number:** A TIFN  $\tilde{A}^i$  is a subset of IFN in  $R$  with following membership function and non membership function as follows:

$$\mu_{\tilde{A}^i}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2, \\ \frac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\vartheta_{\tilde{A}^i}(x_1) = \begin{cases} \frac{a_2 - x}{a_2 - a_1} & \text{for } a'_1 \leq x \leq a_2 \\ \frac{x - a_2}{a'_3 - a_2} & \text{for } a_2 \leq x \leq a'_3 \\ 1 & \text{otherwise} \end{cases}$$

where  $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$  and TIFN is denoted by  $\tilde{A}^i_{TIFN} = (a_1, a_2, a_3; a'_1, a_2, a'_3)$

**Definition 2.3: Generalized Intuitionistic Fuzzy Number:** An IFN  $\tilde{A}^i$  is defined as follows:

- i) an intuitionistic fuzzy subject of real line;
- ii) normal, i.e., there is any  $x_0 \in R$  such that  $\mu_{\tilde{A}^i}(x_0) = \omega$  (so  $\vartheta_{\tilde{A}^i}(x_0) = 0$ ) for  $0 < \omega \leq 1$ ;
- iii) a convex set for the membership function  $\mu_{\tilde{A}^i}(x)$ , i.e.,  $\mu_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \in R, \lambda \in [0, \omega]$ ;
- iv) a concave set for the non-membership function  $\vartheta_{\tilde{A}^i}(x)$ , i.e.,  $\vartheta_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda)x_2) \geq \max(\vartheta_{\tilde{A}^i}(x_1), \vartheta_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \in R, \lambda \in [0, \omega]$ ;
- v)  $\mu_{\tilde{A}^i}$  is continuous mapping from  $R$  to the closed interval  $[0, \omega]$  and  $x_0 \in R$ , the relation  $0 \leq \mu_{\tilde{A}^i}(x_0) + \vartheta_{\tilde{A}^i}(x_0) \leq \omega$  holds.

**Definition 2.4: Generalized Triangular Intuitionistic Fuzzy number:** A TIFN  $\tilde{A}^i$  is a subset of IFN in  $R$  with following membership function and non-membership function as follows:

$$\mu_{\tilde{A}^i}(x) = \begin{cases} \omega \frac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2, \\ \omega & \text{for } x = a_2 \\ \omega \frac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\vartheta_{\tilde{A}^i}(x) = \begin{cases} \omega \frac{a_2 - x}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2, \\ 0 & \text{for } x = a_2 \\ \omega \frac{x - a_2}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\ \omega & \text{otherwise} \end{cases}$$

where  $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$  and GTIFN is denoted by

$$\tilde{A}^i_{GTIFN} = \left( (a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega \right).$$

**Definition 2.5:** A GTIFN  $\tilde{A}^i_{GTIFN} = \left( (a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega \right)$  is said to be non-negative iff  $a'_1 \geq 0$ .

**Definition 2.6:** Two GTIFN  $\tilde{A}^i_{GTIFN} = \left( (a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega_1 \right)$  and  $\tilde{B}^i_{GTIFN} = \left( (b_1, b_2, b_3), (b'_1, b_2, b'_3); \omega_2 \right)$  are said to be equal iff  $a_1 = b_1, a_2 = b_2, a_3 = b_3, a'_1 = b'_1, a'_3 = b'_3$  and  $\omega_1 = \omega_2$ .

**Definition 2.7:  $\alpha$ -cut set:** A  $\alpha$ -cut set of  $\tilde{A}^i_{GTIFN} = \left( (a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega \right)$  is a crisp subset of  $R$  which is defined as follows

$$A_\alpha = \{x: \mu_{\tilde{A}^i}(x) \geq \alpha\} = [A_1(\alpha), A_2(\alpha)] = [a_1 + \frac{\alpha}{\omega}(a_2 - a_1), a_3 - \frac{\alpha}{\omega}(a_3 - a_2)].$$

**Definition 2.8:  $\beta$ -cut set:** A  $\beta$ -cut set of  $\tilde{A}^i_{GTIFN} = \left( (a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega \right)$  is a crisp subset of  $R$  which is defined as follows

$$A_\beta = \{x: \vartheta_{\tilde{A}^i}(x) \leq \beta\} = [A'_1(\beta), A'_2(\beta)] = [a_2 - \frac{\beta}{\omega}(a_2 - a'_1), a_2 + \frac{\beta}{\omega}(a'_3 - a_2)].$$

**Definition 2.9:  $(\alpha, \beta)$ -cut set:** A  $(\alpha, \beta)$ -cut set of  $\tilde{A}^i_{GTIFN} = \left( (a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega \right)$  is a crisp subset of  $R$  which is defined as follows

$$A_{\alpha, \beta} = \{[A_1(\alpha), A_2(\alpha)]; [A'_1(\beta), A'_2(\beta)]\}, \alpha + \beta \leq \omega, \alpha, \beta \in [0, \omega].$$

**Definition 2.10: Addition of two GTIFN:** Let  $\tilde{A}^i_{GTIFN} = \left( (a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega_1 \right)$  and  $\tilde{B}^i_{GTIFN} = \left( (b_1, b_2, b_3), (b'_1, b_2, b'_3); \omega_2 \right)$  be two GTIFN, then the addition of two GTIFN is given by

$$\tilde{A}^i_{GTIFN} \oplus \tilde{B}^i_{GTIFN} = \left( (a_1 + b_1, a_2 + b_2, a_3 + b_3), (a'_1 + b'_1, a_2 + b_2, a'_3 + b'_3); \omega \right),$$

where  $0 < \omega \leq 1, \omega = \min(\omega_1, \omega_2)$ .

**Definition 2.11: Subtraction of two GTIFN:** Let  $\tilde{A}^i_{GTIFN} = ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega_1)$  and  $\tilde{B}^i_{GTIFN} = ((b_1, b_2, b_3), (b'_1, b_2, b'_3); \omega_2)$  be two GTIFN, then the subtraction of two GTIFN is given by

$$\tilde{A}^i_{GTIFN} \ominus \tilde{B}^i_{GTIFN} = ((a_1 - b_3, a_2 - b_2, a_3 - b_1), (a'_1 - b'_3, a_2 - b_2, a'_3 - b'_1); \omega),$$

where  $0 < \omega \leq 1, \omega = \min(\omega_1, \omega_2)$ .

**Definition 2.12: Multiplication by a scalar:** Let  $\tilde{A}^i_{GTIFN} = ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega)$  and  $k$  is a scalar then  $k\tilde{A}^i_{GTIFN}$  is also a GTIFN and is defined as

$$k\tilde{A}^i_{GTIFN} = \begin{cases} ((ka_1, ka_2, ka_3), (ka'_1, ka_2, ka'_3); \omega), & \text{if } k > 0 \\ ((ka_3, ka_2, ka_1), (ka'_3, ka_2, ka'_1); \omega), & \text{if } k < 0 \end{cases}$$

where  $0 < \omega \leq 1$ .

**Definition 2.13: Multiplication of two GTIFN:** Let

$\tilde{A}^i_{GTIFN} = ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega_1)$  and  $\tilde{B}^i_{GTIFN} = ((b_1, b_2, b_3), (b'_1, b_2, b'_3); \omega_2)$  be two GTIFN, then the subtraction of two GTIFN is given by

$$\tilde{A}^i_{GTIFN} \otimes \tilde{B}^i_{GTIFN} = ((a_1b_1, a_2b_2, a_3b_3), (a'_1b'_1, a_2b_2, a'_3b'_3); \omega),$$

where  $0 < \omega \leq 1, \omega = \min(\omega_1, \omega_2)$ .

**Definition 2.14: Division of two GTIFN:** Let  $\tilde{A}^i_{GTIFN} = ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega_1)$  and  $\tilde{B}^i_{GTIFN} = ((b_1, b_2, b_3), (b'_1, b_2, b'_3); \omega_2)$  be two GTIFN, then the division of two GTIFN is given by

$$\tilde{A}^i_{GTIFN} \div \tilde{B}^i_{GTIFN} = \left( \left( \frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1} \right), \left( \frac{a'_1}{b'_3}, \frac{a_2}{b_2}, \frac{a'_3}{b'_1} \right); \omega \right),$$

where  $0 < \omega \leq 1, \omega = \min(\omega_1, \omega_2)$ .

**Definition 2.15: Inverse of a GTIFN:** Let  $\tilde{A}^i_{GTIFN} = ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega)$  be a GTIFN, then its inverse is given by

$$\frac{1}{\tilde{A}^i_{GTIFN}} = \left( \left( \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1} \right), \left( \frac{1}{a'_3}, \frac{1}{a_2}, \frac{1}{a_1} \right); \omega \right)$$

### 3 Non-linear operation of GTIFN

#### 3.1 Modulus of a GTIFN

Let  $\tilde{A}^i_{GTIFN} = ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega)$  be a GTIFN, then its modulus is given by

$$|\tilde{A}^i_{GTIFN}| = \left| ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega) \right| = \begin{cases} ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega) & \tilde{A}^i_{GTIFN} \geq 0 \\ ((-a_3, -a_2, -a_1), (a'_1, a_2, a'_3); \omega) & \tilde{A}^i_{GTIFN} < 0 \end{cases}$$

#### 3.2 Square root of a GTIFN

Square root of a GTIFN  $\tilde{A}^i_{GTIFN} = ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega) > 0$  is obtained as follows

$$\sqrt{\tilde{A}^i_{GTIFN}} = \sqrt{((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega)} = ((d, e, f), (l, m, n); \omega)$$

$$\text{Or, } \left( ((d, e, f), (l, m, n); \omega) \right)^2 = ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega)$$

Now applying the multiplication rule we get

$$\sqrt{((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega)} = \left( (\sqrt{a_1}, \sqrt{a_2}, \sqrt{a_3}), (\sqrt{a'_1}, \sqrt{a_2}, \sqrt{a'_3}); \omega \right)$$

#### 3.3 A general recursive formula for $\left( ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega) \right)^n$

Using the multiplication of two positive generalized Intuitionistic fuzzy number, we have

$$\left( ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega) \right)^n = \left( ((a_1)^n, (a_2)^n, (a_3)^n), ((a'_1)^n, (a_2)^n, (a'_3)^n); \omega \right)$$

Now we find a general recursive formulae for  $(-\tilde{A}^i_{GTIFN})^n$  :

$$\begin{aligned} & \left( -((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega) \right)^n \\ &= \begin{cases} \left( ((a_3)^n, (a_2)^n, (a_1)^n), ((a'_3)^n, (a_2)^n, (a'_1)^n); \omega \right), & n \text{ is even} \\ \left( (-(a_3)^n, -(a_2)^n, -(a_1)^n), (-(a'_3)^n, -(a_2)^n, -(a'_1)^n); \omega \right), & n \text{ is odd} \end{cases} \end{aligned}$$

#### 3.4 Exponential of a non-negative GTIFN

We use the Taylor series expansion method.

We know that  $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, -\infty < x < \infty$

For  $x = \tilde{A}^i_{GTIFN} = ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega)$  we have

$$e^{\tilde{A}^i_{GTIFN}} = 1 + \frac{\tilde{A}^i_{GTIFN}}{1!} + \frac{\tilde{A}^i_{GTIFN}^2}{2!} + \frac{\tilde{A}^i_{GTIFN}^3}{3!} + \dots, x \geq 0$$

Now,  $\left( ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega) \right)^n = \left( ((a_1)^n, (a_2)^n, (a_3)^n), ((a'_1)^n, (a_2)^n, (a'_3)^n); \omega \right)$

Therefore,  $e^{\tilde{A}^i_{GTIFN}} = ((1,0,0), (0,0,0); \omega) + \sum_{i=1}^{\infty} \frac{\left( ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega) \right)^i}{i!}$

i.e.,  $e^{\tilde{A}^i_{GTIFN}} = \left( \left( \left( 1 + \frac{a_1}{1!} + \frac{a_1^2}{2!} + \dots \right), \left( \frac{a_2}{1!} + \frac{a_2^2}{2!} + \dots \right), \left( \frac{a_3}{1!} + \frac{a_3^2}{2!} + \dots \right) \right), \left( \left( \frac{a'_1}{1!} + \frac{a_1'^2}{2!} + \dots \right), \left( \frac{a_2}{1!} + \frac{a_2^2}{2!} + \dots \right), \left( \frac{a'_3}{1!} + \frac{a_3'^2}{2!} + \dots \right) \right); \omega \right)$

Or,  $e^{\tilde{A}^i_{GTIFN}} = \left( (e^{a_1}, e^{a_2} - 1, e^{a_3} - 1), (e^{a'_1} - 1, e^{a_2} - 1, e^{a'_3} - 1); \omega \right)$

### 3.5 Inverse Exponential of a non-negative GTIFN

$$e^{-\tilde{A}^i_{GTIFN}} = \frac{1}{e^{\tilde{A}^i_{GTIFN}}} = \left( \left( \frac{1}{e^{a_1}}, \frac{1}{e^{a_2-1}}, \frac{1}{e^{a_3-1}} \right), \left( \frac{1}{e^{a'_1-1}}, \frac{1}{e^{a_2-1}}, \frac{1}{e^{a'_3-1}} \right); \omega \right)$$

**Corollary:**  $e^{\tilde{A}^i_{GTIFN}} \cdot e^{\tilde{B}^i_{GTIFN}} = e^{\tilde{A}^i_{GTIFN} + \tilde{B}^i_{GTIFN}}$  if  $\tilde{A}^i_{GTIFN}, \tilde{B}^i_{GTIFN} \geq 0$

**Corollary:**  $(e^{\tilde{A}^i_{GTIFN}})^a = e^{a\tilde{A}^i_{GTIFN}}$  if  $\tilde{A}^i_{GTIFN} \geq 0$  and  $a \in R^+$

**Corollary:**  $\frac{e^{\tilde{A}^i_{GTIFN}}}{e^{\tilde{B}^i_{GTIFN}}} = e^{\tilde{A}^i_{GTIFN} - \tilde{B}^i_{GTIFN}}$  if  $\tilde{A}^i_{GTIFN}, \tilde{B}^i_{GTIFN} \geq 0$ .

### 3.6 Logarithm of a non-negative GTIFN

Let  $\log_e \left( ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega) \right) = ((x_1, x_2, x_3), (x'_1, x_2, x'_3); \omega)$

Therefore,  $\left( (a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega \right) = e^{((x_1, x_2, x_3), (x'_1, x_2, x'_3); \omega)} = \left( (e^{x_1}, e^{x_2} - 1, e^{x_3} - 1), (e^{x'_1} - 1, e^{x_2} - 1, e^{x'_3} - 1); \omega \right)$

$$e^{x_1} = a_1 \text{ or } x_1 = \log_e a_1$$

$$e^{x_2} - 1 = a_2 \text{ or } x_2 = \log_e(1 + a_2)$$

$$e^{x_3} - 1 = a_3 \text{ or } x_3 = \log_e(1 + a_3)$$

$$e^{x'_1} - 1 = a'_1 \text{ or } x'_1 = \log_e(1 + a'_1)$$

$$e^{x'_3} - 1 = a'_3 \text{ or } x'_3 = \log_e(1 + a'_3)$$

Hence,  $\log_e \left( ((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega) \right) = \left( (\log_e a_1, \log_e(1 + a_2), \log_e(1 + a_3)), (\log_e(1 + a'_1), \log_e(1 + a_2), \log_e(1 + a'_3)); \omega \right)$

**Corollary:**  $\log_e \tilde{A}^i_{GTIFN} + \log_e \tilde{B}^i_{GTIFN} = \log_e (\tilde{A}^i_{GTIFN} \tilde{B}^i_{GTIFN})$  if  $\tilde{A}^i_{GTIFN}, \tilde{B}^i_{GTIFN} > 0$

**Corollary:**  $\log_e \tilde{A}^i_{GTIFN} - \log_e \tilde{B}^i_{GTIFN} = \log_e \left( \frac{\tilde{A}^i_{GTIFN}}{\tilde{B}^i_{GTIFN}} \right)$  if  $\tilde{A}^i_{GTIFN} \geq \tilde{B}^i_{GTIFN} > 0$

**Corollary:**  $\log_e (\tilde{A}^i_{GTIFN})^a = a \log_e \tilde{A}^i_{GTIFN}$  if  $\tilde{A}^i_{GTIFN} > 0, a \in I^+$

### 3.7 Positive solution of $\left(\left((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega\right)\right)^{\frac{1}{n}}$

By using multiplication rule we also find the  $n^{th}$  ( $n > 0$ ) positive root of a fuzzy number as

$$\left(\left((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega\right)\right)^{\frac{1}{n}} \simeq \left(\left((a_1)^{\frac{1}{n}}, (a_2)^{\frac{1}{n}}, (a_3)^{\frac{1}{n}}\right), \left((a'_1)^{\frac{1}{n}}, (a_2)^{\frac{1}{n}}, (a'_3)^{\frac{1}{n}}\right); \omega\right).$$

### 3.8 $\tilde{A}^i_{GTIFN} \tilde{B}^i_{GTIFN}$ if $\tilde{A}^i_{GTIFN} > \mathbf{0}$ , $\tilde{B}^i_{GTIFN} \geq \mathbf{0}$

Let  $\tilde{A}^i_{GTIFN} = \left((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega\right)$  and  $\tilde{B}^i_{GTIFN} = \left((b_1, b_2, b_3), (b'_1, b_2, b'_3); \omega\right)$

$$\begin{aligned} \text{Therefore, } & \left((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega\right)^{\left((b_1, b_2, b_3), (b'_1, b_2, b'_3); \omega\right)} \\ &= e^{\left((b_1, b_2, b_3), (b'_1, b_2, b'_3); \omega\right) \ln \left((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega\right)} \\ &= e^{\left((b_1, b_2, b_3), (b'_1, b_2, b'_3); \omega\right) \left(\ln a_1, \ln(1+a_2), \ln(1+a_3), (\ln(1+a'_1), \ln(1+a_2), \ln(1+a'_3)); \omega\right)} \\ &= e^{\left((b_1 \ln a_1, b_2 \ln(1+a_2), b_3 \ln(1+a_3)), (b'_1 \ln(1+a'_1), b_2 \ln(1+a_2), b'_3 \ln(1+a'_3)); \omega\right)} \\ &= \left(\left(e^{b_1 \ln a_1}, e^{b_2 \ln(1+a_2)} - 1, e^{b_3 \ln(1+a_3)} - 1\right), \left(e^{b'_1 \ln(1+a'_1)} - 1, e^{b_2 \ln(1+a_2)} - 1, e^{b'_3 \ln(1+a'_3)} - 1\right); \omega\right) \\ &= \left(\left(a_1^{b_1}, (1+a_2)^{b_2} - 1, (1+a_3)^{b_3} - 1\right), \left((1+a'_1)^{b'_1} - 1, (1+a_2)^{b_2}, (1+a'_3)^{b'_3} - 1\right); \omega\right). \end{aligned}$$

### 3.9 $a^{\tilde{A}^i_{GTIFN}}$ where $a \geq 1$ and $\tilde{A}^i_{GTIFN} \geq \mathbf{0}$

Let  $\tilde{A}^i_{GTIFN} = \left((a_1, a_2, a_3), (a'_1, a_2, a'_3); \omega\right)$

$$\begin{aligned} a^{\tilde{A}^i_{GTIFN}} &= e^{\tilde{A}^i_{GTIFN} \ln a} = \exp \left(\left((a_1 \ln a, a_2 \ln a, a_3 \ln a), (a'_1 \ln a, a_2 \ln a, a'_3 \ln a); \omega\right)\right) \\ &= \left(\left(a^{a_1}, a^{a_2} - 1, a^{a_3} - 1\right), \left(a^{a'_1} - 1, a^{a_2} - 1, a^{a'_3} - 1\right); \omega\right). \end{aligned}$$

## 4 Numerical example

### Example 4.1

Find the value of  $\sqrt{\tilde{x} + \sqrt{\tilde{x} + \sqrt{\tilde{x} + \dots}}}$  where  $\tilde{x} = ((3,4,6), (2,4,7); 0.7)$ .

**Solution:** Let the value of the above is  $\tilde{p} = ((a, b, c), (r, b, t); 0.7)$ .

Therefore  $\sqrt{\tilde{x} + \tilde{p}} = \tilde{p}$ .

Or,  $\sqrt{((3+a, 4+b, 6+c), (2+r, 4+b, 7+t); 0.7)} = ((a, b, c), (r, b, t); 0.7)$ .

This implies,

$$\sqrt{3+a} = a \text{ or, } a^2 - a - 3 = 0 \text{ or, } a = 2.30$$

$$\sqrt{4+b} = b \text{ or, } b^2 - b - 4 = 0 \text{ or, } b = 2.56$$

$$\sqrt{6+c} = c \text{ or, } c^2 - c - 6 = 0 \text{ or, } c = 3$$

$$\sqrt{2+r} = r \text{ or, } r^2 - r - 2 = 0 \text{ or, } r = 2$$

$$\sqrt{7+t} = t \text{ or, } t^2 - t - 7 = 0 \text{ or, } t = 3.14$$

Hence the value of the above is  $((2.30, 2.56, 3), (2, 2.56, 3.14); 0.7)$ .

### Example 4.2

Find all the positive solution of  $e^{\tilde{x}} = \sqrt{((3.24, 4, 4.84), (2.56, 4, 5.76); 0.8)}$ .

**Solution:**

$$e^{\tilde{x}} = \sqrt{((3.24, 4, 4.84), (2.56, 4, 5.76); 0.8)} = ((\sqrt{3.24}, \sqrt{4}, \sqrt{4.84}), (\sqrt{2.56}, \sqrt{4}, \sqrt{5.76}); 0.8)$$

$$\text{Or, } e^{\tilde{x}} = ((1.8, 2, 2.2), (1.6, 2, 2.4); 0.8).$$

$$\text{Or, } \tilde{x} = ((\ln 1.8, \ln 3, \ln 3.2), (\ln 2.6, \ln 3, \ln 3.4); 0.8).$$

$$\text{Or, } \tilde{x} = ((0.58, 1.09, 1.16), (0.95, 1.09, 1.22); 0.8).$$

### Example 4.3

Find all the solutions of the equation  $\sqrt{|\tilde{x}|} = ((4, 6, 7), (3, 6, 8); 0.6)$ .

$$\text{Solution: } |\tilde{x}| = ((4, 6, 7), (3, 6, 8); 0.6)^2 = ((16, 36, 49), (9, 36, 64); 0.6)$$

The equation has two solutions as

$$((16, 36, 49), (9, 36, 64); 0.6)$$

and

$$((-16, -36, -49), (9, 36, 64); 0.6).$$

### Example 4.4

Evaluate the fuzzy solution of  $((2.197, 3.375, 4.913), (1.728, 3.375, 5.832); 0.9)^{\frac{1}{3}}$ .

**Solution:** The positive fuzzy solution is  $((1.3, 1.5, 1.7), (1.2, 1.5, 1.8); 0.9)$ .

### Example 4.5

Compute the value of  $((3, 4, 5), (2, 4, 6); 0.8)^{((4, 5, 7), (3, 5, 8); 0.8)}$ .

**Solution:** The value of above is given by  $((81, 3124, 279935), (26, 3124, 5764800); 0.8)$ .



## 5 Application

### Bank Account Problem, [33]

The Balance  $B(t)$  of a bank account grows under continuous process given by  $\frac{dB}{dt} = rB$ , where  $r$  the constant of proportionality is the annual interest rate. If there are initially  $B(t) = B_0$  balance, solve the above problem in fuzzy environment when

$$\widetilde{B}_0 = ((\$850, \$1000, \$1100), (\$800, \$1000, \$1200); 0.7)$$

and

$$\tilde{r} = ((3.7, 4, 4.5), (3.5, 4, 5); 0.7)\%.$$

Find the solution after  $t = 3$  years.

**Solution:** The solution is given by  $B(t = 3) = \widetilde{B}_0 e^{\frac{3}{100}\tilde{r}}$

$$\begin{aligned} &= ((\$850, \$1000, \$1100), (\$800, \$1000, \$1200); 0.7)e^{((.111, .12, .135), (.105, .12, .15); 0.7)} \\ &= ((\$850, \$1000, \$1100), (\$800, \$1000, \$1200); 0.7) \\ &= ((1.1174, 0.1275, 0.1445), (0.1107, 0.1275, 0.1618); 0.7) \\ &= ((\$949.79, \$127.50, \$158.95), (\$88.56, \$127.50, \$194.16); 0.7) \end{aligned}$$

## 6 Conclusion

In this paper, we discussed some nonlinear operation (such as logarithm, exponential) on generalized triangular intuitionistic fuzzy number. Some example and an application are given. A imprecise bank account problem are given in generalized triangular intuitionistic fuzzy environment.

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