NIFS 15 (2009), 3, 20-24

REMARK ON OPERATIONS "SUBTRACTION" OVER INTUITIONISTIC FUZZY SETS

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1 On intuitionistic fuzzy versions of operation "negation"

During the last four years 36 different versions of operation "negation" were introduced over the Intuitionistic Fuzzy Sets (IFS, see [1]). First, we will give the forms of these versions of operation "negation".

In some of these definitions we shall use functions sg and $\overline{\text{sg}}$:

$$\text{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}, \qquad \overline{\text{sg}}(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$$

The negations have the following forms (see, e.g., [3]):

$$\neg_1 A = \{\langle \nu_A(x), \mu_A(x)\rangle | x \in E\},$$

$$\neg_2 A = \{\langle \overline{\text{sg}}(\mu_A(x)), \text{sg}(\mu_A(x))\rangle | x \in E\},$$

$$\neg_3 A = \{\langle \nu_A(x), \mu_A(x).\nu_A(x) + \mu_A(x)^2\rangle | x \in E\},$$

$$\neg_4 A = \{\langle \nu_A(x), 1 - \nu_A(x)\rangle | x \in E\},$$

$$\neg_5 A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(1 - \nu_A(x)))\rangle | x \in E\},$$

$$\neg_6 A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \text{sg}(\mu_A(x)))\rangle | x \in E\},$$

$$\neg_7 A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \mu_A(x)\rangle\rangle | x \in E\},$$

$$\neg_8 A = \{\langle 1 - \mu_A(x), \mu_A(x)\rangle | x \in E\},$$

$$\neg_9 A = \{\langle \overline{\text{sg}}(\mu_A(x)), \mu_A(x)\rangle | x \in E\},$$

$$\neg_{10} A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), 1 - \nu_A(x)\rangle\rangle | x \in E\},$$

$$\neg_{11} A = \{\langle \text{sg}(\nu_A(x)), \overline{\text{sg}}(\nu_A(x)))\rangle | x \in E\},$$

$$\neg_{12} A = \{\langle \nu_A(x).(\mu_A(x) + \nu_A(x)), \mu_A(x).(\mu_A(x) + \nu_A(x)^2)\rangle | x \in E\},$$

$$\neg_{13} A = \{\langle \text{sg}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \mu_A(x)))\rangle | x \in E\},$$

$$\neg_{14} A = \{\langle \text{sg}(\nu_A(x)), \overline{\text{sg}}(1 - \mu_A(x)))\rangle | x \in E\},$$

$$\neg_{15} A = \{\langle \overline{\text{sg}}(1 - \nu_A(x)), \overline{\text{sg}}(1 - \mu_A(x)))\rangle | x \in E\},$$

$$\neg_{16}A = \{\langle \overline{\text{sg}}(\mu_A(x)), \overline{\text{sg}}(1-\mu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{17}A = \{\langle \overline{\text{sg}}(1-\nu_A(x)), \overline{\text{sg}}(\nu_A(x))\rangle\rangle | x \in E\},$$

$$\neg_{18}A = \{\langle x, \nu_A(x).\text{sg}(\mu_A(x)), \mu_A(x).\text{sg}(\nu_A(x))\rangle | x \in E\},$$

$$\neg_{19}A = \{\langle x, \nu_A(x).\text{sg}(\mu_A(x)), 0\rangle | x \in E\},$$

$$\neg_{20}A = \{\langle x, \nu_A(x), 0\rangle | x \in E\},$$

$$\neg_{21}A = \{\langle x, \nu_A(x), \mu_A(x).\nu_A(x) + \mu_A(x)^n\rangle | x \in E\},$$

where real number $n \in [2, \infty)$,

$$\neg_{22}A = \{\langle x, \nu_A(x), \mu_A(x).\nu_A(x) + \overline{\text{sg}}(1-\mu_A(x))\rangle | x \in E\},$$

$$\neg_{23}A = \{\langle x, (1-\mu_A(x)).\text{sg}(\mu_A(x)), \mu_A(x).\text{sg}(1-\nu_A(x))\rangle | x \in E\},$$

$$\neg_{24}A = \{\langle x, (1-\mu_A(x)).\text{sg}(\mu_A(x)), 0\rangle | x \in E\},$$

$$\neg_{25}A = \{\langle x, 1-\nu_A(x), 0\rangle | x \in E\},$$

$$\neg_{26}A = \{\langle x, \nu_A(x), \mu_A(x).\nu_A(x) + \overline{\text{sg}}(1-\mu_A(x))\rangle | x \in E\},$$

$$\neg_{27}A = \{\langle x, 1-\mu_A(x), \mu_A(x).(1-\mu_A(x)) + \overline{\text{sg}}(1-\mu_A(x))\rangle | x \in E\},$$

$$\neg_{28}A = \{\langle x, \nu_A(x), (1-\nu_A(x)).\nu_A(x) + \overline{\text{sg}}(\nu_A(x))\rangle | x \in E\},$$

$$\neg_{29}A = \{\langle x, \max(0, \nu_A(x).\mu_A(x) + \overline{\text{sg}}(1-\nu_A(x))),$$
$$\min(1, \mu_A(x).(\nu_A(x).\mu_A(x) + \overline{\text{sg}}(1-\nu_A(x))) + \overline{\text{sg}}(1-\mu_A(x)))\rangle | x \in E\},$$

$$\neg_{30}A = \{\langle x, \nu_A(x).\mu_A(x),$$
$$\mu_A(x).(\nu_A(x).\mu_A(x) + \overline{\text{sg}}(1-\nu_A(x))) + \overline{\text{sg}}(1-\mu_A(x))\rangle | x \in E\},$$

$$\neg_{31}A = \{\langle x, \max(0, (1-\mu_A(x)).\mu_A(x) + \overline{\text{sg}}(\mu_A(x))),$$
$$(\min(1, \mu_A(x).((1-\mu_A(x)).\mu_A(x) + \overline{\text{sg}}(\mu_A(x))) + \overline{\text{sg}}(1-\mu_A(x)))\rangle | x \in E\},$$

$$\neg_{32}A = \{\langle x, (1-\mu_A(x)).\mu_A(x),$$
$$\mu_A(x).((1-\mu_A(x)).\mu_A(x) + \overline{\text{sg}}(\mu_A(x))) + \overline{\text{sg}}(1-\mu_A(x))\rangle | x \in E\},$$

$$\neg_{33}A = \{\langle x, \max(0, ((\nu_A(x).(1-\nu_A(x))) + \overline{\text{sg}}(1-\nu_A(x)))),$$
$$(\min(1, (((1-\nu_A(x)).((\nu_A(x).(1-\nu_A(x))) + \overline{\text{sg}}(1-\nu_A(x)))) + \overline{\text{sg}}(\nu_A(x)))))\rangle | x \in E\},$$

$$\neg_{34}A = \{\langle x, \nu_A(x).(1-\nu_A(x)),$$
$$(1-\nu_A(x)).(\nu_A(x).(1-\nu_A(x)) + \overline{\text{sg}}(1-\nu_A(x))) + \overline{\text{sg}}(\nu_A(x))\rangle | x \in E\},$$

$$\neg^{\varepsilon}A = \{\langle x, \min(1, \nu_A(x) + \varepsilon), \max(0, \mu_A(x) - \varepsilon)\rangle | x \in E\},$$

where $\varepsilon \in [0, 1]$,

$$\neg^{\varepsilon,\eta}A = \{\langle x, \min(1, \nu_A(x) + \varepsilon), \max(0, \mu_A(x) - \eta)\rangle | x \in E\},$$

where $0 \le \varepsilon \le \eta \le 1$.

2 On intuitionistic fuzzy versions of operation "subtraction"

In [4, 5] two versions of operation "subtraction" were defined.

Here, new versions of operation "subtraction" will be introduced for a first time. As a basis, we will use the well-known formula from set theory:

$$A - B = A \cap \neg B,$$

where A and B are given sets. In the IFS-case, if the sets

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\}$$

are given (for the description of their components see [1]) we can define the following versions of operation "subtraction":

$$A -'_i B = A \cap \neg_i B, \tag{1}$$

where $i = 1, 2, ..., 34$.

We see immediately that operation $-_1$ coincides with operation from [5], while the one from [4] does not have an analogue among the 34 operations.

On the other hand, as we discussed in [2], the Law for Excluding Middle is not always valid in IFS theory. By this reason we can introduce a new series of "subtraction" operations, that will have the form:

$$A -''_i B = \neg\neg A \cap \neg_i B, \tag{2}$$

where $i = 1, 2, ..., 34$.

Of course, for every two IFSs A and B it will be valid that:

$$A -'_1 B = A -''_1 B,$$

because the first negation will satisfy the Law for Excluding Middle, but in the rest cases this equality will not be valid.

Below we will make the first step of the research on the new IF-operations, starting with the first two of them: $-'_2$ and $-''_2$.

3 Basic properties of operation $-'_2$

Using (1), we obrain the following form of the operation $-'_2$:

$$A -'_i B = \{\langle x, \min(\mu_A(x), \overline{\text{sg}}(\mu_B(x))), \max(\nu_A(x), \text{sg}(\mu_B(x)))\rangle | x \in E\}.$$

First, we must check that in a result of the operation we obtain an IFS. Really, for two given IFSs A and B and for each $x \in E$ we obtain that if $\mu_B(x) = 0$, then

$$\min(\mu_A(x), \overline{\text{sg}}(\mu_B(x))) + \max(\nu_A(x), \text{sg}(\mu_B(x)))$$

$$= \min(\mu_A(x), 1) + \max(\nu_A(x), 0) = \mu_A(x) + \nu_A(x) \leq 1;$$

if $\mu_B(x) = 1$, then

$$\min(\mu_A(x), \overline{\text{sg}}(\mu_B(x))) + \max(\nu_A(x), \text{sg}(\mu_B(x)))$$

$$= \min(\mu_A(x), 0) + \max(\nu_A(x), 1) = 0 + 1 = 1.$$

Let us define the *empty IFS,* the *totally uncertain IFS,* and the *unit IFS* (see [1]) by:

$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$

$$U^* = \{\langle x, 0, 0\rangle | x \in E\},$$

$$E^* = \{\langle x, 1, 0\rangle | x \in E\}.$$

By analogy, we can prove the following assertions.

Theorem 1: For every two IFSs A and B:

(a) $A -_2' E^* = O^*$,

(b) $A -_2' O^* = A$,

(c) $E^* -_2' A = \neg_2 A$,

(d) $O^* -_2' A = O^*$,

(e) $(A -_2' B) \cap C = (A \cap C) -_2' B = A \cap (C -_2' B)$,

(f) $(A \cap B) -_2' C = (A -_2' C) \cap (B -_2' C)$,

(g) $(A \cup B) -_2' C = (A -_2' C) \cup (B -_2' C)$,

(h) $(A -_2' B) -_2' C = (A -_2' C) -_2' B$.

Proof. (c)

$$E^* -_2' A = \{\langle x, \min(1, \overline{\text{sg}}(\mu_A(x))), \max(0, \text{sg}(\mu_A(x)))\rangle | x \in E\}$$

$$= \{\langle \overline{\text{sg}}(\mu_A(x)), \text{sg}(\mu_A(x))\rangle | x \in E\} = \neg_2 A.$$

(g)

$$(A \cup B) -_2' C = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle | x \in E\} -_2' C$$

$$= \{\langle x, \min(\max(\mu_A(x), \mu_B(x)), \overline{\text{sg}}(\mu_C(x))), \max(\min(\nu_A(x), \nu_B(x)), \text{sg}(\mu_C(x)))\rangle | x \in E\}$$

$$= \{\langle x, \max(\min(\mu_A(x), \overline{\text{sg}}(\mu_C(x))), \min(\mu_B(x), \overline{\text{sg}}(\mu_C(x)))),$$

$$\min(\max(\nu_A(x), \text{sg}(\mu_C(x))), \max(\nu_B(x)), \text{sg}(\mu_B(x)))\rangle | x \in E\}$$

$$= \{\langle x, \min(\mu_A(x), \overline{\text{sg}}(\mu_C(x))), \max(\nu_A(x), \text{sg}(\mu_C(x)))\rangle | x \in E\}$$

$$\cup \{\langle x, \min(\mu_B(x), \overline{\text{sg}}(\mu_C(x))), \max(\nu_B(x)), \text{sg}(\mu_B(x)))\rangle | x \in E\}$$

$$= (A -_2' C) \cup (B -_2' C).$$

Obviously

$$O^* -_2' U^* = O^*,\ O^* -_2' E^* = O^*,\ U^* -_2' O^* = U^*,$$

$$U^* -_2' E^* = O^*,\ E^* -_2' O^* = E^*,\ E^* -_2' U^* = O^*.$$

4 Basic properties of operation $-_2''$

First, we shall note that for each real number x the equalities:

$$\overline{\text{sg}}(\overline{\text{sg}}(x)) = \text{sg}(x) \quad \text{and} \quad \text{sg}(\overline{\text{sg}}(x)) = \overline{\text{sg}}(x)$$

hold. Now, using (2) and having in mind that

$$\neg_2\neg_2 A = \{\langle \text{sg}(\mu_A(x)), \overline{\text{sg}}(\mu_A(x))\rangle | x \in E\},$$

we obrain the following form of the operation $-''_2$:

$$A -''_i B = \{\langle x, \min(\text{sg}(\mu_A(x)), \overline{\text{sg}}(\mu_B(x))), \max(\overline{\text{sg}}(\mu_A(x)), \text{sg}(\mu_B(x)))\rangle | x \in E\}.$$

The check that in a result of the operation we obtain an IFS and the proofs of the next assertions are similar to above ones.

Theorem 3: For every two IFSs A and B:

(a) $A -''_2 E^* = O^*$,
(b) $A -''_2 O^* = \neg_2\neg_2 A$,
(c) $E^* -''_2 A = \neg_2 A$,
(d) $O^* -''_2 A = O^*$,
(e) $(A -''_2 B) \cap C = (A \cap C) -''_2 B = A \cap (C -''_2 B)$,
(f) $(A \cap B) -''_2 C = (A -''_2 C) \cap (B -''_2 C)$,
(g) $(A \cup B) -''_2 C = (A -''_2 C) \cup (B -''_2 C)$,
(h) $(A -''_2 B) -''_2 C = (A -''_2 C) -''_2 B$.

Obviously,

$$O^* -''_2 U^* = O^*,\ O^* -''_2 E^* = O^*,\ U^* -''_2 O^* = U^*,$$
$$U^* -''_2 E^* = O^*,\ E^* -''_2 O^* = E^*,\ E^* -''_2 U^* = O^*.$$

5 Conclusion

In the next author's research the properties of the separate versions of the operations "subtraction" will be discussed by the above manner.

References

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