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# *GENERALIZED CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES*

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***Abstract:*** In this paper we introduce and study the concept of intuitionistic fuzzy g-continuous mappings in intuitionistic fuzzy topological spaces.

***Keywords :*** Intuitionistic fuzzy topology, Intuitionistic fuzzy points, Intuitionistic fuzzy g-closed sets and Intuitionistic fuzzy g-open sets.

***AMS Subject classification : 54A.***

## *1. Introduction*

After the introduction of fuzzy sets by Zadeh [16] in 1965 and fuzzy topology by chang [5] in 1967. Several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets . In the last 20 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [6] introduced the concept of intuitionistic fuzzy topological spaces. Recently many fuzzy topological concepts such as fuzzy compactness [8], fuzzy connectedness [15], fuzzy separation axioms [4], fuzzy metric spaces [14], fuzzy continuity [9] fuzzy multifunctions [11] have been generalized for intuitionistic fuzzy topological spaces. In [13] the authores of this paper extend the concepts of fuzzy g-closed sets due to Thakur and Malviya [12] in intuitionistic fuzzy topological space.In the present paper we introduce and study the concepts of intuitionistic fuzzy g-continuous mappings in intuitionistic fuzzy topological space.

## *2.Preliminaries*

***Definition 2.1***: [1] Let X be a nonempty fixed set. An intuitionistic fuzzy set A in X is an object having the form

$$A=\{<x,\mu_A(x), \gamma_A(x)> : x\in X \}$$

where the functions $\mu_A$:X→$I$ and $\gamma_A$: X→$I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element x∈X to the set A, respectively, and $0 \leq \mu_A(x)+ \gamma_A(x) \leq 1$ for each x∈X.

***Definition 2.2***: [1] Let $X$ be a nonempty set and the intuitionistic fuzzy sets A and B be in the form $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$, $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X\}$ and let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic fuzzy sets in X. Then

(a) $A \subseteq B$ if $\forall x \in X\, [\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)]$;
(b) $A = B$ if $A \subseteq B$ and $B \subseteq A$;
(c) $A^c = \{<x, \gamma_A(x), \mu_A(x)> : x \in X\}$;
(d) $\cap A_i = \{<x, \wedge\mu_{Ai}(x), \vee\gamma_{Ai}(x)> : x \in X\}$;
(e) $\cup A_i = \{<x, \vee\mu_{Ai}(x), \wedge\gamma_{Ai}(x)> : x \in X\}$;
(f) $\underset{\sim}{0} = \{<x, 0, 1> : x \in X\}$ and $\underset{\sim}{1} = \{<x, 1, 0> : x \in X\}$;

***Definition 2.3*** [6]***:*** Two intuitionistic fuzzy sets A and B of X said to be q-coincident (AqB for short) if and only if there exits an element x∈X such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

***Definition 2.4***[6]***:*** Let $(X, \Im)$ be an intuitionistic fuzzy topological space and $A = <x, \mu_A(x),\gamma_A(x)>$ be an intuitionistic fuzzy set in $X$. Then the fuzzy interior and fuzzy closure of $A$ are defined by

$$cl(A) = \cap\{K : K \text{ is an intuitionistic fuzzy closed set in } X \text{ and } A \subseteq K\},$$
$$int(A) = \cup\{G : G \text{ is an intuitionistic fuzzy open set in } X \text{ and } G \subseteq K\}.$$

***Definition 2.5*** [7 ] ***:*** Let X be a nonempty set and c∈X a fixed element in X. If $\alpha\in(0,1]$ and $\beta\in[0,1)$ are two real numbers such that $\alpha+\beta\leq 1$ then $c(\alpha,\beta) = < x,c_\alpha, c_{1-\beta} >$ is called an intuitionistic fuzzy point in X, where $\alpha$ denotes the degree of membership of $c(\alpha,\beta)$, and $\beta$ denotes the degree of nonmembership of $c(\alpha,\beta)$.

***Definition 2.6*** [6]***:*** An intuitionistic fuzzy topology on a nonempty set X is a family $\Im$ of intuitionistic fuzzy sets in $X$ satisfy the following axioms:

($T_1$) $\underset{\sim}{0}, \underset{\sim}{1} \in \Im$,
($T_2$) $G_1\cap G_2 \in \Im$ for any $G_1, G_2 \in \Im$,
($T_3$) $\cup G_i \in \Im$ for any arbitrary family $\{ G_i : i \in J\}\subseteq \Im$.

In this case the pair $(X, \Im)$ is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in $\Im$ is known as an intuitionistic fuzzy open set in $X$.

***Definition 2.7*** [6]***:*** The complement $A^c$ of an intuitionistic fuzzy open set A in an intuitionistic fuzzy topological space $(X, \Im)$ is called an intuitionistic fuzzy closed set in $X$.

***Definition 2.8***[6]***:*** Let $X$ *and* $Y$ be two nonempty sets and $f : X \to Y$ be a function.Then

(a) If B = $\{<y, \mu_B(y), \gamma_B(y)> : y \in Y\}$ is an intuitionistic fuzzy set in $Y$, then the preimage of B under $f$ denoted by $f^{-1}$(B), is the IFS in $X$ defined by

$f^{-1}(B) = <x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)> : x \in X\}$

(b) If A = $\{<x, \lambda_A(x), \nu_A(x)> : x \in X\}$ is an intuitionistic fuzzy set in $X$, then the image of A under $f$ denoted by $f$(A) is the intuitionistic fuzzy set in $Y$ defined by

$f(A) = \{<y, f(\lambda_A)(y), f_-(\nu_A)(y)> : y \in Y\}$

where $f_-(\nu_A) = 1 - f(1 - \nu_A)$.

***Definition 2.9***[9] ***:*** Let (X,$\Im$) and (Y,$\Phi$) be two intuitionistic fuzzy topological spaces and let $f$: X$\to$Y be a function. Then $f$ is said to be intuitionistic fuzzy continuous if and only if the preimage of each intuitionistic fuzzy open set in Y is an intuitionistic fuzzy open set in X.

***Definition 2.10*** [8] ***:*** A family { $G_i$ : i$\in\wedge$} of intuitionistic fuzzy sets in X is said to be an intuitionistic fuzzy open cover of X if $\cup$\{ $G_i$ : i$\in\wedge$\} = $\tilde{1}$ and a finite subfamily of an intuitionistic fuzzy open cover of X which also an intuitionistic fuzzy open cover of X is called a finite subcover { $G_i$: i$\in\wedge$}.

***Definition 2.11*** [8] ***:***An intuitionistic fuzzy topological space (X,$\Im$) is called fuzzy compact if each intuitionistic fuzzy open cover has a finite subcover.

***Definition 2.12***[13]***:*** An intuitionistic fuzzy set A of a intuitionistic fuzzy topological space (X,$\Im$) is called an intuitionistic fuzzy generalized closed (intuitionistic fuzzy g-closed) if cl(A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.

***Definition 2.13***[13]***:***Complement of an intutionistic fuzzy g-closed set is called intutionistic fuzzy g-open set.

***Remark 2.1*** [13]***:*** Every intuitionistic fuzzy closed set (intutionistic fuzzy open set) is intuitionistic fuzzy g-closed(intutionistic fuzzy open set) but its converse may not be true.

Throughout this paper f : (X, $\Im$) $\to$ (Y, $\sigma$) denotes a mapping from an intuitionistic fuzzy topological space (X, $\Im$) to another intuitionistic topological space (Y, $\sigma$).

## 3.*Intuitionistic Fuzzy g-Continuous Mapping*

***Definition 3.1***: A mapping f : (X, $\Im$) $\to$ (Y, $\sigma$) is said to be intuitionistic fuzzy g-continuous if the inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy g-closed in X.

***Remark 3.1***: Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy g-continuous, but the converse may not be true. For,

***Example 3.1:*** Let X = {a, b}, Y = (x, y} and the intuitionistic fuzzy set U and V are defined as follows:

$U = \langle x, (0.5/a, 0.6/b), (0.4/a, 0.4/b)\rangle$

$V = \langle x, (0.3/a, 0.4/b), (0.6/a, 0.6/b)\rangle$

Let $\Im = \{\underset{\sim}{0}, U, \underset{\sim}{1}\}$ and $\sigma = \{\underset{\sim}{0}, V, \underset{\sim}{1}\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f : (X, \Im) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is intuitionistic fuzzy g-continuous but not intuitionistic fuzzy continuous.

***Theorem 3.1***: A mapping $f : (X, \Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy g-continuous if and only if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic fuzzy g-open in X.

***Proof***: It is obvious because $f^{-1}(U^c) = (f^{-1}(U))^c$ for every intuitionistic fuzzy set U of Y.

***Theorem 3.2***: If $f : (X, \Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy g-continuous then for each intuitionistic fuzzy point $c(\alpha, \beta)$ of X and each fuzzy open set V, $f(c(\alpha, \beta)) \subseteq V$ there exist a intuitionistic fuzzy g-open set U such that $c(\alpha, \beta) \subseteq U$ and $f(U) \subseteq V$.

***Proof:*** Let $c(\alpha, \beta)$ be a intuitionistic fuzzy point of X and V be a intuitionistic fuzzy open set such that $c(\alpha, \beta) \subseteq V$, put $U = f^{-1}(V)$ then by hypothesis U is intuitionistic fuzzy g-open set of X such that $c(\alpha, \beta) \subseteq U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

***Theorem 3.3***: If $f : (X, \Im) \rightarrow (Y, \sigma)$ is fuzzy g-continuous, then for each intuitionistic fuzzy point $c(\alpha, \beta)$ in X and each intuitionistic fuzzy open set V of Y such that $c(\alpha, \beta)_q V$, there exists $c(\alpha, \beta)$ in an intuitionistic g-open set U of X such that $c(\alpha, \beta)_q U$ and $f(U) \subseteq V$.

***Proof***: Let $c(\alpha, \beta)$ be an intuitionistic fuzzy point of X and V be an intuitionistic fuzzy open set of Y such that $f(c(\alpha, \beta))_q V$. Put $U = f^{-1}(V)$. Then by hypothesis U is an intuitionistic fuzzy g-open set of X such that $c(\alpha, \beta)_q U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

***Definition 3..2***: Let $(X, \Im)$ be an intuitionistic fuzzy topological space. The generalized closure of a intuitionistic fuzzy set A of X denoted by gcl(A) is the intersection of all intuitionistic fuzzy g-closed sets of X which contains A.

***Remark: 3. 2***: It is clear that, $A \subseteq gcl(A) \subseteq cl(A)$ for any intuitionistic fuzzy set A of X.

***Theorem 3. 4***: If $f : (X, \Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy g-continuous, then $f(gcl(A) \subseteq cl(f(A))$ for every intuitionistic fuzzy set A of X.

***Proof:*** Let A be an intuitionistic fuzzy set of X. Then cl(f(A)) is an intuitionistic fuzzy closed set of Y. Since f is fuzzy g-continuous $f^{-1}(cl(f(A)))$ is intuitionistic fuzzy g-closed in X. Clearly $A \subseteq f^{-1}(cl(f(A)))$. Therefore
$gcl(A) \subseteq gcl(f^{-1}(cl(f(A))) = f^{-1}(cl(f(A)))$. Hence $f(gcl(A)) \subseteq cl(f(A))$.

***Remark 3. 3***: The converse of Theorem 3. 4 may not be true. For,

***Example 3.2***: Let X = {a, b, c}, Y = {x, y, z} and the intuitionistic fuzzy set U and V are defined as:

U = <x, (1/a, 0/b, 0/c), (0/a, 1/b, 1/c)>

V = <x, (1/x, 0/y, 1/z), (0/x, 1/y, 0/z)>

Let $\Im = \{\underset{\sim}{0}, U, \underset{\sim}{1}\}$ and $\sigma = \{\underset{\sim}{0}, V, \underset{\sim}{1}\}$ be intuitionistic fuzzy topologies on X and Y respectively and $f : (X, \Im) \rightarrow (Y, \sigma)$ be a mapping defined by f(a) = y, f(b) = x, f(c) = z. Then $f(gcl(A)) \subseteq cl(f(A))$ holds for every intuitionistic fuzzy set A of X, but f is not fuzzy g-continuous.

***Definition 3.3***: An intuitionistic fuzzy topological space $(X, \Im)$ is said to be intuitionistic fuzzy $T_{1/2}$ if every intuitionistic fuzzy g-closed set in X is intuitionistic fuzzy closed in X.

***Theorem 3.5***: A mapping f from an intuitionistic fuzzy $T_{1/2}$-space $(X, \Im)$ to an intuitionistic fuzzy topological space $(Y, \sigma)$ is intuitionistic fuzzy continuous if and only if it is intuitionistic fuzzy g-continuous.

***Proof***: Obvious.

***Remark 3.4***: The composition of two intuitionistic fuzzy g-continuous mappings may not be intuitionistic fuzzy g-continuous. For,

***Example 3.3***: Let X = {a, b}, Y = (x, y} and Z = {p, q} and the fuzzy sets U, V, and W are defined as follows:

U = <x, (0.5/a, 0.7/b), (0.5/a, 0.3/b)>

V = <x, (0.3/a, 0.2/b), (0.7/a, 0.8/b)>

W = <x, (0.6/a, 0.4/b), (0.4/a, 0.6/b)>

Let $\tau = \{\underset{\sim}{0}, U, \underset{\sim}{1}\}$, $\sigma = \{0, V, 1\}$, and $\eta = \{\underset{\sim}{0}, W, \underset{\sim}{1}\}$ be intuitionistic fuzzy topologies on X, Y and Z respectively. Let the mapping $f : (X, \Im) \rightarrow (Y, \sigma)$ be defined by f(a) = x, f(b) = y and the mapping $g : (Y, \sigma) \rightarrow (Z, \eta)$ be defined by g(x) = p and g(y) = q. Then f and g are intuitionistic fuzzy g-continuous but gof is not intuitionistic fuzzy g-continuous.

**Theorem 3. 6**: If $f : (X, \Im) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy g-continuous and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is intutionistic fuzzy continuous. Then $gof : (X, \Im) \rightarrow (Z, \eta)$ is intutionistic fuzzy g-continuous.

***Proof***: If A is intuitionistic fuzzy closed in Z, then $f^{-1}(A)$ is intuitionistic fuzzy closed in Y

because g is intuitionistic fuzzy continuous. Therefore
$(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is intuitionistic fuzzy g-closed in X. Hence gof is intuitionistic fuzzy g-continuous.

***Theorem 3.7***: If f :(X, $\Im$)→(Y, σ) and g :(Y, σ)→(Z, η) are two intutionistic fuzzy g-continuous mappings and (Y, σ) is intuitionistic fuzzy $T_{1/2}$ space then gof : (X, $\Im$)→ (Z, η) is intuitionistic fuzzy g-continuous.

***Proof***: Obvious.

***Definition 3.4***: An intuitionistic fuzzy topological space (X,$\Im$) is said to be intuitionistic fuzzy GO- compact if every intuitionistic fuzzy g-open cover of X has a finite subcover.

***Theorem 3.8*** : Intuitionistic fuzzy g-continuous image of an intuitionistic fuzzy GO-compact space is intuitionistic fuzzy compact.

***Proof*** : Let f : (X, $\Im$) → (Y, σ) be an intuitionistic fuzzy g-continuous map from an intuitionistic fuzzy GO-compact space (X, $\Im$) onto an intuitionistic fuzzy topological space (Y, σ). Let $\{A_i: i\in\wedge\}$ be an intuitionistic fuzzy g-open cover of Y then $\{f^{-1}(Ai) : i \in\wedge\}$is an intuitionistic fuzzy g-open cover of X. Since X is intuitionistic fuzzy GO-compact it has finite intuitionistic fuzzy subcover say
$\{f^{-1}(A_1),\ldots\ldots f^{-1}(A_n)\}$.Since f is onto $\{A_1, \ldots\ldots,A_n\}$ is an intuitionistic fuzzy open cover of Y and so (Y,σ) is intuitionistic fuzzy compact.

***Definition 3.5*** : An intuitionistic fuzzy topological space X is called intuitionistic fuzzy GO-connected if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy g-open and intuitionistic fuzzy g-closed.

***Remark 3.5***: Every intuitionistic fuzzy GO-connected space is intuitionistic fuzzy $C_5$-connected [15] , but the converse may not be true. For,

***Example 3. 4*** Let X = {a,b}, U = <x,(.5/a,.7/b),(.5/a,.3/b)>,

and $\Im = \{\tilde{0},U,\tilde{1}\}$ be an intuitionistic topology on X, then (X, $\Im$) is intutionistic fuzzy connected but not fuzzy GO-connected.

***Theorem 3.9***: An intuitionistic fuzzy $T_{1/2}$-space is intuitionistic fuzzy $C_5$- connected if and only if it is intuitionistic fuzzy GO-connected.

***Proof***: Obvious.

***Theorem 3. 10*** : If f : (X, $\Im$) → (Y, σ) is an intuitionistic fuzzy g-continuous surjection and X is intuitionistic fuzzy GO-connected then Y is intuitionistic fuzzy $C_5$-connected.

***Proof*** : Suppose Y is not intuitionistic fuzzy connected. Then there exists a proper intuitionistic fuzzy set G of Y which is both intuitionistic fuzzy open and intuitionistic fuzzy closed. Therefore $f^{-1}(G)$ is a proper intuitionistic fuzzy closed and intuitionistic fuzzy open set of X, because f intuitionistic fuzzy g-continuous surjection. Hence X is not intuitionistic fuzzy GO-connected.

# References


[1] K. Atanassov, Intuitionistic Fuzzy Sets, In VII ITKR's Session, (V.Sgurev,Ed.) Sofia, Bulgaria, (1983)

[2] K. Atanassov and S. Stoeva., Intuitionistic Fuzzy Sets, In Polish Symposium on Interval and Fuzzy Mathematics , Poznan, (1983), 23-26

[3] K. Atanassov, Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, 20(1986), 87-96.

[4] Bayhan Sadik, On Seperation Axioms in Intuitionistic Topological Spaces.

[5] Intern. Jour. Math. Math. Sci. 27(2001), no.10, 621-630.

[6] C.L. Chang, Fuzzy Topological Spaces, J.Math.Anal.Appl. 24(1968) 182-190.

[7] D. Coker, An Introduction to Intuitionistic Fuzzy Topological Spaces, Fuzzy Sets and Systems .88(1997), 81-89.

[8] D. Coker and M. Demirci, On Intuitionistic Fuzzy Points. Notes On IFS:2-1(1995), 78-83

[9] D. Coker and A.Es. Hyder., On Fuzzy Compactness in Intuitionistic Fuzzy Topological Spaces ,The Journal of Fuzzy Mathematics ,3-4(1995), 899-909.

[10] H. Gurcay, D. Coker and Es., A.Haydar, On Fuzzy Continuity in Intuitionistic Fuzzy Topological Spaces. The Journal of Fuzzy Mathematics Vol.5, no.2, 1997, 365-378.

[11] N. Levine., Generalized Closed Sets In Topology, Rend. Cerc. Mat. Palermo.19(2), 1970, 571-599.

[12] O.Ozbakir and D. Coker, Fuzzy Multifunctions in Intuitionistic Fuzzy Topological Spaces Notes On IFS 5(1999) No.3.

[13] S.S. Thakur and Malviya R., Generalized Closed Sets In Fuzzy Topology, Math. Notae 38(1995), 137-140.

[14] S.S Thakur and Rekha Chaturvedi, Generelized Closed Sets in Intuitionistic Fuzzy Topology. (Submitted).

[15] B. Tripathy, Intuitionistic Fuzzy Metric Spaces. Notes on IFS, Vol.5, (1999), No.2.

[16] N. Turnali and D. Coker, Fuzzy Connectedness in Intuitionistic Fuzzy Topological Spaces. Fuzzy Sets And Systems 116(2000) (3), 369-375.

[17] L.A.Zadeh, Fuzzy Sets, Information and Control, 18(1965), 338-353.