Remark on Dworniczak's intuitionistic fuzzy implications. Part 1

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Abstract: Four new intuitinistic fuzzy implications are introduced. They are modal forms of Dworniczak's intuitionistic fuzzy implication $\rightarrow_{150,\lambda}$. Their basic properties are studied.
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1   Introduction

In [7, 8, 9], Piotr Dworniczak introduce three intuitionistic fuzzy implications, that generalized the defined by the author intuitionistic fuzzy implications in [4, 5, 6]. Some of their properties are discussed in [1].

Here, following the idea from [2], we introduce four new intuitionistic fuzzy implications, modifying Dworniczak's intuitionistic fuzzy implication

$$\langle a, b \rangle \rightarrow_{150,\lambda} \langle c, d \rangle = \langle \frac{b + c + \lambda - 1}{2\lambda}, \frac{a + d + \lambda - 1}{2\lambda} \rangle,$$

where $\lambda \geq 1$.

2   Main results

Let everywhere below the variables x and y have truth values $\langle a, b \rangle$ and $\langle C, D \rangle$, where $a, b, c, d \in [0, 1], a + b \leq 1, c + d \leq 1$.

In [2, 3] the following operations and operators are defined:

$$\neg \langle a, b \rangle \equiv \neg_1 \langle a, b \rangle = \langle b, a \rangle,$$

$$\langle a, b \rangle @ \langle c, d \rangle = \langle \frac{a+c}{2}, \frac{b+d}{2} \rangle,$$

$$\Box \langle a, b \rangle = \langle a, 1-a \rangle,$$

$$\Diamond \langle a, b \rangle = \langle 1-b, b \rangle,$$

$$\boxplus_{\alpha,\beta,\gamma} \langle a, b \rangle = \langle \alpha a, \beta b + \gamma \rangle,$$

$$\boxtimes_{\alpha,\beta,\gamma} \langle a, b \rangle = \langle \alpha a + \gamma, \beta b \rangle,$$

where $\alpha, \beta, \gamma \in [0, 1]$ and $\max(\alpha, \beta) + \gamma \leq 1$,

$$\boxed{\bullet}_{\alpha,\beta,\gamma,\delta} \langle a, b \rangle = \langle \alpha a + \gamma, \beta b + \delta \rangle,$$

where $\alpha, \beta, \gamma, \delta \in [0, 1]$ and $\max(\alpha, \beta) + \gamma + \delta \leq 1$.

We use the following formulas:

$$\langle a, b \rangle \rightarrow^1_{150,\lambda} \langle c, d \rangle = \Box \langle a, b \rangle \rightarrow_{150,\lambda} \Box \langle c, d \rangle,$$

$$\langle a, b \rangle \rightarrow^2_{150,\lambda} \langle c, d \rangle = \Box \langle a, b \rangle \rightarrow_{150,\lambda} \Diamond \langle c, d \rangle,$$

$$\langle a, b \rangle \rightarrow^3_{150,\lambda} \langle c, d \rangle = \Diamond \langle a, b \rangle \rightarrow_{150,\lambda} \Diamond \langle c, d \rangle,$$

$$\langle a, b \rangle \rightarrow^4_{150,\lambda} \langle c, d \rangle = \Diamond \langle a, b \rangle \rightarrow_{150,\lambda} \Box \langle c, d \rangle.$$

So, we obtain the explicit forms of the new four implications as follows:

$$\langle a, b \rangle \rightarrow^1_{150,\lambda} \langle c, d \rangle = \langle \frac{-a+c+\lambda}{2\lambda}, \frac{a-c+\lambda}{2\lambda} \rangle,$$

$$\langle a, b \rangle \rightarrow^2_{150,\lambda} \langle c, d \rangle = \langle \frac{1-a-d+\lambda}{2\lambda}, \frac{a+d+\lambda-1}{2\lambda} \rangle,$$

$$\langle a, b \rangle \rightarrow^3_{150,\lambda} \langle c, d \rangle = \langle \frac{b+c+\lambda-1}{2\lambda}, \frac{1-b-c+\lambda}{2\lambda} \rangle,$$

$$\langle a, b \rangle \rightarrow^4_{150,\lambda} \langle c, d \rangle = \langle \frac{b-d+\lambda}{2\lambda}, \frac{-b+d+\lambda}{2\lambda} \rangle.$$

First, we chack that for every $i = 1, 2, 3, 4$ and for every $\lambda \geq 1$:

$$\langle 0, 1 \rangle \rightarrow^i_{150,\lambda} \langle 0, 1 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle,$$

$$\langle 0, 1 \rangle \rightarrow^i_{150,\lambda} \langle 1, 0 \rangle = \langle \frac{\lambda+1}{2}, \frac{\lambda-1}{2} \rangle,$$

$$\langle 1, 0 \rangle \rightarrow^i_{150,\lambda} \langle 0, 1 \rangle = \langle \frac{\lambda-1}{2}, \frac{\lambda+1}{2} \rangle,$$

$$\langle 1, 0 \rangle \rightarrow^i_{150,\lambda} \langle 0, 1 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle.$$

Second, we chack that for every $\lambda \geq 1$:

$$\langle 0, 0 \rangle \rightarrow^1_{150,\lambda} \langle 0, 1 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle,$$

$$\langle 0,0 \rangle \rightarrow_{150,\lambda}^{2} \langle 0,1 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle,$$

$$\langle 0,0 \rangle \rightarrow_{150,\lambda}^{3} \langle 0,1 \rangle = \langle \frac{\lambda-1}{2}, \frac{\lambda+1}{2} \rangle,$$

$$\langle 0,0 \rangle \rightarrow_{150,\lambda}^{4} \langle 0,1 \rangle = \langle \frac{\lambda-1}{2}, \frac{\lambda+1}{2} \rangle,$$

$$\langle 0,0 \rangle \rightarrow_{150,\lambda}^{1} \langle 0,0 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle,$$

$$\langle 0,0 \rangle \rightarrow_{150,\lambda}^{2} \langle 0,0 \rangle = \langle \frac{\lambda+1}{2}, \frac{\lambda-1}{2} \rangle,$$

$$\langle 0,0 \rangle \rightarrow_{150,\lambda}^{3} \langle 0,0 \rangle = \langle \frac{\lambda-1}{2}, \frac{\lambda+1}{2} \rangle,$$

$$\langle 0,0 \rangle \rightarrow_{150,\lambda}^{4} \langle 0,0 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle,$$

$$\langle 0,0 \rangle \rightarrow_{150,\lambda}^{1} \langle 1,0 \rangle = \langle \frac{\lambda+1}{2}, \frac{\lambda-1}{2} \rangle,$$

$$\langle 0,0 \rangle \rightarrow_{150,\lambda}^{2} \langle 1,0 \rangle = \langle \frac{\lambda+1}{2}, \frac{\lambda-1}{2} \rangle,$$

$$\langle 0,0 \rangle \rightarrow_{150,\lambda}^{3} \langle 1,0 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle,$$

$$\langle 0,0 \rangle \rightarrow_{150,\lambda}^{4} \langle 1,0 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle,$$

$$\langle 0,1 \rangle \rightarrow_{150,\lambda}^{1} \langle 0,0 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle,$$

$$\langle 0,1 \rangle \rightarrow_{150,\lambda}^{2} \langle 0,0 \rangle = \langle \frac{\lambda+1}{2}, \frac{\lambda-1}{2} \rangle,$$

$$\langle 0,1 \rangle \rightarrow_{150,\lambda}^{3} \langle 0,0 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle,$$

$$\langle 0,1 \rangle \rightarrow_{150,\lambda}^{4} \langle 0,0 \rangle = \langle \frac{\lambda+1}{2}, \frac{\lambda-1}{2} \rangle,$$

$$\langle 1,0 \rangle \rightarrow_{150,\lambda}^{1} \langle 0,0 \rangle = \langle \frac{\lambda-1}{2}, \frac{\lambda+1}{2} \rangle,$$

$$\langle 1,0 \rangle \rightarrow_{150,\lambda}^{2} \langle 0,0 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle,$$

$$\langle 1,0 \rangle \rightarrow_{150,\lambda}^{3} \langle 0,0 \rangle = \langle \frac{\lambda-1}{2}, \frac{\lambda+1}{2} \rangle,$$

$$\langle 1,0 \rangle \rightarrow_{150,\lambda}^{4} \langle 0,0 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle.$$

Using definition from [2, 3]

$$\langle a,b \rangle \geq \langle c,d \rangle \text{ if and only if } a \geq c \text{ and } b \leq d,$$

we can prove the validity of the following theorem.

Theorem 1. For every $a, b, c, d \in [0, 1]$, so that $a + b \leq 1$ and $c + d \leq 1$ and for every $\lambda \geq 1$:

$$\langle a, b \rangle \rightarrow^2_{150,\lambda} \langle c, d \rangle \geq \langle a, b \rangle \rightarrow^1_{150,\lambda} \langle c, d \rangle, \tag{1}$$

$$\langle a, b \rangle \rightarrow^4_{150,\lambda} \langle c, d \rangle \geq \langle a, b \rangle \rightarrow^3_{150,\lambda} \langle c, d \rangle, \tag{2}$$

$$\langle a, b \rangle \rightarrow^2_{150,\lambda} \langle c, d \rangle \geq \langle a, b \rangle \rightarrow^4_{150,\lambda} \langle c, d \rangle, \tag{3}$$

$$\langle a, b \rangle \rightarrow^1_{150,\lambda} \langle c, d \rangle \geq \langle a, b \rangle \rightarrow^3_{150,\lambda} \langle c, d \rangle. \tag{4}$$

Proof: For example, let us check the validity of the fourth inequality.
First, we see, that

$$-a + c + \lambda - b - c - \lambda + 1 = 1 - a - b \geq 0,$$

$$1 - b - c + \lambda - a + c - \lambda = 1 - a - b \geq 0.$$
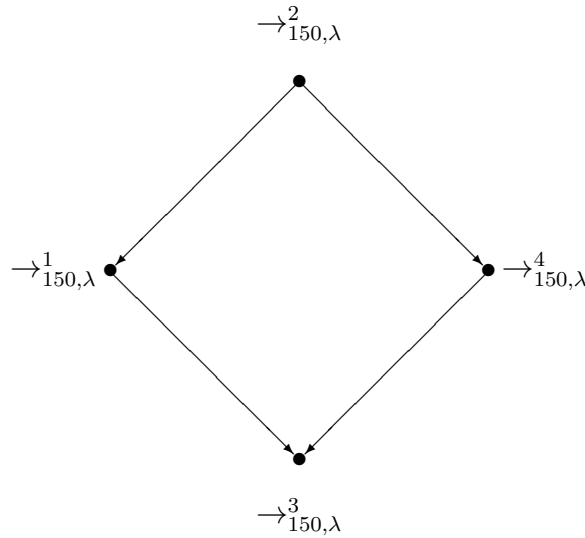
Therefore,

$$\langle \frac{-a + c + \lambda}{2\lambda}, \frac{a - c + \lambda}{2\lambda} \rangle \geq \langle \frac{b + c + \lambda - 1}{2\lambda}, \frac{1 - b - c + \lambda}{2\lambda} \rangle,$$

i.e.,

$$\langle a, b \rangle \rightarrow^1_{150,\lambda} \langle c, d \rangle \geq \langle a, b \rangle \rightarrow^3_{150,\lambda} \langle c, d \rangle.$$

Hence, (4) is valid. (1) – (3) are proved by analogy. $\square$

Now, we can construct the following diagram

Now, we show that the new implications can be represented by a part of the above operators.

Theorem 2. For every $a, b, c, d \in [0, 1]$, so that $a + b \leq 1$ and $c + d \leq 1$ and for every $\lambda \geq 1$:

$$\langle a, b \rangle \rightarrow^1_{150,\lambda} \langle c, d \rangle = \boxtimes_{\frac{1}{\lambda}, \frac{1}{\lambda}, \frac{\lambda-1}{\lambda}} \Diamond \neg \langle a, b \rangle @ \boxplus_{\frac{1}{\lambda}, \frac{1}{\lambda}, \frac{\lambda-1}{\lambda}} \square \langle c, d \rangle,$$

$$\langle a, b \rangle \rightarrow^2_{150,\lambda} \langle c, d \rangle = \boxtimes_{\frac{1}{\lambda}, \frac{1}{\lambda}, \frac{\lambda-1}{\lambda}} \Diamond \neg \langle a, b \rangle @ \boxplus_{\frac{1}{\lambda}, \frac{1}{\lambda}, \frac{\lambda-1}{\lambda}} \square \neg \langle c, d \rangle,$$

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$$\langle a, b\rangle \rightarrow^1_{150,\lambda} \langle c, d\rangle = \boxplus_{\frac{1}{\lambda}, \frac{1}{\lambda}, \frac{\lambda-1}{\lambda}} \square \neg \langle a, b\rangle @ \boxtimes_{\frac{1}{\lambda}, \frac{1}{\lambda}, \frac{\lambda-1}{\lambda}} \Diamond \langle c, d\rangle,$$

$$\langle a, b\rangle \rightarrow^3_{150,\lambda} \langle c, d\rangle = \boxtimes_{\frac{1}{\lambda}, \frac{1}{\lambda}, \frac{\lambda-1}{\lambda}} \square \neg \langle a, b\rangle @ \boxplus_{\frac{1}{\lambda}, \frac{1}{\lambda}, \frac{\lambda-1}{\lambda}} \Diamond \langle c, d\rangle.$$

The proofs of these assertions are similar to the above one. □

Corollary. For every $a, b, c, d \in [0, 1]$, so that $a + b \leq 1$ and $c + d \leq 1$ and for every $\lambda \geq 1$:

$$\langle a, b\rangle \rightarrow^1_{150,\lambda} \langle c, d\rangle = \boxdot_{\frac{1}{\lambda}, \frac{1}{\lambda}, \frac{\lambda-1}{\lambda}, 0} \Diamond \neg \langle a, b\rangle @ \boxdot_{\frac{1}{\lambda}, \frac{1}{\lambda}, 0, \frac{\lambda-1}{\lambda}} \square \langle c, d\rangle,$$

$$\langle a, b\rangle \rightarrow^2_{150,\lambda} \langle c, d\rangle = \boxdot_{\frac{1}{\lambda}, \frac{1}{\lambda}, \frac{\lambda-1}{\lambda}, 0} \Diamond \neg \langle a, b\rangle @ \boxdot_{\frac{1}{\lambda}, \frac{1}{\lambda}, 0, \frac{\lambda-1}{\lambda}} \square \neg \langle c, d\rangle,$$

$$\langle a, b\rangle \rightarrow^1_{150,\lambda} \langle c, d\rangle = \boxdot_{\frac{1}{\lambda}, \frac{1}{\lambda}, 0, \frac{\lambda-1}{\lambda}} \square \neg \langle a, b\rangle @ \boxdot_{\frac{1}{\lambda}, \frac{1}{\lambda}, \frac{\lambda-1}{\lambda}, 0} \Diamond \langle c, d\rangle,$$

$$\langle a, b\rangle \rightarrow^3_{150,\lambda} \langle c, d\rangle = \boxdot_{\frac{1}{\lambda}, \frac{1}{\lambda}, 0, \frac{\lambda-1}{\lambda}} \square \neg \langle a, b\rangle @ \boxdot_{\frac{1}{\lambda}, \frac{1}{\lambda}, \frac{\lambda-1}{\lambda}, 0} \Diamond \langle c, d\rangle.$$

3   Conclusion

In the next parts of the research, we will study from one side other properties of the new implications and from another – the modifications of the two other Dworniczak's implications and their properties.

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