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Cartesian products over intuitionistic fuzzy number subgroups

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Abstract: In this paper, we have verified all the five versions of Cartesian products over intuitionistic fuzzy number subgroups. We have also established some additional properties relevant to Cartesian products of intuitionistic fuzzy number subgroups.
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1 Introduction

The concept of fuzzy set was introduced by Zadeh [5] to handle the problem of uncertainty. As an extension to the fuzzy sets Atanassov [1] introduced intuitionistic fuzzy set. Some five versions of Cartesian products of two intuitionistic fuzzy sets introduced by Stoyanova [4]. Palanivelrajan, Gunasekaran and Kaliraju [4] defined intuitionistic fuzzy number subgroups.

In this paper, we will discuss all the five versions of Cartesian products over intuitionistic fuzzy number subgroups and prove the results.

2 Preliminaries

In this section we state some definitions and properties related to intuitionistic fuzzy set, intuitionistic fuzzy number and intuitionistic fuzzy number subgroups.

Let R be the set of all real numbers and F(R) be the set of all fuzzy subsets defined on R. We denote $F^*(R)$ the set of all fuzzy number and also denote the set of all intuitionistic fuzzy number $IF^*(R)$.

Definition 2.1: An Intuitionistic Fuzzy Set (IFS) A assigns to each element x of the universe X a membership degree $\mu_A(x) \in [0, 1]$ and a non-membership degree $\nu_A(x) \in [0,1]$ such that IFS A is mathematically represented as $\{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$. The value $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the degree of hesitancy or the Intuitionistic index of x to A.

Definition 2.2: An Intuitionistic fuzzy subset $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in R\}$ of the real line is called an intuitionistic fuzzy number if:

(a) $x_{0,} x_1 \in X$ such that $\mu_A(x_0) = 1$ and $\nu_A(x_1) = 1$.

(b) A is if-convex (i.e. its membership function μ is fuzzy convex and its non-membership function ν is fuzzy concave).

(c) μ_A is upper semi-continuous and ν_A is lower semi-continuous.

(d) Supp A = cl $(\{x \in X \mid \nu_A(x)\})$ is bounded.

Remark 2.1: An intuitionistic fuzzy number A represented as $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in R\}$

$$\mu_A(x) = \begin{cases} 1, & if \quad x < a_1 \\ f_A(x), & if \quad a_1 \le x < a_2 \\ 0, & if \quad a_2 \le x \le a_3 \\ g_A(x), & if \quad a_3 < x \le a_4 \\ 1, & if \quad a_4 < x \end{cases}$$

$$\nu_A(x) = \begin{cases} 0, & if \quad x < b_1 \\ h_A(x), & if \quad b_1 \le x < b_2 \\ 1, & if \quad b_2 \le x \le b_3 \\ k_A(x), & if \quad b_3 < x \le b_4 \\ 0, & if \quad b_4 < x \end{cases}$$

where $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in R$ such that $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$ and the function $f_A, g_A, h_A, k_A : R \to [0,1]$.

Example 2.1: The following is an example of intuitionistic fuzzy number

$$\mu_A(x) = \begin{cases} 1, & if \quad x < -2 \\ \frac{-(1+2x)}{3}, & if \quad -2 \le x < \frac{-1}{2} \\ 0, & if \quad \frac{-1}{2} \le x \le \frac{1}{2} \\ \frac{4x^2 - 1}{4}, & if \quad \frac{1}{2} < x \le \frac{\sqrt{5}}{2} \\ 1, & if \quad \frac{\sqrt{5}}{2} < x \end{cases}$$

$$\nu_A(x)=\begin{cases} 0, & if \quad x<-2 \\ \dfrac{4+2x}{3}, & if \quad -2\le x<\dfrac{-1}{2} \\ 1, & if \quad \dfrac{-1}{2}\le x\le\dfrac{1}{2} \\ \dfrac{-4x^2+5}{4}, & if \quad \dfrac{1}{2}<x\le\dfrac{\sqrt{5}}{2} \\ 0, & if \quad \dfrac{\sqrt{5}}{2}<x \end{cases}$$

Definition 2.3: An intuitionistic fuzzy number A of $IF^*(R)$ is said to be intuitionistic fuzzy number subgroup if the following conditions are satisfied.

(i) $\mu_A(xy)\ge \min(\mu_A(x),\mu_A(y))$

(ii) $\mu_A(x^{-1})\ge \mu_A(x)$

(iii) $\nu_A(xy)\le \max(\nu_A(x),\ \nu_A(y))$

(iv) $\nu_A(x^{-1})\le \nu_A(x)$, for all $x, y \in IF^*(R)$.

Definition 2.4. Let E_1 and E_2 be two universes and let $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in E_1\}$, $B=\{\langle y,\mu_B(y),\nu_B(y)\rangle \mid y\in E_2\}$ be two intuitionistic fuzzy sets. The five Cartesian products of two intuitionistic fuzzy sets A and B are defined as follows:

$(i).$ *The Cartesian product* $"\times_1"$

$$A\times_1 B=\{\langle\langle x,y\rangle,\mu_A(x).\mu_B(y),\nu_A(x).\nu_B(y)\rangle \mid x\in E_1\ \&\ y\in E_2\},$$

$(ii).$ *The Cartesian product* $"\times_2"$

$$A\times_2 B=\{\langle\langle x,y\rangle,\mu_A(x)+\mu_B(y)-\mu_A(x).\mu_B(y),\nu_A(x).\nu_B(y)\rangle \mid x\in E_1\ \&\ y\in E_2\},$$

$(iii).$ *The Cartesian product* $"\times_3"$

$$A\times_3 B=\{\langle\langle x,y\rangle,\mu_A(x).\mu_B(y),\nu_A(x)+\nu_B(y)-\nu_A(x).\nu_B(y)\rangle \mid x\in E_1\ \&\ y\in E_2\},$$

$(iv).$ *The Cartesian product* $"\times_4"$

$$A\times_4 B=\{\langle\langle x,y\rangle,\min(\mu_A(x).\mu_B(y)),\max(\nu_A(x).\nu_B(y))\rangle \mid x\in E_1\ \&\ y\in E_2\},$$

$(v).$ *The Cartesian product* $"\times_5"$

$$A\times_5 B=\{\langle\langle x,y\rangle,\max(\mu_A(x).\mu_B(y)),\min(\nu_A(x).\nu_B(y))\rangle \mid x\in E_1\ \&\ y\in E_2\}.$$

from $0\le \mu_A(x).\mu_B(y)+\nu_A(x).\nu_B(y)\le \mu_A(x)+\nu_A(x)\le 1.$

3 Cartesian product over intuitionistic fuzzy number subgroups

Theorem 3.1. If A and B are two intuitionistic fuzzy number subgroups then $A\times_1 B$ is also an intuitionistic fuzzy number subgroup.

Proof: Let A and B are two intuitionistic fuzzy number subgroups in $IF^*(R)$. Consider $(x_1, y_1), (x_2, y_2)\in A\times_1 B$ for every $x_1, x_2\in A$ and $y_1, y_2\in B$.

$$(i)\ \mu_{A\times_1 B}((x_1,y_1),(x_2,y_2)) = \mu_{A\times_1 B}(x_1x_2, y\ y_2)$$
$$= \mu_A(x_1x_2).\mu_B(y_1y_2)$$
$$\geq \min(\mu_A(x_1),\mu_A(x_2)).\min(\mu_B(y_1),\mu_B(y_2))$$
$$= \min(\mu_A(x_1).\mu_B(y_1),\mu_A(x_2).\mu_B(y_2))$$
$$= \min(\mu_{A\times_1 B}(x_1,y_1),\ \mu_{A\times_1 B}(x_2,y_2))$$
$$\therefore\ \mu_{A\times_1 B}((x_1,y_1),(x_2,y_2)) \geq \min(\mu_{A\times_1 B}(x_1,y_1),\ \mu_{A\times_1 B}(x_2,y_2)).$$

$$(ii)\quad \mu_{A\times_1 B}(x_1^{-1},y_1^{-1}) = \mu_A(x_1^{-1}).\mu_B(y_1^{-1})$$
$$\geq \mu_A(x_1).\mu_B(y_1)$$
$$= \mu_{A\times_1 B}(x_1,y_1)$$
$$\therefore\ \mu_{A\times_1 B}(x_1^{-1},y_1^{-1}) \geq \mu_{A\times_1 B}(x_1,y_1).$$

$$(iii)\quad \nu_{A\times_1 B}((x_1,y_1),(x_2,y_2)) = \nu_{A\times_1 B}(x_1x_2, y_1y_2)$$
$$= \nu_A(x_1x_2).\nu_B(y_1y_2)$$
$$\leq \max(\nu_A(x_1),\nu_A(x_2)).\max(\nu_B(y_1),\nu_B(y_2))$$
$$= \max(\nu_A(x_1).\nu_B(y_1),\nu_A(x_2),\nu_B(y_2))$$
$$= \max(\nu_{A\times_1 B}(x_1,y_1),\nu_{A\times_1 B}(x_2,y_2))$$
$$\therefore\ \nu_{A\times_1 B}((x_1,y_1),(x_2,y_2)) \leq \max(\nu_{A\times_1 B}(x_1,y_1),\nu_{A\times_1 B}(x_2,y_2)).$$

$$(iv)\quad \nu_{A\times_1 B}(x_1^{-1},y_1^{-1}) = \nu_A(x_1^{-1}).\nu_B(y_1^{-1})$$
$$\leq \nu_A(x_1).\nu_B(y_1)$$
$$= \nu_{A\times_1 B}(x_1,y_1)$$
$$\therefore\quad \nu_{A\times_1 B}(x_1^{-1},y_1^{-1}) \leq \nu_{A\times_1 B}(x_1,y_1).$$

Therefore $A \times_1 B$ is an intuitionistic fuzzy number subgroup. □

Theorem 3.2: If A and B are two intuitionistic fuzzy number subgroups then $A \times_2 B$ is also an intuitionistic fuzzy number subgroup.

Proof: Let A and B are two intuitionistic fuzzy number subgroups in $IF^*(R)$. Consider $(x_1, y_1), (x_2, y_2) \in A \times_2 B$ for every $x_1, x_2 \in A$ and $y_1, y_2 \in B$.

$$(i)\ \mu_{A\times_2 B}((x_1,y_1),(x_2,y_2)) = \mu_{A\times_2 B}(x_1x_2, y_1y_2)$$
$$= \mu_A(x_1x_2) + \mu_B(y_1y_2) - \mu_A(x_1x_2).\mu_B(y_1y_2)$$
$$\geq \min(\mu_A(x_1),\mu_A(x_2)) + \min(\mu_B(y_1),\mu_B(y_2))$$
$$-\min(\mu_A(x_1),\mu_A(x_2)).\min(\mu_B(y_1),\mu_B(y_2))$$
$$= \min(\mu_A(x_1)+\mu_B(y_1)-\mu_A(x_1).\mu_B(y_1),\mu_A(x_1)+\mu_B(y_1)-\mu_A(x_1).\mu_B(y_1))$$
$$= \min(\mu_{A\times_2 B}(x_1,y_1),\mu_{A\times_2 B}(x_2,y_2))$$
$$\therefore\ \mu_{A\times_2 B}((x_1,y_1),(x_2,y_2)) \geq \min(\mu_{A\times_2 B}(x_1,y_1),\mu_{A\times_2 B}(x_2,y_2)).$$

(*ii*) $\mu_{A\times_2 B}(x_1^{-1}, y_1^{-1}) = (\mu_A(x_1^{-1}) + \mu_B(y_1^{-1}) - \mu_A(x_1^{-1}).\mu_B(y_1^{-1})$

$$\geq \mu_A(x_1) + \mu_B(y_1) - \mu_A(x_1).\mu_B(y_1)$$

$$= \mu_{A\times_2 B}(x_1, y_1)$$

$$\therefore \mu_{A\times_2 B}(x_1^{-1}, y_1^{-1}) \geq \mu_{A\times_2 B}(x_1, y_1).$$

(*iii*) $\nu_{A\times_2 B}((x_1, y_1),(x_2, y_2)) = \nu_{A\times_2 B}(x_1 x_2, y_1 y_2)$

$$= \nu_A(x_1 x_2).\nu_B(y_1 y_2)$$

$$\leq \max(\nu_A(x_1), \nu_A(x_2)).\max(\nu_B(y_1), \nu_B(y_2))$$

$$= \max(\nu_A(x_1).\nu_B(y_1), \nu_A(x_2).\nu_B(y_2))$$

$$= \max(\nu_{A\times_2 B}(x_1, y_1), \nu_{A\times_2 B}(x_2, y_2))$$

$$\therefore \nu_{A\times_2 B}((x_1, y_1),(x_2, y_2)) \leq \max(\nu_{A\times_2 B}(x_1, y_1), \nu_{A\times_2 B}(x_2, y_2)).$$

(*iv*) $\nu_{A\times_2 B}(x_1^{-1}, y_1^{-1}) = \nu_A(x_1^{-1}).\nu_B(y_1^{-1})$

$$\leq \nu_A(x_1).\nu_B(y_1)$$

$$= \nu_{A\times_2 B}(x_1, y_1)$$

$$\therefore \nu_{A\times_2 B}(x_1^{-1}, y_1^{-1}) \leq \nu_{A\times_2 B}(x_1, y_1).$$

Therefore $A \times_2 B$ is an intuitionistic fuzzy number subgroup. □

Theorem 3.3: If A and B are two intuitionistic fuzzy number subgroups then $A \times_3 B$ is also an intuitionistic fuzzy number subgroup.

Proof: Let A and B be two intuitionistic fuzzy number subgroups in $IF^*(R)$. Consider $(x_1, y_1), (x_2, y_2) \in A \times_3 B$ for every $x_1, x_2 \in A$ and $y_1, y_2 \in B$.

(*i*) $\mu_{A\times_3 B}((x_1, y_1),(x_2, y_2)) = \mu_{A\times_3 B}(x_1 x_2, y_1 y_2)$

$$= \mu_A(x_1 x_2).\mu_B(y_1 y_2)$$

$$\geq \min(\mu_A(x_1), \mu_B(x_2)).\min(\mu_A(y_1), \mu_B(y_2))$$

$$= \min(\mu_A(x_1).\mu_B(y_1), \mu_A(x_2).\mu_B(y_2))$$

$$= \min(\mu_{A\times_3 B}(x_1, y_1), \mu_{A\times_3 B}(x_2, y_2))$$

$$\therefore \mu_{A\times_2 B}((x_1, y_1),(x_2, y_2)) \geq \min(\mu_{A\times_3 B}(x_1, y_1), \mu_{A\times_3 B}(x_2, y_2)).$$

(*ii*) $\mu_{A\times_3 B}(x_1^{-1}, y_1^{-1}) = \mu_A(x_1^{-1}) + \mu_B(y_1^{-1}) - \mu_A(x_1^{-1}).\mu_B(y_1^{-1})$

$$\geq \mu_A(x_1) + \mu_B(y_1) - \mu_A(x_1).\mu_B(y_1)$$

$$= \mu_{A\times_3 B}(x_1, y_1)$$

$$\therefore \mu_{A\times_3 B}(x_1^{-1}, y_1^{-1}) \geq \mu_{A\times_3 B}(x_1, y_1).$$

(iii) $\nu_{A\times_3 B}((x_1,y_1),(x_2,y_2)) = \nu_{A\times_3 B}(x_1x_2, y_1y_2)$

$$= \nu_A(x_1x_2)+\nu_B(y_1y_2)-\nu_A(x_1x_2).\nu_B(y_1y_2)$$
$$\le \max(\nu_A(x_1),\nu_A(x_2))+\max(\nu_B(y_1),\nu_B(y_2))$$
$$-\max(\nu_A(x_1),\nu_A(x_2)).\max(\nu_B(y_1),\nu_B(y_2))$$
$$= \max(\nu_A(x_1)+\nu_B(y_1)-\nu_A(x_1).\nu_B(y_1),\nu_A(x_2)+\nu_B(y_2)-\nu_A(x_2).\nu_B(y_2))$$
$$= \max(\nu_{A\times_3 B}(x_1,y_1),\nu_{A\times_3 B}(x_2,y_2))$$

$\therefore$ $\nu_{A\times_3 B}((x_1,y_1),(x_2,y_2) \le \max(\nu_{A\times_3 B}(x_1,y_1),\nu_{A\times_3 B}(x_2,y_2))$.

(iv) $\nu_{A\times_3 B}(x_1^{-1},y_1^{-1}) = \nu_A(x_1^{-1})+\nu_B(y_1^{-1})-\nu_A(x_1^{-1}).\nu_B(y_1^{-1})$

$$\le \nu_A(x_1)+\nu_B(y_1)-\nu_A(x_1).\nu_B(y_1)$$
$$= \nu_{A\times_3 B}(x_1,y_1)$$

$\therefore$ $\nu_{A\times_3 B}(x_1^{-1},y_1^{-1}) \le \nu_{A\times_3 B}(x_1,y_1)$.

Therefore $A \times_3 B$ is also an intuitionistic fuzzy number subgroup. □

Theorem 3.4: If A and B are two intuitionistic fuzzy number subgroups then $A \times_4 B$ is also an intuitionistic fuzzy number subgroup.

Proof: Let A and B be two intuitionistic fuzzy number subgroups in $IF^*(R)$. Consider $(x_1, y_1), (x_2, y_2) \in A \times_4 B$ for every $x_1, x_2 \in A$ and $y_1, y_2 \in B$.

(i) $\mu_{A\times_4 B}((x_1,y_1),(x_{2,}y_2)) = \mu_{A\times_4 B}(x_1x_2, y_1y_2)$

$$= \min(\mu_A(x_1x_2).\mu_B(y_1y_2))$$
$$\ge \min(\min(\mu_A(x_1),\mu_A(x_2)).\min(\mu_B(y_1),\mu_B(y_2)))$$
$$= \min(\min(\mu_A(x_1).\mu_B(y_1)),\min(\mu_A(x_2).\mu_B(y_2)))$$
$$= \min(\mu_{A\times_4 B}(x_1,y_1),\mu_{A\times_4 B}(x_{2,}y_2))$$

$\therefore$ $\mu_{A\times_4 B}((x_1,y_1),(x_{2,}y_2)) \ge \min(\mu_{A\times_4 B}(x_1,y_1),\mu_{A\times_4 B}(x_{2,}y_2))$.

(ii) $\mu_{A\times_4 B}(x_1^{-1},y_1^{-1}) = \min(\mu_A(x_1^{-1}).\mu_B(y_1^{-1}))$

$$\ge \min(\mu_A(x_1).\mu_B(y_1))$$
$$= \mu_{A\times_4 B}(x_1,y_1)$$

$\therefore \mu_{A\times_4 B}(x_1^{-1},y_1^{-1}) \ge \mu_{A\times_4 B}(x_1,y_1)$.

(iii) $\nu_{A\times_4 B}((x_1,y_1),(x_{2,}y_2)) = \nu_{A\times_4 B}(x_1x_2, y_1y_2)$

$$= \max(\nu_A(x_1x_2).\nu_B(y_1y_2))$$
$$\le \max(\max(\nu_A(x_1),\nu_A(x_2)).\max(\nu_B(y_1),\nu_B(y_2)))$$
$$= \max(\max(\nu_A(x_1).\nu_B(y_1)),\max(\nu_A(x_2).\nu_B(y_2)))$$
$$= \max(\nu_{A\times_4 B}(x_1,y_1),\nu_{A\times_4 B}(x_{2,}y_2))$$

$\therefore$ $\nu_{A\times_4 B}((x_1,y_1),(x_{2,}y_2)) \le \max(\nu_{A\times_4 B}(x_1,y_1),\nu_{A\times_4 B}(x_{2,}y_2))$.

(iv) $\nu_{A\times_4 B}(x_1^{-1}, y_1^{-1}) \quad = \max(\nu_A(x_1^{-1}).\nu_B(y_1^{-1}))$

$$\leq \max(\nu_A(x_1).\nu_B(y_1))$$
$$= \nu_{A\times_4 B}(x_1, y_1)$$

$\therefore \quad \nu_{A\times_4 B}(x_1^{-1}, y_1^{-1}) \leq \nu_{A\times_4 B}(x_1, y_1)$.

Therefore $A \times_4 B$ is also an intuitionistic fuzzy number subgroup. □

Theorem 3.5: If A and B are two intuitionistic fuzzy number subgroups then $A \times_5 B$ is also an intuitionistic fuzzy number subgroup.

Proof: Let A and B be two intuitionistic fuzzy number subgroups in $IF^*(R)$. Consider $(x_1, y_1), (x_2, y_2) \in A \times_5 B$ for every $x_1, x_2 \in A$ and $y_1, y_2 \in B$.

(i) $\mu_{A\times_5 B}((x_1, y_1),(x_{2,}y_2)) = \mu_{A\times_5 B}(x_1x_{2,}, y_1y_2)$

$$= \max(\mu_A(x_1x_2).\mu_B(y_1y_2))$$
$$\geq \max(\min(\mu_A(x_1),\mu_A(x_2)).\min(\mu_B(y_1),\mu_B(y_2)))$$
$$= \min(\max(\mu_A(x_1).\mu_B(y_1)),\max(\mu_A(x_2).\mu_B(y_2)))$$
$$= \min(\mu_{A\times_5 B}(x_1, y_1),\mu_{A\times_5 B}(x_{2,}y_2))$$

$\therefore \mu_{A\times_5 B}((x_1, y_1),(x_{2,}y_2)) \geq \min(\mu_{A\times_5 B}(x_1, y_1),\mu_{A\times_5 B}(x_{2,}y_2))$.

(ii) $\quad \mu_{A\times_5 B}(x_1^{-1}, y_1^{-1}) = \max(\mu_A(x_1^{-1}).\mu_B(y_1^{-1}))$

$$\geq \max(\mu_A(x_1).\mu_B(y_1))$$
$$= \mu_{A\times_5 B}(x_1, y_1)$$

$\therefore \mu_{A\times_5 B}(x_1^{-1}, y_1^{-1}) \geq \mu_{A\times_5 B}(x_1, y_1)$.

(iii) $\nu_{A\times_5 B}((x_1, y_1),(x_{2,}y_2)) = \nu_{A\times_5 B}(x_1x_2, y_1y_2)$

$$= \min(\nu_A(x_1x_2).\nu_B(y_1, y_2))$$
$$\leq \min(\max(\nu_A(x_1),\nu_A(x_2)).\max(\nu_B(y_1),\nu_B(y_2)))$$
$$= \max(\min(\nu_A(x_1).\nu_B(y_1)),\min(\nu_A(x_2).\nu_B(y_2)))$$
$$= \max(\nu_{A\times_5 B}(x_1, y_1),\nu_{A\times_5 B}(x_{2,}y_2))$$

$\therefore \nu_{A\times_5 B}((x_1, y_1),(x_{2,}y_2)) \leq \max(\nu_{A\times_5 B}(x_1, y_1),\nu_{A\times_5 B}(x_{2,}y_2))$.

(iv) $\nu_{A\times_5 B}(x_1^{-1}, y_1^{-1}) \quad = \min(\nu_A(x_1^{-1}).\nu_B(y_1^{-1}))$

$$\leq \min(\nu_A(x_1).\nu_B(y_1))$$
$$= \nu_{A\times_5 B}(x_1, y_1)$$

$\therefore \nu_{A\times_5 B}(x_1^{-1}, y_1^{-1}) \leq \nu_{A\times_5 B}(x_1, y_1)$.

Therefore $A \times_5 B$ is an intuitionistic fuzzy number subgroup. □

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