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On Topological Structures Using Intuitionistic Fuzzy Sets

Doğan Çoker

Akdeniz University, Department of Mathematics, 07058-Antalya/TURKEY

Abstract: The purpose of this note is to give a short introduction to "intuitionistic fuzzy topological spaces" and the recent literature on these subjects.

Keywords: Intuitionistic fuzzy set; intuitionistic fuzzy topology; intuitionistic fuzzy topological space; fuzzy continuity; intuitionistic (fuzzy special) set; intuitionistic (fuzzy special) topology; intuitionistic (fuzzy special) topological space.

1. Basic Notions in Intuitionistic Fuzzy Topological Spaces

After the introduction of the concept of fuzzy sets by Zadeh [26] several generalizations of the notion of fuzzy set were made, one of which is given Krassimir T. Atanassov [1], and many works by the same author and his colleagues appeared in the literature [2,3,4,5].

Definition 1.1. [4] Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS for short) A is an object having the form $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$ where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

For brevity, we shall use the symbol $A =< x, \mu_A, \gamma_A >$ for the IFS $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$. In addition to the ones presented by Atanassov, we can define several other operations between IFS's as follows.

Definition 1.2. [11] Let $\{A_i : i \in J\}$ be an arbitrary family of IFS's in X. Then
(a) $\cap_{i\in J} A_i = \{< x, \wedge_{i\in J}\mu_{A_i}(x), \vee_{i\in J}\gamma_{A_i}(x) >: x \in X\}$;
(b) $\cup_{i\in J} A_i = \{< x, \vee_{i\in J}\mu_{A_i}(x), \wedge_{i\in J}\gamma_{A_i}(x) >: x \in X\}$.

Since our main purpose is to construct the tools for developing intuitionistic fuzzy topological spaces, we must introduce the IFS's $\underset{\sim}{0}$ and $\underset{\sim}{1}$ in X as follows:

Definition 1.3. [11] $\underset{\sim}{0} = \{< x, 0, 1 >: x \in X\}$ and $\underset{\sim}{1} = \{< x, 1, 0 >: x \in X\}$.

Now we shall define the image and preimage of IFS's. Let X and Y be two nonempty sets and $f : X \to Y$ be a function.

Definition 1.4. [11] (a) If $B = \{< y, \mu_B(y), \gamma_B(y) >: y \in Y\}$ is an IFS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B) = \{< x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) >: x \in X\}$.

(b) If $A = \{< x, \lambda_A(x), \eta_B(x) >: x \in X\}$ is an IFS in X, then the image of A under f, denoted by $f(A)$, is the IFS in Y defined by $f(A) = \{< y, f(\lambda_B)(y), (1 - f(1 - \eta_B))(y) >: y \in Y\}$.

Here we generalize the concept of fuzzy topological space, first initiated by Chang [7], to the case of intuitionistic fuzzy sets.

Definition 1.5. [11] An intuitionistic fuzzy topology (IFT for short) on a nonempty set X is a family τ of IFS's in X containing $\underset{\sim}{0}$, $\underset{\sim}{1}$, and closed under finite infima and arbitrary suprema. In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X.

Any fuzzy topological space (X, τ) in the sense of Chang is obviously an IFTS in the form $\tau = \{A : \mu \in \tau\}$ whenever we identify a fuzzy set in X whose membership function is μ with its counterpart $A = \{< x, \mu(x), 1 - \mu(x) >: x \in X\}$ as before.

Let (X, τ) be an IFTS. Then $\tau_1 = \{\mu_G : G \in \tau\}$ is a fuzzy topological space on X in Chang's sense, and $\tau_2' = \{\gamma_G : G \in \tau\}$ is the family of all fuzzy closed sets of the fuzzy topological space $\tau_2 = \{1 - \gamma_G : G \in \tau\}$ on X in Chang's sense [11].

An IFTS (X, τ) is, of course, in the sense of Chang. Now we can obtain the definition of an IFTS in the sense of Lowen [20] in a natural way: An intuitionistic fuzzy topological space in the sense of Lowen is a pair (X, τ) where (X, τ) is an IFTS and each IFS in the form $C_{\alpha,\beta} = \{< x, \alpha, \beta >: x \in X\}$, where $\alpha, \beta \in I$ are arbitrary and $\alpha + \beta \leq 1$, belongs to τ [11].

Definition 1.6. [4] The complement $\bar{A}$ of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 1.7. [11] Let (X, τ) and (Y, Φ) be two IFTS's and let $f : X \to Y$ be a function. Then f is said to be fuzzy continuous iff the preimage of each IFS in ΦF is an IFS in τ.

Proposition 1.8. [11] Let $f : (X, \tau) \to (Y, \Phi)$ be a function. Then the following expressions are equivalent:

(a) $f : (X, \tau) \to (Y, \Phi)$ is fuzzy continuous.

(b) The preimage of each IFCS in (Y, Φ) is an IFCS in (X, τ).

(c) $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ for each IFS B in B.

(d) $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for each IFS B in Y.

2. Future Prospects

Some introductory results on intuitionistic fuzzy topological spaces are already obtained by several researchers:

* Definition of intuitionistic fuzzy topological spaces, fuzzy continuity and fuzzy compactness in IFTS's [11].

* More results in fuzzy compactness [17,18], in fuzzy continuity [19], fuzzy connectedness [25] and fuzzy subspaces [9] in IFTS's.

* Definitions of intuitionistic fuzzy points [14].

* Separation axioms in IFTS's especially making use of intuitionistic fuzzy points [6].

* Fuzzy inclusion in the intuitionistic sense [15] and introduction to IFTS's in Sostak's sense [16, cf. 24].

* Relations between fuzzy rough sets [21] and intuitionistic L-fuzzy sets [12].

On the other hand, Çoker [10] gave also the discrete case of intuitionistic fuzzy sets, originally under the name of "intuitionistic set", and later it it was renamed as "intuitionistic fuzzy special set".

Definition 2.1. [10] Let X be a nonempty fixed set. An intuitionistic (fuzzy special) set (IS or IFSS for short) A is an object having the form $A =< X, A_1, A_2 >$ where the subsets $A_1, A_2 \subseteq X$ satisfy the property $A_1 \cap A_2 = \emptyset$, and are called set of members of A and set of nonmembers of A, respectively.

In this paper, two new concepts called "intuitionistic (fuzzy special) points" were also defined. Here we list some contributions related to intuitionistic (fuzzy special) topological spaces (ITS or IFSTS for short) which are defined similar to Definition 1.5:

* Definition of intuitionistic (fuzzy special) topological spaces, continuity and compactness in IFSTS's [13].

* Neighborhoods in IFSTS's [8] and connectedness in IFSTS's [22,23].

* Separation axioms in IFSTS's especially defined in terms of intuitionistic fuzzy special points [10].

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