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A study on irregular intuitionistic fuzzy graphs of second type

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Abstract: In this paper, we define the Irregular and Complement of intuitionistic fuzzy graphs of second type. Also we establish some of their properties.
Keywords: Intuitionistic fuzzy graphs, Intuitionistic fuzzy graphs of second type, Degree, Irregular, Complement.
AMS Classification: 03B20, 03D45, 03E72, 03F55.

1 Introduction

Fuzzy sets were introduced by Lotfi A. Zadeh [10] in 1965 as a generalisation of classical (crisp) sets. Further the fuzzy sets are generalised by Krassimir T. Atanassov [1] in which he has taken non-membership values also into consideration and introduced Intuitionistic Fuzzy sets (IFS) and their extensions like Intuitionistic Fuzzy sets of second type (IFSST), Intuitionistic L-Fuzzy sets (ILFS) and Temporal Intuitionistic Fuzzy sets (TIFS). R. Parvathi and N. Palaniappan [5] introduced some Operations on IFSST and A. Shannon and K. T. Atanassov [6] discussed the theory of Intuitionistic Fuzzy Graphs. R. Parvathi and M. G. Karunambigai [4] introduced Intuitionistic Fuzzy Graphs (IFG) and elaborately and analyzed various components. Further A. Nagoor Gani and S. Shajitha Begum [2] introduced the concepts of degree, regular and irregular IFG.

In this paper we further study the intuitionistic fuzzy graphs of second type and some of their properties. In Section 2, we give some basic definitions and in Section 3, we define the irregular and complement of intuitionistic fuzzy graphs of second type. Also establish some of their properties. The paper is concluded in Section 4.

2 Preliminaries

In this section, we give some basic definitions.

Definition 2.1. [4] An Intuitionistic Fuzzy Graph (IFG) is of the form $G = [V, E]$ where
(i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1 : V \longrightarrow [0, 1]$ and $\nu_1 : V \longrightarrow [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$ for every $v_i \in V$, $(i = 1, 2, \ldots, n)$,
(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \longrightarrow [0, 1]$ and $\nu_2 : V \times V \longrightarrow [0, 1]$ are such that $\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$, $\nu_2(v_i, v_j) \leq \max[\nu_1(v_i), \nu_1(v_j)]$ and $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$, $(i, j = 1, 2, \ldots, n)$.

Definition 2.2. [2] Let $G = [V, E]$ be an IFG then the degree of a vertex v is defined by $d(v) = (d_\mu(v), d_\nu(v))$ where $d_\mu(v) = \sum_{u \neq v} \mu_2(v, u)$ and $d_\nu(v) = \sum_{u \neq v} \nu_2(v, u)$.

Definition 2.3. [2] The minimum degree of G is $\delta(G) = [\delta_\mu(G), \delta_\nu(G)]$, where $\delta_\mu(G)$ $= \min\{d_\mu(v)/v \in V\}$ and $\delta_\nu(G) = \min\{d_\nu(v)/v \in V\}$.

Definition 2.4. [2] The maximum degree of G is $\Delta(G) = [\Delta_\mu(G), \Delta_\nu(G)]$ where $\Delta_\mu(G)$ $= \max\{d_\mu(v)/v \in V\}$ and $\Delta_\nu(G) = \max\{d_\nu(v)/v \in V\}$

Definition 2.5. [2] An IFG $G = [V, E]$ is said to be regular, if every vertex adjacent to vertices with same degrees.

Definition 2.6. [3] Let $G = [V, E]$ be an IFG. Then G is irregular, if there is a vertex which is adjacent to vertices with distinct degrees.

Definition 2.7. [3] Let $G = [V, E]$ be a connected IFG. Then G is neighbourly irregular IFG, if every two adjacent vertices of G have distinct degree.

Definition 2.8. [3] Let $G = [V, E]$ be a connected IFG. Then G is highly irregular IFG, if every vertex of G is adjacent to vertices with distinct degrees.

Definition 2.9. [4] The complement of an IFG $G = [V, E]$ is denoted by $\overline{G} = [\overline{V}, \overline{E}]$ and is defined as
(i) $\overline{\mu_1}(v) = \mu_1(v)$ and $\overline{\nu_1}(v) = \nu_1(v)$ for every $v \in V$;
(ii) $\overline{\mu_2}(v_1, v_2) = \min(\mu_1(v_1), \mu_1(v_2)) - \mu_2(v_1, v_2)$ and $\overline{\nu_2}(v_1, v_2) = \max(\nu_1(v_1), \nu_1(v_2)) - \nu_2(v_1, v_2)$ for all $(v_1, v_2) \in E$.

Definition 2.10. [7] An Intuitionistic Fuzzy Graphs of Second Type (IFGST) is of the form $G = [V, E]$ where
(i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1 : V \longrightarrow [0, 1]$ and $\nu_1 : V \longrightarrow [0, 1]$ denote the

degree of membership and non-membership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i)^2 + \nu_1(v_i)^2 \leq 1$ for every $v_i \in V$, $(i = 1, 2, \ldots, n)$,
(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \longrightarrow [0, 1]$ and $\nu_2 : V \times V \longrightarrow [0, 1]$ are such that $\mu_2(v_i, v_j) \leq \min[\mu_1(v_i)^2, \mu_1(v_j)^2]$, $\nu_2(v_i, v_j) \leq \max[\nu_1(v_i)^2, \nu_1(v_j)^2]$ and $0 \leq \mu_2(v_i, v_j)^2 + \nu_2(v_i, v_j)^2 \leq 1$ for every $(v_i, v_j) \in E$, $(i, j = 1, 2, \ldots, n)$.

Definition 2.11. [8] Let $G = [V, E]$ is an IFGST then the degree of a vertex in G is denoted by $d(v)$ and defined as, $d(v) = [d_\mu(v), d_\nu(v)]$ where $d_\mu(v) = \sum_{u \neq v} \mu_2(v, u)$ and $d_\nu(v) = \sum_{u \neq v} \nu_2(v, u)$ for all $u, v \in V$.

Definition 2.12. [8] Let $G = [V, E]$ is an IFGST then the minimum degree of G is denoted by $\delta(G)$ and defined as, $\delta(G) = [\delta_\mu(G), \delta_\nu(G)]$ where $\delta_\mu(G) = \min\{d_\mu(v)/v \in V\}$ and $\delta_\nu(G) = \min\{d_\nu(v)/v \in V\}$.

Definition 2.13. [8] Let $G = [V, E]$ is an IFGST then the maximum degree of G is denoted by $\Delta(G)$ and defined as, $\Delta(G) = [\Delta_\mu(G), \Delta_\nu(G)]$ where $\Delta_\mu(G) = \max\{d_\mu(v)/v \in V\}$ and $\Delta_\nu(G) = \max\{d_\nu(v)/v \in V\}$.

Definition 2.14. [9] An IFGST $G = [V, E]$ is said to be regular, if every vertex adjacent to vertices with same degree.

3 Irregular and complement of intuitionistic fuzzy graphs of second type

In this section, we define the irregular, complement of intuitionistic fuzzy graphs of second type and establish some of their properties.

Definition 3.1. An IFGST $G = [V, E]$ is said to be irregular, if there is a vertex which is adjacent to vertices with distinct degrees.

Example 3.1. In Figure 1. $d(v_1) = (0.40, 0.51)$, $d(v_2) = (0.20, 0.50)$, $d(v_3) = (0.44, 0.61)$ and $d(v_4) = (0.20, 0.50)$. Here, v_2 is adjacent to the vertices v_1 and v_3 with distinct degrees. Hence Figure 1 is an example of Irregular IFGST.

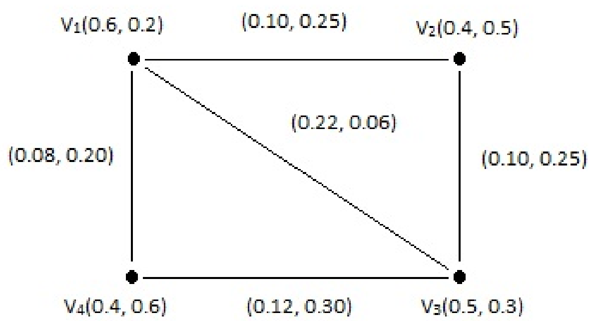

Figure 1. Irregular IFGST.

Definition 3.2. A connected IFGST $G = [V, E]$ is said to be neighbourly irregular IFGST, if every two adjacent vertices of G have distinct degree.

Example 3.2. In Figure 1. $d(v_1) = (0.40, 0.51)$, $d(v_2) = (0.20, 0.50)$, $d(v_3) = (0.44, 0.61)$ and $d(v_4) = (0.20, 0.50)$. Here, every two adjacent vertices having distinct degrees. Hence Figure 1. is also an example of neighbourly irregular IFGST.

Definition 3.3. An IFGST $G = [V, E]$ is said to be highly irregular IFGST, if every vertex of G is adjacent to vertices with distinct degrees.

Example 3.3. In Figure 2. $d(v_1) = (0.30, 0.43)$, $d(v_2) = (0.38, 0.27)$, $d(v_3) = (0.40, 0.20)$, $d(v_4) = (0.58, 0.17)$ and $d(v_5) = (0.54, 0.27)$. Here, every vertex of Figure 2 is adjacent to vertices with distinct degrees. Hence Figure 2 is an example of highly irregular IFGST.

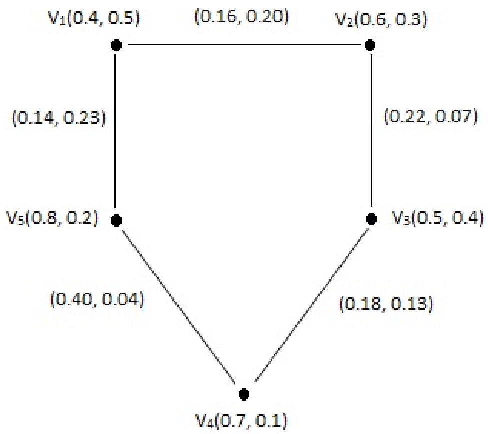

Figure 2. Highly Irregular IFGST.

Proposition 3.1. A highly irregular IFGST need not be a neighbourly irregular IFGST.

Proof. In Figure 3. $d(v_1) = (0.12, 0.56)$, $d(v_2) = (0.17, 0.46)$, $d(v_3) = (0.17, 0.46)$ and $d(v_4) = (0.12, 0.56)$. Here, every vertex of Figure 3 is adjacent to vertices with distinct degrees. Hence the graph is highly irregular IFGST. Also the adjacent vertices v_2 and v_3 has the same degree, i.e., $d(v_2) = d(v_3) = (0.17, 0.46)$, hence Figure 3 is not neighbourly irregular IFGST.

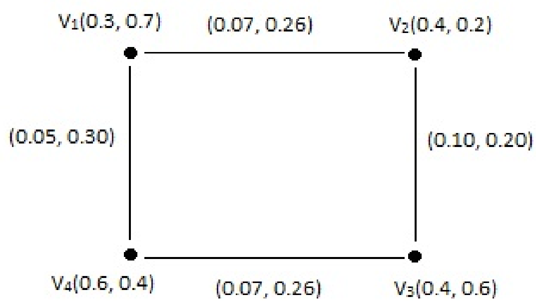

Figure 3.

Therefore a highly irregular IFGST need not be a neighbourly irregular IFGST. □

Proposition 3.2. A neighbourly irregular IFGST need not be a highly IFGST.

Proof. In Figure 1, $d(v_1) = (0.40, 0.51)$, $d(v_2) = (0.20, 0.50)$, $d(v_3) = (0.44, 0.61)$ and $d(v_4) = (0.20, 0.50)$. Here, every two adjacent vertices having distinct degrees, hence the graph is neighbourly irregular IFGST. But the adjacent vertices of v_1, has the same degree (i.e) $d(v_2) = d(v_4) = (0.20, 0.50)$. Therefore a neighbourly irregular IFGST need not be a highly IFGST. □

Theorem 3.1. Let $G = [V, E]$ be highly and neighbourly irregular IFGST if and only if the degrees of all vertices of G are distinct.

Proof. Let $G = [V, E]$ be an IFGST with $v_1, v_2, \ldots, v_n \in V$. Assume that G is highly irregular IFGST and neighbourly irregular IFGST.

Consider the adjacent vertices of v_1 be $v_2, v_3, \ldots, v_n$ with the degrees $(l_2, m_2), (l_3, m_3), \ldots, (l_n, m_n)$ respectively. $d(v_1)$ is not equal to either of $(l_2, m_2), (l_3, m_3), \ldots, (l_n, m_n)$ because G is neighbourly irregular IFGST. Since G is highly irregular IFGST, we have $l_2 \neq l_3 \neq, \ldots, \neq l_n$ and $m_2 \neq m_3 \neq, \ldots, \neq m_n$.

Therefore the degrees of all vertices of G are distinct.

Conversely, assume that the degrees of all vertices of G are distinct. Which implies that, every two adjacent vertices have distinct degrees and to every vertex, the adjacent vertices have distinct degrees. That is, G is neighbourly irregular IFGST and highly irregular IFGST. This completes the proof of the theorem. □

Definition 3.4. The complement of an IFGST $G = [V, E]$ is denoted by $\overline{G} = [\overline{V}, \overline{E}]$ and is defined as

(i) $\overline{\mu_1}(v) = \mu_1(v)$ and $\overline{\nu_1}(v) = \nu_1(v)$ for every $v \in V$

(ii) $\overline{\mu_2}(v_1, v_2) = \min(\mu_1^2(v_1), \mu_1^2(v_2)) - \mu_2(v_1, v_2)$ and $\overline{\nu_2}(v_1, v_2) = \max(\nu_1^2(v_1), \nu_1^2(v_2)) - \nu_2(v_1, v_2)$ for all $(v_1, v_2) \in E$.

Example 3.4.

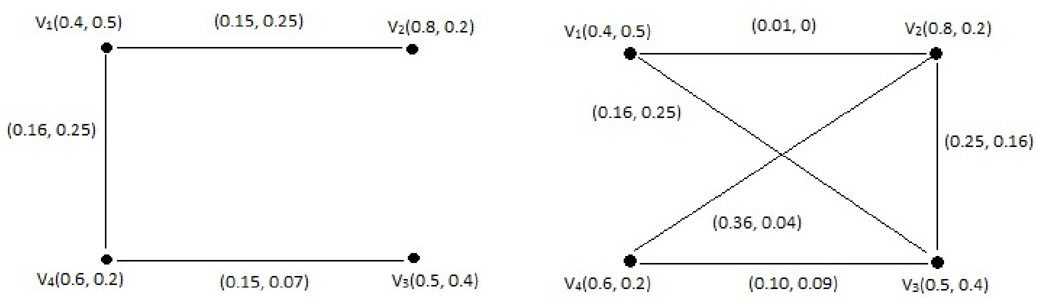

Figure 4. IFGST G and its Complement $\overline{G}$.

Proposition 3.3. Let G be an IFGST. If $G = [V, E]$ is neighbourly irregular then its complement $\overline{G}$ need not be neighbourly irregular.

Proof. In Figure 5. $d(v_1) = (0.12, 0.35)$, $d(v_2) = (0.18, 0.44)$, $d(v_3) = (0.22, 0.25)$ and $d(v_4) = (0.16, 0.16)$. Here, every two adjacent vertices has distinct degrees that is G is neighbourly irregular IFGST.

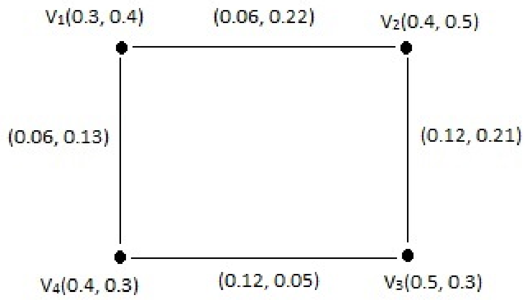

Figure 5. IFGST G.

In Figure 6. $d(v_1) = (0.36, 0.46)$, $d(v_2) = (0.47, 0.57)$, $d(v_3) = (0.38, 0.48)$ and $d(v_4) = (0.47, 0.57)$. Here, the vertices v_2 and v_4 has the same degree (i.e) $d(v_2) = d(v_4) = (0.47, 0.57)$. Hence the complement of G is not neighbourly irregular IFGST.

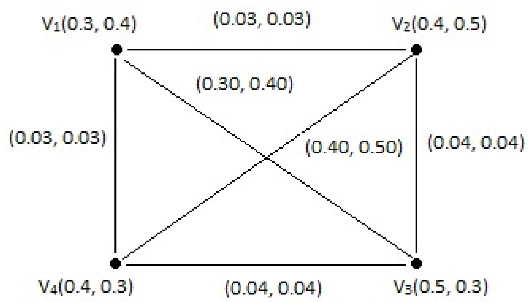

Figure 6. Complement of Figure 5.

Therefore, the complement of a neighbourly irregular IFGST need not be neighbourly irregular. □

4 Conclusion

In this paper, we have defined the irregular, complement of intuitionistic fuzzy graphs of second type. Also established some of their properties. In future, we will study some more properties and applications of IFGST.

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