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InterCriteria Analysis of Bat Algorithm with Parameter Adaptation Using Type-1 and Interval Type-2 Fuzzy Systems

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Abstract: In this paper, InterCriteria Analysis (ICrA) is considered for analysis of the results from application of Type-1 and Interval Type-2 fuzzy logic to dynamic adaptation of Bat Algorithm (BA) parameters. BA is applied to different optimization problems – five benchmark functions. The modification of the BA integrating Type-1 and Interval Type-2 fuzzy logic systems was successfully applied. The both fuzzy systems perform well even the increase of the benchmark functions complexity. Obtained results from ICrA show that the fuzzy systems, Type-1 and Interval Type-2, have similar performance in dynamic BA parameter adaptation.

Keywords: InterCriteria Analysis, Type-1 Fuzzy System, Interval Type-2 Fuzzy System, Bat Algorithm.

AMS Classification: 03E72.

1 Introduction

InterCriteria Analysis (ICrA) is an approach [1] aiming to go beyond the nature of the criteria involved in a process of evaluation of multiple objects against multiple criteria, and, on this basis, to discover any existing correlations between the criteria themselves. Given in details in [1], ICrA has been developed further in [8, 19].

Up to now ICrA has been applied in several problem fields, namely:

- for the purposes of temporal, threshold and trends analyses of an economic case-study of European Union member states' competitiveness [7, 9];
- analysis of Genetic Algorithms (GA) performance for parameter identification problems [14, 17];
- for evaluation of the performance of hybrid schemes using GA and Ant Colony Optimization (ACO) [16];
- for evaluation of pollution indicators of rivers [13];
- to universities ranking [10];
- in radar detection threshold analysis [11];
- for Neural Network preprocessing procedure [18];
- etc.

In this paper ICrA is applied for analysis of dynamic parameter adaptation for the Bat Algorithm (BA) using a Type-1 fuzzy system (T1FS) and Interval Type-2 fuzzy system (IT2FS) [15]. The modification of the BA with T1FS and IT2FS is a deeper integration of a fuzzy system that works in conjunction with the algorithm and its parameters. The modification of BA aims to reach optimal results leveraging the convergence speed of the algorithm and avoiding premature convergence to a local minimum. It is feasible to proceed with the study of the BA.

The BA is a metaheuristic optimization method proposed by Yang in 2010 [22]. tTis algorithm is based on the behavior of micro bats which use echolocation pulses with different emission and sound. There are some modifications of the BA in which the main feature of the algorithm is a convergence speed making it ideal for problem solving, where a quick solution is required [12, 20, 21, 23, 24].

This paper is organized as follows: in Section 2 we describe the background of ICrA, in Section 3 we present the results from T1FS and IT2FS parameter adaptation of BA, in Section 4 we propose the ICrA of T1FS and IT2FS parameter adaptation, in Section 5 we describe the conclusions.

2 Background of InterCriteria Analysis

Following [1] and [5] we will obtain an Intuitionistic Fuzzy Pair (IFP) [2] as the degrees of "agreement" and "disagreement" between two criteria applied on different objects. We remind briefly that an IFP is an ordered pair of real non-negative numbers $\langle a,b \rangle$ such that: $a+b \le 1$.

By O we denote the set of all objects $O_1, O_2, \ldots, O_n$ being evaluated, and by $C(O)$ the set of values assigned by a given criteria C to the objects, i.e.,

$$O \overset{\text{def}}{=} \{O_1, O_2, \ldots, O_n\},\ \ C(O) \overset{\text{def}}{=} \{C(O_1), C(O_2), \ldots, C(O_n)\}.$$

Let $x_i = C(O_i)$. Then the following set can be defined:

$$C^*(O) \overset{\text{def}}{=} \{\langle x_i, x_j \rangle \mid i \ne j\ \&\ \langle x_i, x_j \rangle \in C(O) \times C(O)\}.$$

Further, if $x = C(O_i)$ and $y = C(O_j)$, $x \prec y$ will be written iff $i < j$.

In order to compare two criteria we must construct the vector of all internal comparisons of each criteria, which fulfill exactly one of three relations R, $\bar{R}$ and $\tilde{R}$. In other words, we require that for a fixed criterion C and any ordered pair $\langle x, y\rangle \in C^*(O)$ it is true:

$$\langle x, y\rangle \in R \Leftrightarrow \langle y, x\rangle \in \bar{R}, \tag{1}$$

$$\langle x, y\rangle \in \tilde{R} \Leftrightarrow \langle x, y\rangle \notin (R \cup \bar{R}), \tag{2}$$

$$R \cup \bar{R} \cup \tilde{R} = C^*(O). \tag{3}$$

From the above it is seen that we need only consider a subset of $C(O) \times C(O)$ for the effective calculation of the vector of internal comparisons (denoted further by $V(C)$) since from (1) - (3) it follows that if we know what is the relation between x and y we also know what is the relation between y and x. Thus we will only consider lexicographically ordered pairs $\langle x, y\rangle$. Let, for brevity $C_{i,j} = \langle C(O_i), C(O_j)\rangle$. Then for a fixed criterion C we construct the vector:

$$V(C) = \{C_{1,2}, C_{1,3}, \ldots, C_{1,n}, C_{2,3}, C_{2,4}, \ldots, C_{2,n}, C_{3,4}, \ldots, C_{3,n}, \ldots, C_{n-1,n}\}.$$

It can be easily seen that it has exactly $n(n-1)/2$ elements. Further, to simplify our considerations, we replace the vector $V(C)$ with $\hat{V}(C)$, where for each $1 \le k \le n(n-1)/2$ for the k-th component it is true:

$$\widehat{V_k}(C) = \begin{cases} -1 \text{ iff } V_k(C) \in R, \\ -1 \text{ iff } V_k(C) \in \bar{R}, \\ \ 0 \text{ otherwise.} \end{cases}$$

Then when comparing two criteria we determine the degree of "agreement" between the two as the number of matching components (divided by the length of the vector for normalization purposes). This can be done in several ways, e.g. by counting the matches or by taking the complement of the Hamming distance. The degree of "disagreement" is the number of components of opposing signs in the two vectors (again normalized by the length). This also may be done in various ways. A pseudocode of the **Algorithm 1** [17] used in this study for calculating the degrees of agreement and disagreement between two criteria C and C' is presented in Fig. 1.

It is obvious (from the way of calculation) that for $\mu_{C,C'}$, $\nu_{C,C'}$, we have $\mu_{C,C'} = \mu_{C',C}$, $\nu_{C,C'} = \nu_{C',C}$. Also, $\langle \mu_{C,C'}, \nu_{C,C'}\rangle$ is an IFP. In the most of the obtained pairs $\langle \mu_{C,C'}, \nu_{C,C'}\rangle$, the sum $\mu_{C,C'} + \nu_{C,C'}$ is equal to 1. However, there may be some pairs, for which this sum is less than 1. The difference

$$\pi_{C,C'} = 1 - \mu_{C,C'} - \nu_{C,C'}$$

is considered as a degree of "uncertainty".

Require: Vectors $\hat{V}(C)$ and $\hat{V}(C')$

Degree of Agreement ($\hat{V}(C), \hat{V}(C')$)
    $V \leftarrow \hat{V}(C) - \hat{V}(C')$
    $\mu_{C,C'} \leftarrow 0$
    for $i \leftarrow 1$ to $\frac{n(n-1)}{2}$ **do**
        if $V_i = 0$ **then**
            $\mu_{C,C'} \leftarrow \mu_{C,C'} + 1$
        end if
    end for
    $\mu_{C,C'} \leftarrow \frac{2}{n(n-1)} \mu_{C,C'}$
    return $\mu_{C,C'}$
end function

Degree of Disagreement ($\hat{V}(C), \hat{V}(C')$)
    $V \leftarrow \hat{V}(C) - \hat{V}(C')$
    $\nu_{C,C'} \leftarrow 0$
    for $i \leftarrow 1$ to $\frac{n(n-1)}{2}$ **do**
        if $\text{abs}(V_i) = 2$ **then**
            $\nu_{C,C'} \leftarrow \nu_{C,C'} + 1$
        end if
    end for
    $\nu_{C,C'} \leftarrow \frac{2}{n(n-1)} \nu_{C,C'}$
    return $\nu_{C,C'}$
end function

Figure 1. **Algorithm 1:** Calculating "agreement" and "disagreement" between two criteria [17]

3 T1FS and IT2FS parameter adaptation of Bat Algorithm

3.1 Fuzzy adaptation of the BA parameters

The detailed description of the theoretical background of the applied BA and proposed fuzzy adaptation of the parameters are presented in [15].

For the purpose of fuzzy adaptation of the BA parameters the selected BA parameters are as follows: "*Iteration*", "*Beta*" and "*PulseRate*". These three parameters are integrated into the Type-1 fuzzy system and Interval Type-2 fuzzy system.

The "*Iteration*" variable is defined by the Eq. (4), and has a range from 0 to 1. This variable can be seen as the percentage of the current iteration.

$$Iteration = \frac{Current\ \ Iteration}{Maximum\ of\ \ Iterations} \tag{4}$$

The "*Beta*" variable is located between [0, 1] which is increasing with the stepiterations and the variable "*PulseRate*" value is between [0, 1], which is decreasing with the step iterations.

The main difference between a T1FS and an IT2FS, is that the degree of membership is also fuzzy, and is represented by the footprint of uncertainty (FOU), so if we shift from Type-1 to Type-2, theoretically we need a degree of FOU, so that this degree was manually modified until the best possible FOU is obtained.

Fig. 2 shows the rule set from the original T1FS for parameter adaptation. These rules stay the same in the change from T1FS to IT2FS. The set of If-Then fuzzy rules are granulated into 3 rules in order to cover all iterations and the search space. To start in Low "*Iteration*" the parameter "*Beta*" is Low and the "*PulseRate*" is High, going to the Middle "*Iteration*" the parameter "*Beta*" is Middle and "*PulseRate*" is Middle start to cover much of the search space, in the High "*Iteration*" the same procedure is repeated – the parameter "*Beta*" is High and "*PulseRate*" is Low, in this way achieving the exploitation of the search space.

1. If (*Iteration* is Low) then (*Beta* is Low) (*PulseRate* is High) (1)
2. If (*Iteration* is Middle) then (*Beta* is Middle) (*PulseRate* is Middle) (1)
3. If (*Iteration* is High) then (*Beta* is High) (*PulseRate* is Low) (1)

Figure 2. Rule set from original T1FS for parameter adaptation

The IT2FS is designed for the parameter adaptation. We develop this system manually, this is, we change the levels of FOU of each point of each membership function, but each point has the same level of FOU, also the input and output variables have only interval Type-2 triangular membership functions [15].

3.2 Simulation results

The results of the performed computational exeperiments with the five benchmark mathematical functions [15] used to evaluate the performance of the BA with T1FS are shown in next three tables. Table 1 presents the average results, Table 2 – the best results and Table 3 – the worst results. The row "Dim" represents the benchmark functions dimension.

Table 1. T1FS – average results

Dim	10	20	30	40	50
Sphere	2.45E-07	3.42E-06	7.93E-06	1.68E-05	2.69E-05
Ackley	8.16E-09	1.06E-08	1.56E-08	8.84E-09	6.76E-09
Rastrigin	2.71E-09	1.09E-09	6.43E-10	0.994959	2.33E-09
Zakharov	8.18E-12	1.68E-12	1.70E-11	8.33E-12	7.22E-12
SumSquare	1.56E-06	3.08E-05	0.000156	0.000411	0.001042
Dim	60	70	80	90	100
Sphere	3.99E-05	5.53E-05	7.68E-05	0.000106	9.34E-05
Ackley	1.68E-08	3.32E-08	1.72E-08	5.01E-09	3.51E-09
Rastrigin	1.12E-09	7.03E-10	7.47E-10	0.994959	5.07E-10
Zakharov	3.29E-12	2.27E-11	4.45E-12	2.83E-12	3.70E-12
SumSquare	0.049183	0.00553	0.091078	0.083557	1.288607

Table 2. T1FS – best results

Dim	10	20	30	40	50
Sphere	1.41E-07	3.11E-06	7.50E-06	1.60E-05	2.37E-05
Ackley	8.74E-10	9.90E-10	1.05E-08	3.17E-09	3.37E-09
Rastrigin	2.02E-10	1.66E-10	2.67E-10	0.994959	6.52E-11
Zakharov	2.56E-12	3.48E-13	3.97E-12	3.84E-12	1.95E-12
SumSquare	1.54E-06	2.31E-05	0.000141	0.000353	0.000732
Dim	60	70	80	90	100
Sphere	3.67E-05	4.70E-05	7.16E-05	9.47E-05	8.68E-05
Ackley	1.39E-08	4.66E-09	6.60E-10	4.00E-10	1.19E-09
Rastrigin	2.62E-10	2.05E-11	3.25E-10	0.994959	1.31E-10
Zakharov	1.80E-12	7.16E-12	2.95E-12	9.35E-13	1.26E-12
SumSquare	0.001994	0.003792	0.004906	0.01076	0.009488

Table 3. T1FS – worst results

Dim	10	20	30	40	50
Sphere	8.14E-07	3.76E-06	1.05E-05	2.27E-05	4.08E-05
Ackley	8.81E-09	1.32E-07	2.57E-08	2.44E-08	1.04E-08
Rastrigin	2.24E-08	7.15E-09	1.04E-08	0.994959	1.43E-08
Zakharov	3.74E-11	1.60E-11	1.70E-10	7.89E-11	2.40E-11
SumSquare	1.62E-06	5.61E-05	0.000337	0.001209	0.003227
Dim	60	70	80	90	100
Sphere	5.49E-05	8.25E-05	1.16E-04	1.43E-04	1.33E-04
Ackley	5.69E-08	1.33E-07	3.27E-07	3.50E-08	6.58E-08
Rastrigin	2.01E-08	2.87E-09	3.96E-09	0.994959	1.20E-09
Zakharov	3.93E-12	2.52E-10	1.79E-11	1.26E-11	1.56E-11
SumSquare	1.405737	0.025541	2.527893	2.102634	38.28994

The results of the performed computational exeperiments with the five benchmark mathematical functions [15] used to evaluate the performance of the BA with IT2FS are shown, respectivelly in Table 4 (average results), Table 5 (best results) and Table 6 (worst results).

The application of IT2FS in BA, as shown in the above tables, obtains the best values for the most benchmark functions [15]. Considering the average results the better results for the most benchmark functions shows BA with T1FS. This does not mean that one is better than the other. The results show that the application of IT2FS in the BA parameters adaptation is as effective approach as T1FS in the BA parameters adaptation. Implementation of IT2FS gives us the possibility to attack more complex problems certainty skilled to find good solutions integrating higher level uncertainty.

Table 4. IT2FS – average results

Dim	10	20	30	40	50
Sphere	3.17E-07	2.90E-06	8.88E-06	1.72E-05	2.74E-05
Ackley	3.84E-08	1.20E-08	1.94E-08	1.92E-09	1.50E-08
Rastrigin	9.95E-01	9.95E-01	7.00E-10	1.9899	9.95E-01
Zakharov	6.93E-12	1.06E-11	1.64E-11	7.69E-12	2.90E-12
SumSquare	1.78E-06	2.91E-05	0.001949	0.039674	0.464369
Dim	60	70	80	90	100
Sphere	4.45E-05	5.40E-05	7.25E-05	9.35E-05	1.29E-04
Ackley	1.53E-08	5.51E-09	6.42E-09	9.37E-09	9.82E-09
Rastrigin	1.00E-09	9.95E-01	1.99E+00	1.9899	9.95E-01
Zakharov	7.17E-12	7.86E-12	1.68E-11	3.73E-12	7.63E-12
SumSquare	18.52372	29.50533	101.8667	243.2728	200.1128

Table 5. IT2FS – best results

Dim	10	20	30	40	50
Sphere	1.57E-07	2.57E-06	8.55E-06	1.52E-05	2.52E-05
Ackley	1.54E-08	3.15E-09	8.82E-09	3.25E-10	1.49E-09
Rastrigin	9.95E-01	9.95E-01	8.00E-11	1.9899	9.95E-01
Zakharov	8.30E-14	1.70E-12	3.52E-12	4.53E-13	1.72E-13
SumSquare	1.25E-06	2.54E-05	0.000126	0.000302	0.000726
Dim	60	70	80	90	100
Sphere	3.66E-05	5.10E-05	6.02E-05	8.18E-05	1.14E-04
Ackley	7.03E-09	1.54E-09	2.27E-09	4.20E-09	2.09E-09
Rastrigin	3.00E-10	9.95E-01	1.99E+00	1.9899	9.95E-01
Zakharov	1.31E-12	1.00E-12	2.15E-12	2.20E-13	3.78E-13
SumSquare	0.001424	0.003745	0.007301	0.008817	0.015054

Table 6. IT2FS – worst results

Dim	10	20	30	40	50
Sphere	6.27E-07	4.83E-06	1.22E-05	2.08E-05	4.47E-05
Ackley	3.56E-07	2.23E-08	1.37E-07	2.42E-08	1.69E-07
Rastrigin	9.95E-01	9.95E-01	4.00E-09	1.9899	9.95E-01
Zakharov	6.49E-11	5.13E-11	9.29E-11	6.02E-11	8.21E-11
SumSquare	3.48E-06	7.51E-05	0.054243	1.180622	13.90583
Dim	60	70	80	90	100
Sphere	6.06E-05	7.24E-05	1.19E-04	1.21E-04	1.92E-04
Ackley	3.02E-08	8.98E-09	1.53E-08	3.79E-08	1.55E-07
Rastrigin	8.00E-09	9.95E-01	1.99E+00	1.9899	9.95E-01
Zakharov	1.29E-10	4.95E-11	8.35E-11	2.53E-11	1.55E-10
SumSquare	555.6535	884.9757	3044.999	7186.608	5960.815

4 ICrA of T1FS and IT2FS parameter adaptation of Bat Algorithm

Based on **Algorithm 1** ICrA has been applied on the results presented in Tables 1-6. The ten different function dimensions, 10, 20, 30, 40, 50, 60, 70, 80, 90 and 100, are considered as ICrA "objects". The five benchmark functions are considered as ICrA "criteria". Presented Tables 1-6 are considered as six Index Matrices [3, 4]. Obtained degrees of "agreement" ($\mu_{C,C'}$), degrees of "disagreement" ($\nu_{C,C'}$) and degrees of "uncertainty" ($\pi_{C,C'}$), in the case of BA with T1FS and BA with IT2FS, are as follows:

Table 7. ICrA for T1FS – average results

$\mu_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	1.00	0.49	0.40	0.38	0.93
Ackley	0.49	1.00	0.36	0.67	0.47
Rastrigin	0.40	0.36	1.00	0.47	0.38
Zakharov	0.38	0.67	0.47	1.00	0.40
Sum Square	0.93	0.47	0.38	0.40	1.00

$\nu_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	0.00	0.51	0.58	0.62	0.07
Ackley	0.51	0.00	0.62	0.33	0.53
Rastrigin	0.58	0.62	0.00	0.51	0.60
Zakharov	0.62	0.33	0.51	0.00	0.60
Sum Square	0.07	0.53	0.60	0.60	0.00

$\pi_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	0.00	0.00	0.02	0.00	0.00
Ackley	0.00	0.00	0.02	0.00	0.00
Rastrigin	0.02	0.02	0.00	0.02	0.02
Zakharov	0.00	0.00	0.02	0.00	0.00
Sum Square	0.00	0.00	0.02	0.00	0.00

Table 8. ICrA for T1FS – best results

$\mu_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	1.000	0.444	0.533	0.400	1.000
Ackley	0.444	1.000	0.311	0.644	0.444
Rastrigin	0.533	0.311	1.000	0.489	0.533
Zakharov	0.400	0.644	0.489	1.000	0.400
Sum Square	1.000	0.444	0.533	0.400	1.000

$\nu_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	0.000	0.556	0.444	0.600	0.000
Ackley	0.556	0.000	0.667	0.356	0.556
Rastrigin	0.444	0.667	0.000	0.489	0.444
Zakharov	0.600	0.356	0.489	0.000	0.600
Sum Square	0.000	0.556	0.444	0.600	0.000

$\pi_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	0.00	0.00	0.02	0.00	0.00
Ackley	0.00	0.00	0.02	0.00	0.00
Rastrigin	0.02	0.02	0.00	0.02	0.02
Zakharov	0.00	0.00	0.02	0.00	0.00
Sum Square	0.00	0.00	0.02	0.00	0.00

Table 9. ICrA for T1FS – worst results

$\mu_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	1.000	0.667	0.400	0.333	0.933
Ackley	0.667	1.000	0.244	0.489	0.689
Rastrigin	0.400	0.244	1.000	0.467	0.378
Zakharov	0.333	0.489	0.467	1.000	0.356
Sum Square	0.933	0.689	0.378	0.356	1.000

$\nu_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	0.000	0.333	0.578	0.667	0.067
Ackley	0.333	0.000	0.733	0.511	0.311
Rastrigin	0.578	0.733	0.000	0.511	0.600
Zakharov	0.667	0.511	0.511	0.000	0.644
Sum Square	0.067	0.311	0.600	0.644	0.000

$\pi_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	0.00	0.00	0.02	0.00	0.00
Ackley	0.00	0.00	0.02	0.00	0.00
Rastrigin	0.02	0.02	0.00	0.02	0.02
Zakharov	0.00	0.00	0.02	0.00	0.00
Sum Square	0.00	0.00	0.02	0.00	0.00

Table 10. ICrA for IT2FS – average results

$\mu_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	1.00	0.36	0.47	0.49	0.98
Ackley	0.36	1.00	0.09	0.42	0.33
Rastrigin	0.47	0.09	1.00	0.33	0.49
Zakharov	0.49	0.42	0.33	1.00	0.47
Sum Square	0.98	0.33	0.49	0.47	1.00

$\nu_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	0.000	0.644	0.244	0.511	0.022
Ackley	0.644	0.000	0.622	0.578	0.667
Rastrigin	0.244	0.622	0.000	0.378	0.222
Zakharov	0.511	0.578	0.378	0.000	0.533
Sum Square	0.022	0.667	0.222	0.533	0.000

$\pi_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	0.00	0.00	0.29	0.00	0.00
Ackley	0.00	0.00	0.29	0.00	0.00
Rastrigin	0.29	0.29	0.00	0.29	0.29
Zakharov	0.00	0.00	0.29	0.00	0.00
Sum Square	0.00	0.00	0.29	0.00	0.00

Table 11. ICrA for IT2FS – best results

$\mu_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	1.000	0.400	0.467	0.489	1.000
Ackley	0.400	1.000	0.200	0.556	0.400
Rastrigin	0.467	0.200	1.000	0.267	0.467
Zakharov	0.489	0.556	0.267	1.000	0.489
Sum Square	1.000	0.400	0.467	0.489	1.000

$\nu_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	0.000	0.600	0.244	0.511	0.000
Ackley	0.600	0.000	0.511	0.444	0.600
Rastrigin	0.244	0.511	0.000	0.444	0.244
Zakharov	0.511	0.444	0.444	0.000	0.511
Sum Square	0.000	0.600	0.244	0.511	0.000

$\pi_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	0.00	0.00	0.29	0.00	0.00
Ackley	0.00	0.00	0.29	0.00	0.00
Rastrigin	0.29	0.29	0.00	0.29	0.29
Zakharov	0.00	0.00	0.29	0.00	0.00
Sum Square	0.00	0.00	0.29	0.00	0.00

Table 12. ICrA for IT2FS – worst results

$\mu_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	1.000	0.444	0.467	0.556	0.978
Ackley	0.444	1.000	0.267	0.622	0.422
Rastrigin	0.467	0.267	1.000	0.200	0.489
Zakharov	0.556	0.622	0.200	1.000	0.533
Sum Square	0.978	0.422	0.489	0.533	1.000

$\nu_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	0.000	0.556	0.244	0.444	0.022
Ackley	0.556	0.000	0.444	0.378	0.578
Rastrigin	0.244	0.444	0.000	0.511	0.222
Zakharov	0.444	0.378	0.511	0.000	0.467
Sum Square	0.022	0.578	0.222	0.467	0.000

$\pi_{C,C'}$	Sphere	Ackley	Rastrigin	Zakharov	Sum Square
Sphere	0.00	0.00	0.29	0.00	0.00
Ackley	0.00	0.00	0.29	0.00	0.00
Rastrigin	0.29	0.29	0.00	0.29	0.29
Zakharov	0.00	0.00	0.29	0.00	0.00
Sum Square	0.00	0.00	0.29	0.00	0.00

Presented results show that in case of BA with T1FS there is a very small degree of uncertainty”, $\pi_{C,C'} = 0.2$ for results of *Rastrigin* function. For the same benchmark function, in case of BA with IT2FS the $\pi_{C,C'}$-value is larger, $\pi_{C,C'} = 0.29$. Only in case of *Rastrigin* function both BA (BA with T1FS and BA with IT2FS) fall several times within the same local minimum for the various dimensions – 0.994959.

To compare performance of BA with T1FS and BA with IT2FS for all five benchmark functions, the obtained results for $\mu_{C,C'}$-values are summarized, respectively for average, best and worst values. The results are presented in the Tables 13–15.

The results are analyzed and discussed according to the proposed in [6] scale for a definition of consonance and dissonance between each pair of criteria. The following notations are used: D – dissonance, SD – strong dissonance, PC – positive consonance, WPD – weak positive consonance, WD – weak dissonance, SPC – strong positive consonance, WNC – weak negative consonance and NC – negative consonance.

Table 13. ICrA – $\mu_{C,C'}$-values for average results

Benchmark function relation	T1FS		IT2FS
Ackley-Sphere	0.356	D – SD	0.489
Rastrigin-Sphere	0.467	SD – D	0.400
Zakharov-Sphere	0.489	SD – D	0.378
Sum Square-Sphere	0.978	SPC – PC	0.933
Rastrigin-Ackley	0.089	NC – D	0.356
Zakharov-Ackley	0.422	D – D	0.667
Sum Square-Ackley	0.333	D – SD	0.467
Zakharov-Rastrigin	0.333	D – SD	0.467
Sum Square-Rastrigin	0.489	SD – D	0.378
Sum Square-Zakharov	0.467	D – D	0.400

Table 14. ICrA – $\mu_{C,C'}$-values for best results

Benchmark function relation	T1FS		IT2FS
Ackley-Sphere	0.444	SD – D	0.400
Rastrigin-Sphere	0.533	SD – SD	0.467
Zakharov-Sphere	0.400	D – SD	0.489
Sum Square-Sphere	1.000	SPC – SPC	1.000
Rastrigin-Ackley	0.311	WD – WNC	0.200
Zakharov-Ackley	0.644	D – SD	0.556
Sum Square-Ackley	0.444	SD – D	0.400
Zakharov-Rastrigin	0.489	SD – WD	0.267
Sum Square-Rastrigin	0.533	SD – SD	0.467
Sum Square-Zakharov	0.400	D – SD	0.489

Table 15. ICrA – $\mu_{C,C'}$-values for worst results

Benchmark function relation	T1FS		IT2FS
Ackley-Sphere	0.667	D – SD	0.444
Rastrigin-Sphere	0.400	D – SD	0.467
Zakharov-Sphere	0.333	D – SD	0.556
Sum Square-Sphere	0.933	PC – SPC	0.978
Rastrigin-Ackley	0.244	WNC – WD	0.267
Zakharov-Ackley	0.489	SD – D	0.622
Sum Square-Ackley	0.689	WD – D	0.422
Zakharov-Rastrigin	0.467	SD – WNC	0.200
Sum Square-Rastrigin	0.378	D – SD	0.489
Sum Square-Zakharov	0.356	D – SD	0.533

Based on the obtained $\mu_{C,C'}$-values we can compare the performance of the applied BA with T1FS and IT2FS. According Table 13, the average results the BA with T1FS performs slightly different compared to BA with IT2FS considering benchmark functions *Rastrigin* and *Ackley*. If we refer Table 1 it can be seen that the BA with T1FS performs better compared to BA with IT2FS in case of both functions *Rastrigin* and *Ackley*. For the rest benchmark functions both BA perform identically with the increase of function dimension, i.e., function complexity.

The results in Table 14 show that considering the best results both BA perform again identically with the increase of function dimension, i.e., function complexity.

Analysis of the worst results (Table 15) shows analogical behavior with the average and best results. However, in this case BA with T1FS performs slightly different compared to BA with IT2FS considering benchmark functions *Rastrigin* and *Zakharov*.

5 Conclusion

In this investigation ICrA approach is applied, employing the apparatuses of Index Matrices and Intuitionistic Fuzzy Sets. ICrA works based on an existing index matrix with multiobject multicriteria evaluations aiming to produce a new index matrix that contains intuitionistic fuzzy pairs with the correlations revealed to exist in between the set of evaluation criteria. Here, ICrA is implemented for comparison of the performance of BA modified with T1FS and IT2FS for BA parameter adaptation. The five benchmark function, namely *Sphere*, *Ackley*, *Rastrigin*, *Zakharov* and *SumSquare*, are considered. The ICrA show that both BA – BA with T1FS and BA with IT2FS have the similar performance with the increase of the benchmark function complexity. However, considering the average results BA with T1FS performs better in comparison with BA with IT2FS, but considering the obtained best values BA with IT2FS show better performance in comparison with T1FS BA parameter adaptation.

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