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AN ALGORITHM FOR CONSTRUCTING OF GENERALIZED NETS ON BASE OF CASE STUDIES. Part 1

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1 Introduction

A quarter of century ago the Generalized Nets (GNs, see [1, 4]) were defined as extensions of Petri nets (see, e.g. [5]) and other Petri net extensions and modifications.

The ideas and algorithms for Petri net- and GN-construction are based on expert knowledge for real parallel processes and for the conditions influencing their flow.

Here, and in future authors research, some algorithms, based on case study ideology, will be described.

2 Main idea: definitions and the first algorithm

Let us have a set of data

$$D = \{d_1, d_2, ..., d_m\}$$

for events in the frames of a fixed real process. Let these data be obtained on the basis of observations of one or more objects in different places and in different time-moments. Let the time moments be ordered sequentially and let them be represented by natural numbers. Then the data for the i-th object has the form:

$$d_i = \{(e_{i,1}, c_{i,1}), (e_{i,2}, c_{i,2}), ..., (e_{i,n}, c_{i,n})\},$$

where
$e_{i,j} \in E \cup \{*_e\}$ for $1 \leq i \leq m$ and $1 \leq j \leq n$, E is a set of events, $*_e$ is the empty event (lack of events, non-held event),
$c_{i,j} \in C \cup \{*_c\}$ for $1 \leq i \leq m$ and $1 \leq j \leq n$, C is a set of events characteristics, $*_c$ is the empty characteristic (lack of characteristic).

Let us assume that if for $1 \leq i_1 < i_2 \leq m$ and for $1 \leq j \leq n$: $e_{i_1,j} = e_{i_2,j}$, then $c_{i_1,j} = c_{i_2,j}$.

We can also assume that if for some (i, j) $(1 \leq i \leq m$ and $1 \leq j \leq n)$: $e_{i,j} = *_e$, then $c_{i,j} = *_c$.

Let

$$V_j = \{e_{i,j} \mid 1 \leq i \leq m\},$$
$$Y_j = \{c_{i,j} \mid 1 \leq i \leq m\},$$

for $1 \leq j \leq n$.

When event $e_{i,j}$ has exactly one predecessor and one successor, we define functions for $1 \leq i \leq m$ and $1 \leq j \leq n$:

$$w^-(e_{i,j}) = e_{i-1,j}, \text{ for } i \geq 2,$$
$$w^+(e_{i,j}) = e_{i+1,j}, \text{ for } i \leq m-1.$$

When object $e_{i,j}$ can have more than one predecessor and/or more than one successor, we define functions:

$$W^-(e_{i,j}) = \{e \mid e = w^-(e_{i,j})\}, \text{ for } i \geq 2,$$
$$W^+(e_{i,j}) = \{e \mid e = w^+(e_{i,j})\}, \text{ for } i \leq m-1.$$

Obviously, for each (i, j) $(1 \leq i \leq m$ and $1 \leq j \leq n)$:

$$w^-(e_{i,j}) \in W^-(e_{i,j}),$$

$$w^+(e_{i,j}) \in W^+(e_{i,j}).$$

The graphical form of the data is illustrated on Figure 1, when, e.g.

$$d_1 = \{(v_{1,1}, c_{1,1}), (v_{2,2}, c_{2,2}), (v_{2,3}, c_{2,3}), (v_{2,4}, c_{2,4}\},$$

$$d_2 = \{(v_{1,1}, c_{1,1}), (v_{2,2}, c_{2,2}), (v_{1,3}, c_{1,3}), (v_{1,4}, c_{1,4}\},$$
$$d_3 = \{(v_{2,1}, c_{2,1}), (v_{1,2}, c_{1,2}), (v_{1,3}, c_{1,3}), (v_{1,4}, c_{1,4}\},$$
$$d_4 = \{(v_{3,1}, c_{3,1}), (v_{2,2}, c_{2,2}), (v_{3,3}, c_{3,3}), (v_{2,4}, c_{2,4}\},$$
$$d_5 = \{(v_{4,1}, c_{4,1}), (v_{3,2}, c_{3,2}), (v_{4,3}, c_{4,3}), (v_{2,4}, c_{2,4}\},$$

where $v_{i,j} \in V_j$ for $1 \leq i \leq m$ and $1 \leq j \leq n$.

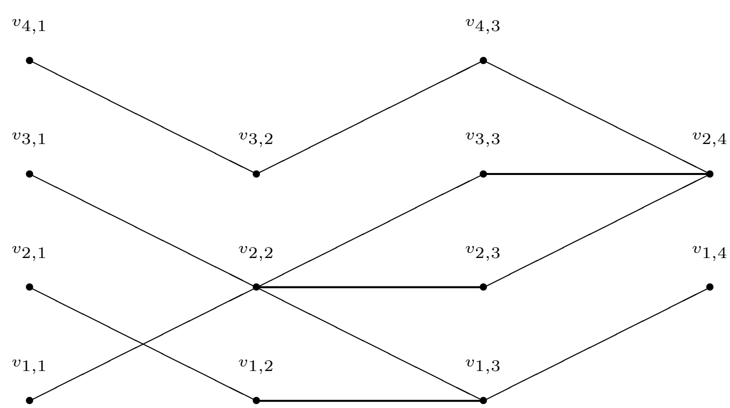

Figure 1.

Here and in future research we shall construct algorithms generating GNs on the basis of the graphs that we can construct, having the above information.

In the present research we shall assume that

1. for each (i, j) $(1 \le i \le m$ and $1 \le j \le n)$: $e_{i,j} \ne *_e$;
2. for every three i, j_1, j_2 $(1 \le i \le m$ and $1 \le j_1, j_2 \le n$ and $j_1 \ne j_2)$: $e_{i,j_1} \ne e_{i,j_2}$;
3. for each (i, j) $(1 \le i \le m$ and $1 \le j \le n)$: $|W^+(e_{i,j})| = 1$, where $|X|$ is the cardinality of set X.

In next authors' research we shall omit some of these restrictions, generalizing the present algorithm.

We shall construct the set of couples

$$\{(P_{j,k}, Q_{j,k}) \mid 1 \le k \le s_j, 1 \le j \le n-1\},$$

where $1 \le s_j \le m$ for $1 \le j \le n-1$. The elements of this set satisfy the condition

$$(\forall j : 1 \le j \le n-1)((\forall k : 1 \le k \le s_j)(\forall l : 1 \le l \le s_j)$$

$$(k \ne l \to (P_{j,k} \cap P_{j,l} = \emptyset \ \& \ Q_{j,k} \cap Q_{j,l} = \emptyset))$$

$$\& \ (\bigcup_{k=1}^{s_j} P_{j,k} = V_j \ \& \ \bigcup_{k=1}^{s_j} Q_{j,k} = V_{j+1})).$$

Therefore, the equality

$$\bigcup_{k=1}^{s_{j+1}} P_{j+1,k} = \bigcup_{k=1}^{s_j} Q_{j,k}$$

holds for each j $(1 \le j \le n-1)$.

The algorithm for construction of $P_{j,k}$ and $Q_{j,k}$ for $1 \le k \le s_j$ and j $(1 \le j \le n-1)$ is the following.

When $J + K = 2$, i.e., $J = K = 1$ we obtain the first step of the algorithm, as it is shown above.

Let us assume that the sets $P_{j,k}$ and $Q_{j,k}$ are constructed for $j < J$ and $k < K$ for $J + K > 2$. Then we construct set

$$X = V_J - \bigcup_{k=1}^{K-1} P_{J,k}$$

and let $e_{u,v}$ is the minimal element of X. Then we construct sequentially the sets

$$Q'_{J,K} = W^+(e_{u,v});$$

$$P_{J,K} = \bigcup_{e_{p,q} \in Q'_{J,K}} W^-(e_{p,q});$$

$$Q_{J,K} = \bigcup_{e_{p,q} \in P_{J,K}} W^+(e_{p,q}).$$

If

$$P_{J,K} = \emptyset$$

or

$$Q_{J,K} = \emptyset$$

we eliminate this step and the next step starts for $J+1$, if $J < n$. If $J = n$ the algorithm stops.

Now, using the ideas of the algorithm from [3], we can construct the graphical structure of the GN-transition

$$Z_{P_{j,k},Q_{j,k}} \langle L'_Z, L"_Z, t_1^Z, t_2^Z, r_Z, M_Z, \Box_Z \rangle,$$

that corresponds to sets $P_{j,k}$ and $Q_{j,k}$ for $1 \leq k \leq s_j$ and j $(1 \leq j \leq n-1)$. For it the set of input places L'_Z and the set of output places $L"_Z$ satisfy the equalities:

$$L'_Z = P_{j,k},$$

$$L"_Z = Q_{j,k}.$$

For this transition

$$t_1^Z = *$$

(i.e., this component is not defined) and

$$t_2^Z = t^o$$

(i.e., this component is equal to the elementary time-step; without problems we can assume that it is equal to 1). Transition condition has the form of an index matrix (see[2]):

$$r = \begin{array}{c|c} & l''_1 \ \ldots \ l''_j \ \ldots \ l''_n \\ \hline l'_1 & \\ \vdots & r_{i,j} \\ l'_i & (r_{i,j} \ \ - \ \text{predicate} \ ) \\ \vdots & (1 \leq i \leq m, 1 \leq j \leq n) \\ l'_m & \end{array},$$

where $r_{i,j}$ is the predicate which corresponds to the i-th input and j-th output places and its form will be determined following [3]. The index matrix M of the capacities of transition's arcs

$$M = \begin{array}{c|c} & l''_1 \ \ldots \ l''_j \ \ldots \ l''_n \\ \hline l'_1 & \\ \vdots & m_{i,j} \\ l'_i & (m_{i,j} \geq 0 - \ \text{natural number} \ ) \\ \vdots & (1 \leq i \leq m, 1 \leq j \leq n) \\ l'_m & \end{array}$$

is determined analogously. Finally,

$$\Box_Z = \bigvee_{e_{p,q} \in P_{J,K}} e_{p,q}.$$

On the basis of the above data $\{d_1, d_2, d_3, d_4, d_5\}$ we can construct the GN from Figure 2.

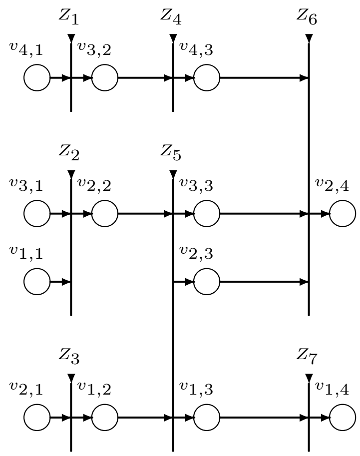

Figure 2.

The characteristic function connected with place $e_{i,j}$ determines value $c_{i,j}$. The transitions, places and tokens do not have priorities, and places and arcs do not have capacities. The GN is without temporal components.

3 Conclusion

In future research we shall construct new algorithms in which some of the above restrictions will be omitted. So, different ways for construction of GNs on the base of case study ideology, will be described.

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