

# Regular weakly generalized locally closed sets in intuitionistic fuzzy topological spaces

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**Abstract:** The purpose of this paper is to introduce and study the concepts of regular weakly generalized locally closed sets, regular weakly generalized locally continuous mappings in intuitionistic fuzzy topological spaces. Some of their properties are explored.

**Keywords:** Intuitionistic fuzzy topology, Intuitionistic fuzzy regular weakly generalized closed set, Intuitionistic fuzzy regular weakly generalized continuous mapping, Intuitionistic fuzzy regular weakly generalized locally closed sets, Intuitionistic fuzzy regular weakly generalized locally continuous mappings.

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## 1 Introduction

Fuzzy set (FS), proposed by Zadeh [13] in 1965, as a framework to encounter uncertainty, vagueness and partial truth, represents a degree of membership for each member of the universe of discourse to a subset of it. Padmapriya et.al [5] introduced and studied the concept of intuitionistic fuzzy locally closed sets, intuitionistic fuzzy  $\pi$ -locally closed sets and intuitionistic fuzzy  $\pi$ - $\beta$ -locally closed sets in 2011.

In this paper I introduce regular weakly generalized locally closed sets, regular weakly generalized locally continuous mappings in intuitionistic fuzzy topological spaces.

## 2 Preliminaries

**Definition 2.1.** [1] Let  $X$  be a non empty fixed set. An intuitionistic fuzzy set (IFS in short)  $A$  in  $X$  is an object having the form  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$  where the functions  $\mu_A(x) : X \rightarrow [0, 1]$  and  $\nu_A(x) : X \rightarrow [0, 1]$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of non-membership (namely  $\nu_A(x)$ ) of each element  $x \in X$  to the set  $A$ , respectively, and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for each  $x \in X$ . Denote by  $\text{IFS}(X)$ , the set of all intuitionistic fuzzy sets in  $X$ .

**Definition 2.2.** [1] Let  $A$  and  $B$  be IFSs of the form  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$  and  $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in X\}$ . Then

- (a)  $A \subseteq B$  if and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$ ,
- (b)  $A = B$  if and only if  $A \subseteq B$  and  $B \subseteq A$ ,
- (c)  $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X\}$ ,
- (d)  $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle \mid x \in X\}$ ,
- (e)  $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle \mid x \in X\}$ .

For the sake of simplicity, we shall use the notation  $A = \langle x, \mu_A, \nu_A \rangle$  instead of  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$ . Also for the sake of simplicity, we shall use the notation  $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$  instead of  $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$ .

The intuitionistic fuzzy sets  $0_{\sim} = \{\langle x, 0, 1 \rangle \mid x \in X\}$  and  $1_{\sim} = \{\langle x, 1, 0 \rangle \mid x \in X\}$  are respectively the empty set and the whole set of  $X$ .

**Definition 2.3.** [3] An intuitionistic fuzzy topology (IFT in short) on a non empty  $X$  is a family  $\tau$  of IFSs in  $X$  satisfying the following axioms:

- (a)  $0_{\sim}, 1_{\sim} \in \tau$ ,
- (b)  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$ ,
- (c)  $\cup G_i \in \tau$  for any arbitrary family  $\{G_i \mid i \in J\} \subseteq \tau$ .

In this case the pair  $(X, \tau)$  is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in  $\tau$  is known as an intuitionistic fuzzy open set (IFOS in short) in  $X$ .

The complement  $A^c$  of an IFOS  $A$  in an IFTS  $(X, \tau)$  is called an intuitionistic fuzzy closed set (IFCS in short) in  $X$ .

**Definition 2.4.** [3] Let  $(X, \tau)$  be an IFTS and  $A = \langle x, \mu_A, \nu_A \rangle$  be an IFS in  $X$ . Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

$$\text{int}(A) = \cup \{ G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$$

$$\text{cl}(A) = \cap \{ K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K \}.$$

Note that for any IFS  $A$  in  $(X, \tau)$ , we have  $\text{cl}(A^c) = (\text{int}(A))^c$  and  $\text{int}(A^c) = (\text{cl}(A))^c$  [13].

**Definition 2.5.** An IFS  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$  in an IFTS  $(X, \tau)$  is said to be an

- (a) [4] intuitionistic fuzzy semi closed set (IFSCS in short) if  $\text{int}(\text{cl}(A)) \subseteq A$ ,

- (b) [4] intuitionistic fuzzy  $\alpha$ -closed set (IF $\alpha$ CS in short) if  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$ ,
- (c) [4] intuitionistic fuzzy pre-closed set (IFPCS in short) if  $\text{cl}(\text{int}(A)) \subseteq A$ ,
- (d) [4] intuitionistic fuzzy regular closed set (IFRCS in short) if  $\text{cl}(\text{int}(A)) = A$ ,
- (e) [12] intuitionistic fuzzy generalized closed set (IFGCS in short) if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS,
- (g) [9] intuitionistic fuzzy  $\alpha$ generalized closed set (IF $\alpha$ GCS in short) if  $\alpha\text{cl}(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is an IFOS.

An IFS  $A$  is called intuitionistic fuzzy semi open set, intuitionistic fuzzy  $\alpha$ -open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy  $\alpha$  generalized open set (IFSOS, IF $\alpha$ OS, IFPOS, IFROS, IFGOS, IFGSOS and I $\alpha$ FGOS) if the complement  $A^c$  is an IFSCS, IF $\alpha$ CS, IFPCS, IFRCS, IFGCS, IFGSCS and IF $\alpha$ GCS respectively.

**Definition 2.6.** [6] An IFS  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$  in an IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy regular weakly generalized closed set (IFRWGCS in short) if  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFROS in  $X$ .

The family of all IFRWGCSs of an IFTS  $(X, \tau)$  is denoted by IFRWGCS( $X$ ).

**Definition 2.7.** [6] An IFS  $A$  is said to be an intuitionistic fuzzy regular weakly generalized open set (IFRWGOS in short) in  $(X, \tau)$  if the complement  $A^c$  is an IFRWGCS in  $X$ .

The family of all IFRWGOSs of an IFTS  $(X, \tau)$  is denoted by IFRWGO( $X$ ).

**Result 2.8.** [6] Every IFCS, IF $\alpha$ CS, IFGCS, IFRCS, IFPCS, IF $\alpha$ GCS is an IFRWGCS but the converses need not be true in general.

**Definition 2.9.** [7] Let  $(X, \tau)$  be an IFTS and  $A = \langle x, \mu_A, \nu_A \rangle$  be an IFS in  $X$ . Then the intuitionistic fuzzy regular weakly generalized interior and an intuitionistic fuzzy regular weakly generalized closure are defined by

$$\text{rwgint}(A) = \cup \{ G \mid G \text{ is an IFRWGOS in } X \text{ and } G \subseteq A \},$$

$$\text{rwgcl}(A) = \cap \{ K \mid K \text{ is an IFRWGCS in } X \text{ and } A \subseteq K \}.$$

**Definition 2.10.** [3] Let  $f$  be a mapping from an IFS  $X$  to an IFS  $Y$ . If  $B = \{ \langle y, \mu_B(y), \nu_B(y) \rangle \mid y \in Y \}$  is an IFS in  $Y$ , then the pre-image of  $B$  under  $f$  denoted by  $f^{-1}(B)$ , is the IFS in  $X$  defined by  $f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x)) \rangle \mid x \in X \}$ .

If  $A = \{ \langle x, \lambda_A(x), \nu_A(x) \rangle \mid x \in X \}$  is an IFS in  $X$ , then the image of  $A$  under  $f$  denoted by  $f(A)$  is the IFS in  $Y$  defined by  $f(A) = \{ \langle y, f(\lambda_A(y)), f_-(\nu_A(y)) \rangle \mid y \in Y \}$  where  $f_-(\nu_A) = 1 - f(1 - \nu_A)$ .

**Definition 2.11.** [8] A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy regular weakly generalized continuous (IFRWG continuous in short) if  $f^{-1}(B)$  is an IFRWGCS in  $(X, \tau)$  for every IFCS  $B$  of  $(Y, \sigma)$ .

**Definition 2.12.** [7] A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy regular weakly generalized irresolute (IFRWG irresolute in short) if  $f^{-1}(B)$  is an IFRWGCS in  $(X, \tau)$  for every IFRWGCS  $B$  of  $(Y, \sigma)$ .

**Definition 2.13.** A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$  is said to be an

- (a) [12] intuitionistic fuzzy closed mapping (IFCM for short) if  $f(A)$  is an IFCS in  $Y$  for every IFCS  $A$  in  $X$ ,
- (b) [4] intuitionistic fuzzy semi closed mapping (IFSCM for short) if  $f(A)$  is an IFSCS in  $Y$  for every IFCS  $A$  in  $X$ ,
- (c) [4] intuitionistic fuzzy pre-closed mapping (IFPCM for short) if  $f(A)$  is an IFPCS in  $Y$  for every IFCS  $A$  in  $X$ ,
- (d) [4] intuitionistic fuzzy  $\alpha$ -closed mapping (IF $\alpha$ CM for short) if  $f(A)$  is an IF $\alpha$ CS in  $Y$  for every IFCS  $A$  in  $X$ ,
- (e) [10] intuitionistic fuzzy  $\alpha$ -generalized closed mapping (IF $\alpha$ GCM for short) if  $f(A)$  is an IF $\alpha$ GCS in  $Y$  for every IFCS  $A$  in  $X$ .

**Definition 2.14.** [6] An IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy  $\text{IF}_{\text{rw}}\text{T}_{1/2}$  space if every IFRWGCS in  $X$  is an IFCS in  $X$ .

**Definition 2.15.** [6] An IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy  $\text{IF}_{\text{rwg}}\text{T}_{1/2}$  space if every IFRWGCS in  $X$  is an IFPCS in  $X$ .

**Result 2.16:** [6] (i) Every IFCS in an IFTS  $(X, \tau)$  is an IFRWGCS.  
(ii) Every IFOS in an IFTS  $(X, \tau)$  is an IFRWGOS.

**Result 2.17:** If an IFTS  $(X, \tau)$  is an  $\text{IF}_{\text{rwg}}\text{T}_{1/2}$  space, then for every subset  $A$  of  $X$ ,  $\text{rwgcl}(A)$  is an IFRWGCS in  $X$ .

*Proof.* By Definition 2.9,  $\text{rwgcl}(A) = \bigcap \{K \mid K \text{ is an IFRWGCS in } X \text{ and } A \subseteq K\}$ . Since  $(X, \tau)$  is an  $\text{IF}_{\text{rwg}}\text{T}_{1/2}$  space,  $\text{rwgcl}(A)$  is an IFCS in  $X$ . By Result 2.16, every IFCS is an IFRWGCS in  $X$ . Hence,  $\text{rwgcl}(A)$  is an IFRWGCS in  $X$ .  $\square$

### 3 Regular weakly generalized locally closed sets in intuitionistic fuzzy topological spaces

In this section, I introduce regular weakly generalized locally closed sets in intuitionistic fuzzy topological spaces and study some of their properties.

**Definition 3.1.** An IFS  $A = \langle x, \mu_A, \nu_A \rangle$  of an IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy regular weakly generalized locally closed set (IFRWGlcS in short) if  $A = B \cap C$  where  $B = \langle x, \mu_B, \nu_B \rangle$  is an IFRWGOS and  $C = \langle x, \mu_C, \nu_C \rangle$  is an IFRWGCS in  $X$ .

The family of all IFRWGlcs of an IFTS  $(X, \tau)$  is denoted by  $\text{IFRWGLC}(X)$ .

**Example 3.2.** Let  $X = \{a, b\}$  be a nonempty set. Let  $T_1 = \langle x, (0.1, 0.2), (0.4, 0.5) \rangle$  and  $T_2 = \langle x, (0.2, 0.3), (0.3, 0.4) \rangle$  be the IFSs of  $X$ . Then the family  $\tau = \{0_-, T_1, T_2, 1_-\}$  is an IFT on  $X$ . The IFS  $A = T_1 \cap T_2^c = \langle x, (0.1, 0.2), (0.4, 0.5) \rangle$  is an IFRWGlcs in  $X$ .

**Proposition 3.3.** Every intuitionistic fuzzy locally closed set is an intuitionistic fuzzy regular weakly generalized locally closed set but not conversely.

*Proof.* Follows from the Result 2.16. □

**Proposition 3.4.** The union of two IFRWGlcs need not be an IFRWGlcs in general as seen from the following example.

**Example 3.5.** Let  $X = \{a, b\}$  be a nonempty set. Let  $T_1 = \langle x, (0.4, 0.3), (0.5, 0.4) \rangle$  be the IFS of  $X$ . Then the family  $\tau = \{0_-, T_1, 1_-\}$  is an IFT on  $X$ . Let the IFSs  $A = \langle x, (0.3, 0.3), (0.7, 0.8) \rangle$  and  $B = \langle x, (0.4, 0.2), (0.5, 0.4) \rangle$  be IFRWGCSs in  $X$ . Then IFS  $E = T_1 \cap A = \langle x, (0.3, 0.3), (0.7, 0.8) \rangle$  and the IFS  $F = T_1 \cap B = \langle x, (0.4, 0.2), (0.5, 0.4) \rangle$  are IFRWGlcs in  $X$ . But the IFS  $E \cup F = \langle x, (0.4, 0.3), (0.5, 0.4) \rangle$  is not an IFRWGlcs in  $X$ .

**Proposition 3.6.** Let  $A$  be an IFRWGlcs in  $X$  and  $B$  be an IFRWGOS in  $X$ . Then  $A \cap B$  is an IFRWGlcs in  $X$  if  $(X, \tau)$  is an  $\text{IF}_{\text{rwg}}T_{1/2}$  space.

*Proof.* Since  $A$  is an IFRWGlcs in  $X$ , we have  $A = P \cap Q$  where  $P$  is an IFRWGOS and  $Q$  is an IFRWGCS in  $X$ .

Now  $A \cap B = (P \cap Q) \cap B = P \cap (Q \cap B) = P \cap (B \cap Q) = (P \cap B) \cap Q$ .

Since  $P$  and  $Q$  are IFRWGOS in  $X$  and  $(X, \tau)$  is an  $\text{IF}_{\text{rwg}}T_{1/2}$  space,  $P \cap B$  is an IFRWGOS in  $X$ . Hence  $A \cap B$  is an IFRWGlcs in  $X$ . □

**Proposition 3.7.** Let  $(X, \tau)$  be an  $\text{IF}_{\text{rwg}}T_{1/2}$  space. Then for any subset  $A$  of an IFTS  $(X, \tau)$ , the following statements are equivalent.

- (i)  $A \in \text{IFRWGLC}(X)$ .
- (ii)  $A = U \cap \text{rwgcl}(A)$  for some IFRWGOS  $U$ .
- (iii)  $\text{rwgcl}(A) \setminus A$  is an IFRWGCS in  $X$ .
- (iv)  $A \cup (X \setminus \text{rwgcl}(A))$  is an IFRWGOS in  $X$ .

*Proof.* (i)  $\Rightarrow$  (ii): Let  $A \in \text{IFRWGLC}(X)$ . Then there exists an IFRWGOS  $U$  and an IFRWGCS  $F$  of  $X$  such that  $A = U \cap F$ . Since  $A \subseteq U$  and  $A \subseteq \text{rwgcl}(A)$ ,  $A \subseteq U \cap \text{rwgcl}(A)$ . By the definition of rwg-closure, we have  $\text{rwgcl}(A) \subseteq F$  and hence  $U \cap \text{rwgcl}(A) \subseteq U \cap F = A$ . Therefore  $A = U \cap \text{rwgcl}(A)$ .

(ii)  $\Rightarrow$  (i): Assume  $A = U \cap \text{rwgcl}(A)$  for some IFRWGOS  $U$ . By Result 2.17,  $\text{rwgcl}(A)$  is an IFRWGCS in  $X$  and hence  $A = U \cap \text{rwgcl}(A) \in \text{IFRWGLC}(X)$ .

(iii)  $\Rightarrow$  (ii): Let  $U = X \setminus \text{rwgcl}(A) \setminus A$ . By (iii),  $U$  is an IFRWGOS in  $X$  and  $A = U \cap \text{rwgcl}(A)$  holds.

(ii)  $\Rightarrow$  (iii):  $A = U \cap \text{rwgcl}(A)$  for some IFRWGOS  $U$  in  $X$ . Now  $\text{rwgcl}(A) \setminus A = \text{rwgcl}(A) \cap A^c = \text{rwgcl}(A) \cap (U \cap \text{rwgcl}(A))^c = \text{rwgcl}(A) \cap (U^c \cup \text{rwgcl}(A))^c = (\text{rwgcl}(A) \cap U^c) \cup (\text{rwgcl}(A) \cap (\text{rwgcl}(A))^c) = \text{rwgcl}(A) \cap U^c$ , which is an IFRWGCS in  $X$ , since  $U^c$  is an IFRWGCS in  $X$  and  $(X, \tau)$  be an  $\text{IF}_{\text{rwg}}\text{T}_{1/2}$  space.

(iii)  $\Rightarrow$  (iv): Let  $F = \text{rwgcl}(A) \setminus A$ . Since  $X \setminus F = A \cup (X \setminus \text{rwgcl}(A))$ ,  $X \setminus F$  is an IFRWGOS. Hence  $A \cup (X \setminus \text{rwgcl}(A))$  is an IFRWGOS in  $X$ .

(iv)  $\Rightarrow$  (iii): Let  $U = A \cup (X \setminus \text{rwgcl}(A))$ . Then  $X \setminus U$  is an IFRWGCS, since  $U$  is an IFRWGOS by (iv) and  $X \setminus U = \text{rwgcl}(A) \setminus A$ . Hence,  $\text{rwgcl}(A) \setminus A$  is an IFRWGCS in  $X$ .  $\square$

**Proposition 3.8.** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be an intuitionistic fuzzy regular weakly generalized irresolute mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $B \in \text{IFRWGLC}(Y)$  implies  $f^{-1}(B) \in \text{IFRWGLC}(X)$ .  $\square$

*Proof.* Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be an intuitionistic fuzzy regular weakly irresolute mapping. Let  $B \in \text{IFRWGLC}(Y)$ . Then there exist IFRWGOS  $G$  and an IFRWGCS  $H$  such that  $B = G \cap H$ , which implies that  $f^{-1}(B) = f^{-1}(G) \cap f^{-1}(H)$ . Since  $f$  is an intuitionistic fuzzy regular weakly generalized irresolute mapping,  $f^{-1}(G)$  and  $f^{-1}(H)$  are IFRWGOS and IFRWGCS in  $X$  respectively. Hence,  $f^{-1}(B) \in \text{IFRWGLC}(X)$ .  $\square$

## 4 Intuitionistic fuzzy regular weakly generalized locally continuous mappings

In this section, I introduce some new classes of intuitionistic fuzzy continuous mappings called intuitionistic fuzzy regular weakly generalized locally continuous mappings, intuitionistic fuzzy regular weakly generalized locally irresolute mapping and study some of their properties.

**Definition 4.1.** A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$  is called an intuitionistic fuzzy regular weakly generalized locally continuous mapping if  $f^{-1}(B)$  is an IFRWGLcs in  $(X, \tau)$  for every IFCS  $B$  of  $(Y, \sigma)$ .

**Proposition 4.2.** Every intuitionistic fuzzy locally continuous mapping is an intuitionistic fuzzy regular weakly generalized locally continuous mapping but not conversely.

*Proof.* Assume that  $f: (X, \tau) \rightarrow (Y, \sigma)$  is an intuitionistic fuzzy locally continuous mapping. Let  $B$  be an IFCS in  $Y$ . Since  $f$  is an intuitionistic fuzzy locally continuous mapping,  $f^{-1}(B)$  is an intuitionistic fuzzy locally closed set in  $X$ . By Proposition 3.3, every intuitionistic fuzzy locally closed set is an intuitionistic fuzzy regular weakly generalized locally closed set. Therefore,  $f^{-1}(A)$  is an IFRWGLcs in  $X$ . Hence  $f$  is an intuitionistic fuzzy regular weakly generalized locally continuous mapping.  $\square$

**Example 4.3.** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $T_1 = \langle x, (0.2, 0.4), (0.7, 0.6) \rangle$ ,  $T_2 = \langle x, (0.4, 0.5), (0.6, 0.5) \rangle$ ,  $T_3 = \langle y, (0.8, 0.7), (0.1, 0.2) \rangle$ . Then the family  $\tau = \{0_{\sim}, T_1, T_2, 1_{\sim}\}$  and  $\sigma = \{0_{\sim}, T_3, 1_{\sim}\}$  are IFTs on  $X$  and  $Y$ , respectively. Define A mapping  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . Then  $f$  is an intuitionistic fuzzy regular weakly generalized locally continuous mapping. But  $f$  is not an intuitionistic fuzzy locally continuous mapping, since the IFS  $T_3^c = \langle y, (0.1, 0.2), (0.8, 0.7) \rangle$  is an IFCS in  $Y$  but  $f^{-1}(T_3^c) = \langle x, (0.1, 0.2), (0.8, 0.7) \rangle$  is not an IFcs in  $X$ .

**Remark 4.4.** Composition of two intuitionistic fuzzy regular weakly generalized locally continuous mappings need not be an intuitionistic fuzzy regular weakly generalized locally mapping in general as seen from the following example.

**Example 4.5.** Let  $X = \{a, b\}$ ,  $Y = \{c, d\}$ ,  $Z = \{u, v\}$  and  $T_1 = \langle x, (0.2, 0.2), (0.7, 0.6) \rangle$ ,  $T_2 = \langle x, (0.3, 0.5), (0.5, 0.5) \rangle$ ,  $T_3 = \langle y, (0.8, 0.7), (0.1, 0.2) \rangle$  and  $T_4 = \langle z, (0.5, 0.5), (0.3, 0.5) \rangle$ . Then the family  $\tau = \{0_{\sim}, T_1, T_2, 1_{\sim}\}$ ,  $\sigma = \{0_{\sim}, T_3, 1_{\sim}\}$  and  $\delta = \{0_{\sim}, T_4, 1_{\sim}\}$  are IFTs on  $X$ ,  $Y$  and  $Z$ , respectively. Define A mapping  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = c$  and  $f(b) = d$  and  $g : (Y, \sigma) \rightarrow (Z, \delta)$  by  $f(c) = u$  and  $f(d) = v$ . Then  $f$  and  $g$  are intuitionistic fuzzy regular weakly generalized locally continuous mappings. But their composition  $g \circ f : (X, \tau) \rightarrow (Z, \delta)$  is not an intuitionistic fuzzy regular weakly generalized locally continuous mapping, since the IFS  $T_4^c = \langle z, (0.3, 0.5), (0.5, 0.5) \rangle$  is an IFCS in  $Z$  but  $(g \circ f)^{-1}(T_4^c) = \langle x, (0.3, 0.5), (0.5, 0.5) \rangle$  is not an intuitionistic fuzzy regular weakly generalized locally closed set in  $X$ .

**Definition 4.6.** A mapping  $f : (X, \tau) \rightarrow (Y, \sigma)$  from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$  is called an intuitionistic fuzzy regular weakly generalized locally irresolute mapping if  $f^{-1}(A)$  is an IFRWGlc in  $(X, \tau)$  for every IFRWGlc  $B$  of  $(Y, \sigma)$ .

**Example 4.7.** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $T_1 = \langle x, (0.2, 0.4), (0.7, 0.6) \rangle$ ,  $T_2 = \langle x, (0.4, 0.5), (0.6, 0.5) \rangle$ ,  $T_3 = \langle y, (0.1, 0.2), (0.8, 0.7) \rangle$ . Then the family  $\tau = \{0_{\sim}, T_1, T_2, 1_{\sim}\}$  and  $\sigma = \{0_{\sim}, T_3, 1_{\sim}\}$  are IFTs on  $X$  and  $Y$  respectively. Define A mapping  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . Then  $f$  is an intuitionistic fuzzy regular weakly generalized locally irresolute mapping.

**Proposition 4.8.** Every intuitionistic fuzzy regular weakly generalized irresolute mapping is an intuitionistic fuzzy regular weakly generalized locally irresolute mapping in general.

*Proof.* Follows from Proposition 3.8. □

**Proposition 4.9.** Every intuitionistic fuzzy regular weakly generalized locally irresolute mapping is an intuitionistic fuzzy regular weakly generalized locally continuous mapping in general.

*Proof.* Follows from the fact that every IFCS is an IFRWGCS and hence it is an IFRWGlc. □

**Proposition 4.10.** Composition of two intuitionistic fuzzy regular weakly generalized locally irresolute mappings is an intuitionistic fuzzy regular weakly generalized locally irresolute mapping in general.

*Proof.* Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  and  $g : (Y, \sigma) \rightarrow (Z, \delta)$  be any two intuitionistic fuzzy regular weakly generalized locally irresolute mappings. Let  $A$  be an IFRWGlc in  $Z$ . By hypothesis,  $g^{-1}(A)$  is an IFRWGlc in  $Y$ . Since  $f$  is an intuitionistic fuzzy regular weakly generalized locally irresolute mapping,  $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$  is an IFRWGlc in  $X$ . Hence,  $g \circ f : (X, \tau) \rightarrow (Z, \delta)$  is an intuitionistic fuzzy regular weakly generalized locally irresolute mapping.  $\square$

**Proposition 4.11.** If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is an intuitionistic fuzzy regular weakly generalized locally irresolute mapping and  $g : (Y, \sigma) \rightarrow (Z, \delta)$  is an intuitionistic fuzzy locally continuous mapping then their composition  $g \circ f : (X, \tau) \rightarrow (Z, \delta)$  is an intuitionistic fuzzy regular weakly generalized locally continuous mapping.

*Proof.* Let  $A$  be an IFCS in  $Z$ . Since  $g$  is an intuitionistic fuzzy locally continuous mapping,  $g^{-1}(A)$  is an IFClc in  $Y$ . By Proposition 3.3, every intuitionistic fuzzy locally closed set is an intuitionistic fuzzy regular weakly generalized locally closed. Therefore  $g^{-1}(A)$  is an IFRWGlc in  $Y$ . Since  $f$  is an intuitionistic fuzzy regular weakly generalized locally irresolute mapping,  $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$  is an IFRWGlc in  $X$ . Hence  $g \circ f : (X, \tau) \rightarrow (Z, \delta)$  is an intuitionistic fuzzy regular weakly generalized locally continuous mapping  $\square$

**Proposition 4.12.** If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is an intuitionistic fuzzy regular weakly generalized locally irresolute mapping and  $g : (Y, \sigma) \rightarrow (Z, \delta)$  an intuitionistic fuzzy regular weakly generalized locally continuous mapping then their composition  $g \circ f : (X, \tau) \rightarrow (Z, \delta)$  is an intuitionistic fuzzy regular weakly generalized locally continuous mapping.

*Proof.* Let  $A$  be an IFCS in  $Z$ . By hypothesis  $g^{-1}(A)$  is an IFRWGlc in  $Y$ . Since  $f$  is an intuitionistic fuzzy regular weakly generalized locally irresolute mapping, then  $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$  is an IFRWGlc in  $X$ . Hence  $g \circ f : (X, \tau) \rightarrow (Z, \delta)$  is an intuitionistic fuzzy regular weakly generalized locally continuous mapping.  $\square$

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