Third Int. Conf. on IFS, Sofia, 16-17 Oct. 1999
NIFS 5 (1999) 3, 6-10

AN IDEA FOR AN INTUITIONISTIC FUZZY APPROACH TO DECISION MAKING

Cecilia Temponi[1], Anthony Shannon[2], Krassimir Atanassov[3] and Adrian Ban[4]

[1] Southwest Texas State University, San Marcos, Texas 78666, e-mail: ct01@swt.edu
[2] KvB College of Visual Communication, 99 Mount Street North Sydney NSW 2060 Australia, e-mail: tony@kvb.edu.au
[3] CLBME - Bulg. Academy of Sci., and MRL, P.O.Box 12, Sofia-1113, Bulgaria, e-mail: krat@bgcict.acad.bg
[4] Deptartment of Mathematics, University of Oradea, Str. Armatei Romane 5, 3700 Oradea, ROMANIA
e-mails: aban@lego.rdsor.ro and aiban@hs.uoradea.ro

In this paper we shall discuss a possibility for using the interpretational triangle of intuitionistic fuzzy sets in the area of the decision making.

Initially, we shall introduce some basic definitions, related to the Intuitionistic Fuzzy Sets (IFSs), following [1].

Let a set E be fixed. An IFS A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where the functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

For every two IFSs A and B many operations, relations and operators are defined (see, e.g. [1-4]), the most important of which, related to the present research, are $(\alpha, \beta \in [0,1])$:

$$\begin{array}{lcl}
A \subset B & iff & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)); \\
A \supset B & iff & B \subset A; \\
A = B & iff & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)); \\
\overline{A} & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}; \\
\Box A & = & \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\}; \\
\Diamond A & = & \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\}; \\
D_\alpha(A) & = & \{\langle x, \mu_A(x) + \alpha \cdot \pi_A(x), \nu_A(x) + (1-\alpha) \cdot \pi_A(x)\rangle \,|\, x \in E\}; \\
F_{\alpha,\beta}(A) & = & \{\langle x, \mu_A(x) + \alpha \cdot \pi_A(x), \nu_A(x) + \beta \cdot \pi_A(x)\rangle \,|\, x \in E\}, \text{ where } \alpha + \beta \leq 1; \\
G_{\alpha,\beta}(A) & = & \{\langle x, \alpha \cdot \mu_A(x), \beta \cdot \nu_A(x)\rangle \,|\, x \in E\};
\end{array}$$

$$
\begin{array}{lcl}
H_{\alpha,\beta}(A) & = & \{\langle x, \alpha\cdot\mu_A(x), \nu_A(x)+\beta\cdot\pi_A(x)\rangle \mid x\in E\};\\
H^*_{\alpha,\beta}(A) & = & \{\langle x, \alpha\cdot\mu_A(x), \nu_A(x)+\beta\cdot(1-\alpha\cdot\mu_A(x)-\nu_A(x))\rangle \mid x\in E\};\\
J_{\alpha,\beta}(A) & = & \{\langle x, \mu_A(x)+\alpha\cdot\pi_A(x), \beta\cdot\nu_A(x)\rangle \mid x\in E\};\\
J^*_{\alpha,\beta}(A) & = & \{\langle x, \mu_A(x)+\alpha\cdot(1-\mu_A(x)-\beta\cdot\nu_A(x)), \beta\cdot\nu_A(x)\rangle \mid x\in E\},\\
C(A) & = & \{\langle x, K, L\rangle | x\in E\};\\
I(A) & = & \{\langle x, k, l\rangle | x\in E\};
\end{array}
$$

$$
W(A) = \{\langle x, \frac{\sum\limits_{y\in E}\mu_A(y)}{card(E)}, \frac{\sum\limits_{y\in E}\nu_A(y)}{card(E)}\rangle | x\in E\},
$$

where

$$
K = \sup_{y\in E}\mu_A(y), \quad L = \inf_{y\in E}\nu_A(y),
$$

$$
k = \inf_{y\in E}\mu_A(y), \quad l = \sup_{y\in E}\nu_A(y)
$$

and $card(E)$ is the number of the elements of the (finite) set E. The case when E is an infinite set is defined analogically.

Let a universe E be given and let the figure F in the Euclidean plane with Cartesian coordinates be given (see Fig. 1).

Let $A \subset E$ be a fixed set. Then we can construct a function f_A from E to F, such that if $x \in E$, then

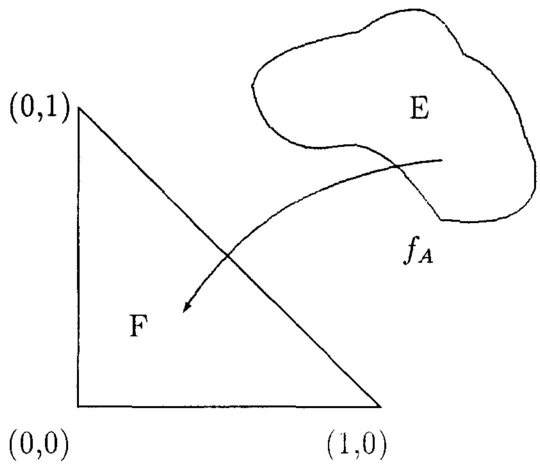

Fig. 1.

$$
p = f_A(x) \in F,
$$

the point p has coordinates $\langle a, b\rangle$ for which:

$$
0 \le a + b \le 1,
$$

and these coordinates are such that:

$$a = \mu_A(x),$$
$$b = \nu_A(x).$$

The geometrical interpretations of the operators C and I are given in Figures 2 and 3, respectively.

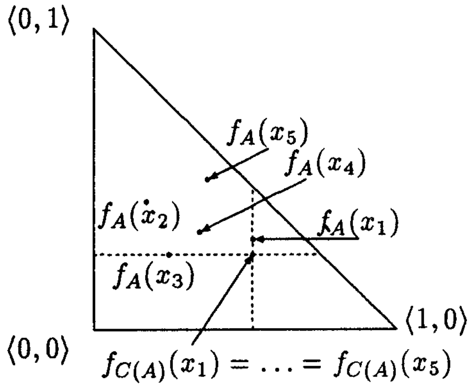

Fig 2.

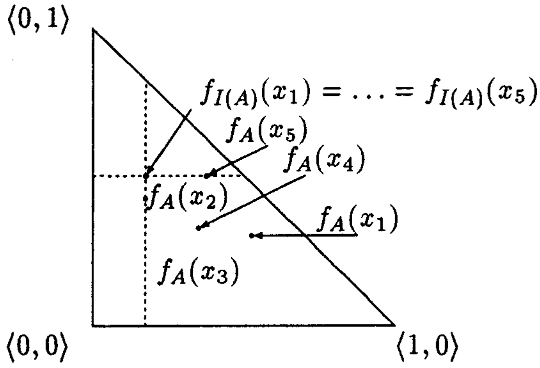

Fig 3.

Therefore, using operators C and I we can construct an area in the interpretational triangle and this area will contain all points which are interpretations of the elements of the given IFS A. Now, we can use the operator W with which we shall determine the weight-center of these points. The place of this centre in the framework of the above mentioned area can be used as a criterion for the validity (correctness) of the elements of A. For this aim we can construct four concentric circles $K_i(W(A), r_i)$ $(1 \leq i \leq 4)$, where the radiuses r_i of the circles correspond to the distances between the point $W(A)$ - the centre of all circles and every one of the vertical or horizontal sides of the area (see Fig. 4).

Let us note by K_u, K_d, K_l, K_r the circles, which are tangent to the upper, bottom, left and right boundaries of the area, respectively.

Because the four circles are concentric, as we mentioned above, these circles can be ordered with respect to their inclusions. This order can be used for decision making.

We must note that the point $W(A)$ can lie on one of the boundaries of the area; then, and only then, does the area degenerate to a segment.

It can be seen easily that:

• if the point $W(A)$ is nearly to the down side of the area, then a bigger number of elements of the IFS A have smaller degrees of non-membership;

• if the point $W(A)$ is nearly to the upper side of the area, then a larger number of elements of the IFS A have larger degrees of non-membership;

• if the point $W(A)$ is nearly to the left side of the area, then a larger number of elements of the IFS A have smaller degrees of membership;

• if the point $W(A)$ is nearly to the right side of the area, then a larger number of elements of the IFS A have larger degrees of membership.

These four criteria can be combined, and in the simplest case they can be reduced to two possibilities:

• if the point $W(A)$ is nearly to the left and upper sides of the area, then a larger number of elements of the IFS A have 'bad' values for their degrees;

• if the point $W(A)$ is nearly to the right and down sides of the area, then a larger number of elements of the IFS A have 'good' values for their degrees (see Fig. 5).

We can obtain an essentially more detailed case if we use the order of inclusions of the four circles. For example, in Fig. 6 is shown the case:

$$K_d \subset K_r \subset K_l \subset K_u. \tag{1}$$

Another order is, for example,

$$K_r \subset K_d \subset K_l \subset K_u. \tag{2}$$

All possible inclusions amount to 24. We can extimate these 24 cases by intuitionistic fuzzy estimations, too, using some conventions for 'goodness' of the estimations, which are related to the manner of the ordering.

If, for example, we decide that the inclusions (1) and (2) are equivalent (i.e., if we decide that it is the same to have elements with high degrees of membership and to have elements with low degrees of non-membership) we shall obtain one configuration of estimations. Another (more detailed) configuration can be obtained if we use an order between cases (1) and (2), too.

An illustration of the intuitionistic fuzzy estimations of the orders is given in the next table.

$$\begin{array}{ll}
K_u \subset K_d \subset K_l \subset K_r & < \frac{1}{8}, \frac{1}{4} > \\
K_u \subset K_d \subset K_r \subset K_l & < \frac{1}{2}, \frac{3}{8} > \\
K_u \subset K_l \subset K_d \subset K_r & < 0, 1 > \\
K_u \subset K_l \subset K_r \subset K_d & < 0, 1 > \\
K_u \subset K_r \subset K_l \subset K_d & < \frac{1}{2}, \frac{3}{8} > \\
K_u \subset K_r \subset K_d \subset K_l & < \frac{1}{4}, \frac{5}{8} > \\
K_d \subset K_u \subset K_l \subset K_r & < \frac{3}{8}, \frac{1}{2} >
\end{array}$$

$$K_d \subset K_u \subset K_r \subset K_l \quad < \tfrac{5}{8}, \tfrac{1}{4} >$$
$$K_d \subset K_l \subset K_u \subset K_r \quad < \tfrac{3}{8}, \tfrac{1}{2} >$$
$$K_d \subset K_l \subset K_r \subset K_u \quad < \tfrac{5}{8}, \tfrac{1}{4} >$$
$$K_d \subset K_r \subset K_l \subset K_u \quad < 1, 0 >$$
$$K_d \subset K_r \subset K_u \subset K_l \quad < 1, 0 >$$
$$K_l \subset K_u \subset K_d \subset K_r \quad < 0, 1 >$$
$$K_l \subset K_u \subset K_r \subset K_d \quad < 0, 1 >$$
$$K_l \subset K_r \subset K_d \subset K_u \quad < \tfrac{3}{8}, \tfrac{1}{2} >$$
$$K_l \subset K_r \subset K_u \subset K_d \quad < \tfrac{1}{4}, \tfrac{5}{8} >$$
$$K_l \subset K_d \subset K_r \subset K_u \quad < \tfrac{3}{8}, \tfrac{1}{2} >$$
$$K_l \subset K_d \subset K_u \subset K_r \quad < \tfrac{1}{4}, \tfrac{5}{8} >$$
$$K_r \subset K_u \subset K_d \subset K_l \quad < \tfrac{5}{8}, \tfrac{1}{4} >$$
$$K_r \subset K_u \subset K_l \subset K_d \quad < \tfrac{1}{2}, \tfrac{3}{8} >$$
$$K_r \subset K_l \subset K_u \subset K_d \quad < \tfrac{1}{2}, \tfrac{3}{8} >$$
$$K_r \subset K_l \subset K_d \subset K_u \quad < \tfrac{5}{8}, \tfrac{1}{4} >$$
$$K_r \subset K_d \subset K_l \subset K_u \quad < 1, 0 >$$
$$K_r \subset K_d \subset K_u \subset K_l \quad < 1, 0 >$$

The above constructed criteria can be used, for instance, in consumer choice in fashion, in which an individual's style preference is related to self-identity, attraction to esteemed groups, desire to be distanced from repulsive groups, and attitude towards change [5].

REFERENCES:

[1] Atanassov K., Intuitionistic fuzzy sets, Fuzzy sets and Systems, Vol. 20 (1986), No. 1, 87-96.

[2] Atanassov K., More on intuitionistic fuzzy sets, Fuzzy Sets and Systems, Vol. 33 (1989), No. 1, 37-45.

[3] Atanassov K., On four intuitionistic fuzzy topological operators, submitted to Mathware.

[4] Atanassov K., A. Ban, On an operator over intuitionistic fuzzy sets, in press in Comptes Rendus de l'Academie bulgare des Sciences.

[5] Shannon, Tony, A mathematical model of consumer choice in fashion, KvB Backgrounders, October (1999), No.27.