On the law of large numbers for IF-events

Beloslav Riečan

Faculty of Natural Sciences, Matej Bel University
Department of Mathematics
Tajovského 40, 97401 Banská Bystrica, Slovakia,
Mathematical Institut, Slovak Academy of Sciences,
Štefánikova 49, 84101 Bratislava, Slovakia.
Email:beloslav.riecan@umb.sk

IF-events

Atanassov K.: Intuitionistic Fuzzy Sets: Theory and Applications, Physica Verlag, New York 1999

IF-event $A = (\mu_A, \nu_A), \mu_A, \nu_A : X \to [0,1]$ $\mu_A + \nu_A \leq 1$.

$$A \odot B = ((\mu_A + \mu_B - 1) \vee 0, (\nu_A + \nu_B) \wedge 1)),$$

$$A \oplus B = ((\mu_A + \mu_B) \wedge 1, (\nu_A + \nu_B - 1) \vee 0),$$

$$\neg A = (1 - \mu_A, 1 - \nu_B).$$

State

$m : \mathcal{F} \to [0, 1]$

$$m((1,0)) = 1, m((0,1)) = 0,$$

$$A \odot B = (0,1) \Longrightarrow m(A \oplus B) = m(A) + m(B),$$

$$A_n \nearrow A \Longrightarrow m(A_n) \nearrow m(A).$$

Observable

Observable $x : \mathcal{B}(R) \to \mathcal{F}$

$$x(R) = (1,0), x(\emptyset) = (0,1),$$

$$A \cap B = \emptyset \Longrightarrow x(A) \odot x(B) = (0,1), x(A \oplus B) = x(A) + x(B),$$

$$A_n \nearrow A \Longrightarrow x(A_n) \nearrow x(A).$$

Example. $\xi : X \to R, x(A) = \xi^{-1}(A)$.

$x : \mathcal{B}(R) \to \mathcal{F}$ - observable

$m : \mathcal{F} \to [0, 1]$ - state

$m_x : \mathcal{B}(R) \to [0, 1], m_x(A) = m(x(A))$

$\Longrightarrow$

m_x is a probability measure

Independence

$x, y : \mathcal{B}(R) \to \mathcal{F}$ independent $\iff$
$\exists$ observable $\kappa : \mathcal{B}(R^2) \to \mathcal{F}, \forall A, B \in \mathcal{B}(R)$

$$m_\kappa(A \times B) = m_x(A) m_y(B)$$

Sum of observables

$x, y : \mathcal{B}(R) \to \mathcal{F}$ independent

$$x + y(A) = \kappa(g^{-1}(A))$$

where

$$g : R^2 \to R, g(u, v) = u + v$$

Motivation.

$$(\xi + \eta)^{-1} C == T^{-1}(g^{-1}(C)), T = (\xi, \eta) : X \to R$$

Moments

$$E(x) = \int_{-\infty}^{\infty} t dm_x(t)$$

$$\sigma^2(x) = \int_{-\infty}^{\infty} (t - E(x))^2 dm_x(t)$$

Law of large numbers

If (x_n) is a sequence of independent observables with the same expectation a and the same dispersion σ^2then for any $\varepsilon > 0$

$$\lim_{n\to\infty} m\left(\left(\frac{1}{n}\sum_{i=1}^{n} x_i - a\right)(-\varepsilon, \varepsilon)\right) = 1.$$