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On some measures of information and knowledge for intuitionistic fuzzy sets

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Abstract

We address the problem of assessing information and knowledge conveyed by an Atanassov's intuitionistic fuzzy set (A-IFS for short). We pay particular attention to the relationship between positive and negative knowledge (expressed by entropy which may be seen as a dual measure to information), and take into account also reliability of the information expressed by the hesitation margin.

Keywords: Intuitionistic fuzzy sets, amount of information, entropy, hesitation margin.

1 Introduction

The information conveyed by a piece of data represented by a fuzzy sets consists of a membership function. Its related knowledge is context dependent and derived from information placed in the context considered. It may be, for example, a dual measure to entropy (as considered in Quinlan [9]). The transformation of information into knowledge is a critical one from a practical point of view (cf. Stewart [10]). It is a crucial task for problem solving and any analysis of a specific situation and/or problem. A notable example may here be the omnipresent problem of decision making.

We consider in this paper information conveyed by a piece of data represented by an A-IFS and its related knowledge that is context dependent.

Information represented by an A-IFS, may be considered just as a generalization of information conveyed by a fuzzy set, and consists from the two terms present in the definition of an A-IFS, i.e., the membership and non-membership functions ("responsible" for the positive and negative information, respectively). But for practical purposes it seems necessary to also take into account a so called hesitation margin (cf. Szmidt and Kacprzyk [14], [15], [18], [16], [20]), [21], Bustince et al. [5], [6], Szmidt and Kukier [19], [22], [23], etc.)

We show in this paper that the entropy alone, although calculated by taking into account the hesitation margin as well (cf. Szmidt and Kacprzyk [16], [20]) may be not a satisfactory dual measure of knowledge useful from the point of view of decision making or any specific problem solving activity that is performed via the A-IFSs. The reason is that an entropy measure answers the question about the fuzziness but does not consider reasons for the fuzziness. So, the two situations, on the one hand, one with the maximal entropy for a membership function equal to a non-membership function (with both of them equal to 0.5), and – on the other hand – when we know absolutely nothing, are equal from the point of view of the entropy measure (in terms of the A-IFSs). However, from the point of view of decision making the two situations are quite different. This is the motivation of this paper as we propose here a new measure of knowledge for A-IFSs. The proposed measure is not going to replace the entropy measures but may capture additional features which are relevant when making decisions. The new measure of knowledge is tested on a simple example taken from the source Quinlan's paper, but solved using different tools than therein. The presented example is not big but it is a challenge to many classification and machine learning methods, and its solution which we have proposed can be an inspiration to the solution of many real world problems.

2 Brief introduction to A-IFSs

One of the possible generalizations of a fuzzy set in X (Zadeh [24]) given by

$$A^{'} = \{< x, \mu_{A'}(x) > | x \in X\} \tag{1}$$

where $\mu_{A'}(x) \in [0, 1]$ is the membership function of the fuzzy set $A^{'}$, is an A-IFS (Atanassov [1], [3]) A is given by

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \tag{2}$$

where: $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{3}$$

and $\mu_A(x)$, $\nu_A(x) \in [0, 1]$ denote a degree of membership and a degree of non-membership of $x \in A$, respectively. (Two approaches to the assigning memberships and non-memberships for A-IFSs are proposed by Szmidt and Baldwin [13]).

Obviously, each fuzzy set may be represented by the following A-IFS

$$A = \{< x, \mu_{A'}(x), 1 - \mu_{A'}(x) > | x \in X\} \tag{4}$$

An additional concept for each A-IFS in X, that is not only an obvious result of (2) and (3) but which is also relevant for applications, we will call (Atanasov [3])

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{5}$$

a *hesitation margin* of $x \in A$ which expresses a lack of knowledge of whether x belongs to A or not (cf. Atanassov [3]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

The hesitation margin turns out to be important while considering the distances (Szmidt and Kacprzyk [14], [15], [18], entropy (Szmidt and Kacprzyk [16], [20]), similarity (Szmidt and Kacprzyk [21]) for the A-IFSs, etc. i.e., the measures that play a crucial role in virtually all information processing tasks.

Hesitation margins turn out to be relevant for applications - in image processing (cf. Bustince et al. [5], [6]) and classification of imbalanced and overlapping classes (cf. Szmidt and Kukier [19], [22], [23]), group decision making, negotiations, voting and other situations (cf. Szmidt and Kacprzyk papers).

In our further considerations we will use the notion of distances. In Szmidt and Kacprzyk [15], [18], Szmidt and Baldwin [11], [12], it is shown why in the calculation of distances between A-IFSs one should use all three terms describing A-IFSs.

The most often used distances between A-IFSs A, B in $X = \{x_{1,}, \ldots, x_n\}$ are:
– the normalized Hamming distance (Szmidt and Baldwin [11], [12], Szmidt and Kacprzyk [15], [18]):

$$l_{IFS}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|) \quad (6)$$

– and the normalized Euclidean distance (Szmidt and Baldwin [11], [12], Szmidt and Kacprzyk [15], [18]):

$$\begin{aligned} &q_{IFS}(A,B) = \\ &= (\frac{1}{2n}\sum_{i=1}^{n}(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2)^{\frac{1}{2}} \end{aligned} \quad (7)$$

For distances (6), and (7) we have $0 \underline{<} l_{IFS}(A,B) \underline{<} 1$, and $0 \underline{<} q_{IFS}(A,B) \underline{<} 1$. Clearly these distances satisfy the conditions of the metric.

Also the notation of a complement set A^C will be used

$$A^C = \{\langle x, \nu_A(x), \mu_A(x), \pi_A(x)\rangle | x \in X\} \quad (8)$$

2.1 Entropy

It is necessary to stress that entropy we examine here is a non-probabilistic-type entropy measure for A-IFSs in the sense of De Luca and Termini [7] axioms which are intuitive and have been widely employed in the fuzzy literature. The axioms were properly reformulated for intuitionistic fuzzy sets (see Szmidt and Kacprzyk [16]).

In our further considerations concerning entropy, in addition to the distances, the concept of cardinality of an intuitionistic fuzzy set will also be useful.

Definition 1 *(Szmidt and Kacprzyk [16], [17]) Let A be an intuitionistic fuzzy set in X. First, we define the following two cardinalities of an intuitionistic fuzzy set:*

- *the least ("sure") cardinality of A is equal to the so-called sigma-count (cf. Zadeh [24], [25], and is called here the $\min \sum Count$:*

$$\min Card(A) = \min \sum Count(A) = \sum_{i=1}^{n} \mu_A(x_i) \quad (9)$$

- *the biggest cardinality of A, which is possible due to π_A, is called the $\max \sum Count$, and is equal to*

$$\max Card(A) \;=\; \max \sum Count(A) = \sum_{i=1}^{n} (\mu_A(x_i) + \pi_A(x_i)) \qquad (10)$$

and, clearly, for A^c (where A^c is a complement of A) we have

$$\min Card(A^c) \;=\; \min \sum Count(A^c) = \sum_{i=1}^{n} \nu_A(x_i) \qquad (11)$$

$$\max Card(A^c) \;=\; \max \sum Count(A^c) = \sum_{i=1}^{n} (\nu_A(x_i) + \pi_A(x_i)) \qquad (12)$$

Then the cardinality of an intuitionistic fuzzy set is defined as a number from the interval:

$$CardA \;\in\; [\min \sum Count(A), \max \sum Count(A)] \qquad (13)$$

Remark: in the above formulas (9)–(13), for $i = 1$, we will use later, for simplicity, the following symbols: $\min Count(A)$ instead $\min \sum Count(A)$, $\max Count(A)$ instead $\max \sum Count(A)$, $\min Count(A^c)$ instead $\min \sum Count(A^c)$, $\max Count\ (A^c)$ instead $\max \sum Count(A^c)$.

The measure of similarity for this special case was called entropy as it answers the question: how fuzzy is a fuzzy set? In other words, entropy $E(x)$ measures the whole missing information which may be necessary to say if an element x fully belongs or fully does not belong to our set.

Definition 2 *A ratio-based measure of fuzziness i.e., entropy of an intuitionistic fuzzy element is given in the following way:*

$$E(x) = \frac{a}{b} \qquad (14)$$

where a is a distance(x, x_{near}) *from x to the nearer point x_{near} among $M(1,0,0)$ and $N(0,1,0)$, and b is the* distance(x, x_{far}) *from x to the farer point x_{far} among $M(1,0,0)$ and $N(0,1,0)$.*

Formula (14) describes the degree of fuzziness for a single element belonging to an intuitionistic fuzzy set. For n elements belonging to an intuitionistic fuzzy set we have

$$E = \frac{1}{n} \sum_{i=1}^{n} E(x_i) \qquad (15)$$

Fortunately enough, while applying the Hamming distances in (14), the entropy of intuitionistic fuzzy sets is the ratio of the biggest cardinalities ($\max \sum Counts$) involving only x and x^c. The following theorem was proven in (Szmidt and Kacprzyk [16]).

Theorem 1 A generalized entropy measure of an intuitionistic fuzzy set with n elements is

$$E = \frac{1}{n} \sum_{i=1}^{n} \frac{\max Count(x_i \cap x_i^c)}{\max Count(x_i \cup x_i^c)} \qquad (16)$$

3 Measure of information and knowledge for A-IFSs

The information concerning a separate element x belonging to an A-IFS is equal to $\mu(x)+\nu(x)$, or, in other words: $1-\pi(x)$. But it is one aspect of information only. For each fixed π there are different possibilities of combination between μ and ν. The combination between them influences strongly the amount knowledge. The knowledge (for a fixed π) is different for the distant values between μ and ν, and for the close values between μ and ν. For example, if $\pi = 0.1$, the knowledge for the situation while $\mu = 0.85$ and $\nu = 0.05$ is bigger than for the case: $\mu = 0.45$ and $\nu = 0.45$. Entropy proposed by Szmidt and Kacprzyk ([16], [17]) is a good measure answering the question how fuzzy is an A-IFS (when considering entropy one is not interested in the reasons of fuzziness). But when decision making, one is also interested in making differences between the following situations:

- we have no information at all, and
- we have a large number of arguments in favor but an equally large number of arguments in favor of the opposite statement.

In other words, we would like to have a measure making a difference between (0.5, 0.5, 0), and (0, 0, 1). To distinguish between these two types of situations, we should take into account, beside entropy measure, also the hesitation margin π.

A good measure of the amount of the knowledge (useful from the point of view of decision making) connected to a separate element $x \in X$ seems to be:

$$K(x) = 1 - 0.5(E(x) + \pi(x)) \tag{17}$$

where $E(x)$ is an entropy measure given by (14) (Szmidt and Kacprzyk [16]), $\pi(x)$ is the hesitation margin.

Measure $K(x)$ (17) makes it possible to meaningfully represent what, in our context, is meant by the amount of knowledge, and is simple both conceptually and numerically, which is a big asset while solving complex real world problems.

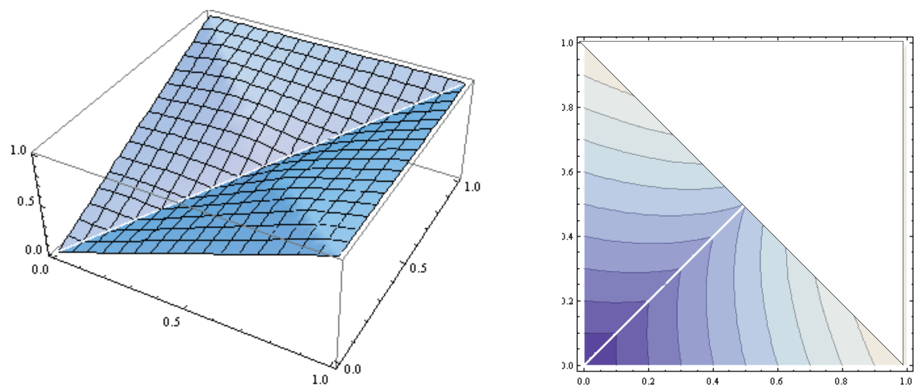
The properties of (17) are the consequence of the properties of entropy measure $E(x)$, and the fact that additionally the lack of information $\pi(x)$ was added, and normalized, namely:

1. $0 \leq K(x) \leq 1$,
2. $K(x) = K(x^C)$
3. For a fixed value of π, $K(x)$ behaves dually to an entropy measure (i.e., as $1 - E(x)$),
4. For a fixed $E(x)$, $K(x)$ increases while π decreases.

In Figure 1 we can see the shape of $K(x)$, and its contour plot.

For n elements, the total amount of knowledge K is:

$$K = \frac{1}{n}\sum_{i=1}^{n}(1 - 0.5(E(x_i) + \pi(x_i))) \tag{18}$$



a) b)

Figure 1: a) - measure $I(x)$; b) - its contourplot

To verify if the proposed measure of knowledge K (17) for A-IFSs gives expected results we use the famous Quinlan's example [9], the so-called "Saturday Morning" in which classification with nominal data is considered. We will verify if we obtain similar results as Quinlan in his ID3 algorithm to select the best attribute to split the training set (Quinlan used in his model Shannon entropy).

3.0.1 Quinlan's example

We have objects described by attributes. Each attribute represent a feature and takes on discrete, mutually exclusive values. For example, if the objects were "Saturday Mornings" and the classification involved the weather, possible attributes might be [9]:

- **outlook**, with values $\{$sunny, overcast, rain$\}$,
- **temperature**, with values $\{$cold, mild, hot$\}$,
- **humidity**, with values $\{$high, normal$\}$, and
- **windy**, with values $\{$true, false$\}$,

Altogether, the above attributes provide a zero-order language for characterizing objects in the universe (the attributes are nominal). A particular Saturday morning, an *example*, might be described as: outlook: overcast; temperature: cold; humidity: normal; windy: false. Each object (example) belongs to one of mutually exclusive classes, C. We assume that there are only two classes, i.e., $C = \{P, N\}$, where: P denotes the set of *positive examples*, and N – that of *negative examples*. There are 14 training examples as shown in Table 1. Each training example e is represented by the attribute-value pairs, i.e., $\{(A_i, a_{i,j}); i = 1, \ldots, l_i\}$ where A_i is an attribute, $a_{i,j}$ is its value – one of possible j values (for each i-th attribute j can be different, e.g., for *outlook*: $j = 3$, for *humidity*: $j = 2$ etc.).

Table 1: The “Saturday Morning” data from [9]

No.	Attributes				Class
	Outlook	Temp	Humidity	Windy	
1	sunny	hot	high	false	N
2	sunny	hot	high	true	N
3	overcast	hot	high	false	P
4	rain	mild	high	false	P
5	rain	cool	normal	false	P
6	rain	cool	normal	true	N
7	overcast	cool	normal	true	P
8	sunny	mild	high	false	N
9	sunny	cool	normal	false	P
10	rain	mild	normal	false	P
11	sunny	mild	normal	true	P
12	overcast	mild	high	true	P
13	overcast	hot	normal	false	P
14	rain	mild	high	true	N

Table 2: The frequencies obtained

	Outlook			Temperature			Humidity		Windy	
	S	O	R	H	M	C	H	N	T	F
Positive	2/9	4/9	3/9	2/9	4/9	3/9	3/9	6/9	3/9	6/9
Negative	3/5	0	2/5	2/5	2/5	1/5	4/5	1/5	3/5	2/5

3.0.2 A-IFS model

First, we make use of frequency description of the problem (see Table 2). The frequency measure (Table 2) used for description of the data (Table 1):

$$f(A_i, a_{i,j}, C) = V(C; A_i = a_{i,j})/p_C \tag{19}$$

where $C = \{P, N\}$; $V(C; A_i = a_{i,j})$ – the number of training examples of C for which $A_i = a_{i,j}$; p_C – the number of the training examples of C.

To describe and classify the “Saturday Morning” data via the intuitionistic fuzzy sets, we use an algorithm proposed in Szmidt and Baldwin [13] to assign the parameters of an intuitionistic fuzzy model which describes the attributes (the relative frequency distribution functions given in Table 2 were the starting point of the algorithm). The assigned description of the attributes in terms of intuitionistic fuzzy sets are given in Table 3 and are used for further calculations.

The main idea is to depart from the traditional assignment of the values: 1, 0, -1 and use the values of all the three functions describing IFSs, i.e. the values of the membership and non-membership degrees, and of the hesitation margin. So we have a counterpart table in which we

Table 3: The counterpart intuitionistic fuzzy model

	Outlook			Temperature			Humidity		Windy	
	S	O	R	H	M	C	H	N	T	F
Hesitation margins	0.67	0	0.69	0.67	1	0.49	0.67	0.4	0.67	0.8
membership values	0	1	0.2	0	0	0.4	0	0.6	0	0.2
non-membership values	0.33	0	0.11	0.33	0	0.11	0.33	0	0.33	0

Table 4: The "Saturday Morning" data in terms of A-IFSs

No.	Attributes				Class
	Outlook	Temperature	Humidity	Windy	
1	(0, 0.33, 0.67)	(0, 0.33, 0.67)	(0, 0.33, 0.67)	(0.2, 0, 0.8)	N
2	(0, 0.33, 0.67)	(0, 0.33, 0.67)	(0, 0.33, 0.67)	(0, 0.33, 0.67)	N
3	(1, 0, 0)	(0, 0.33, 0.67)	(0, 0.33, 0.67)	(0.2, 0, 0.8)	P
4	(0.2, 0.11, 0.69)	(0, 0, 1)	(0, 0.33, 0.67)	(0.2, 0, 0.8)	P
5	(0.2, 0.11, 0.69)	(0.4, 0.11, 0.49)	(0.6, 0, 0.4)	(0.2, 0, 0.8)	P
6	(0.2, 0.11, 0.69)	(0.4, 0.11, 0.49)	(0.6, 0, 0.4)	(0, 0.33, 0.67)	N
7	(1, 0, 0)	(0.4, 0.11, 0.49)	(0.6, 0, 0.4)	(0, 0.33, 0.67)	P
8	(0, 0.33, 0.67)	(0, 0, 1)	(0, 0.33, 0.67)	(0.2, 0, 0.8)	N
9	(0, 0.33, 0.67)	(0.4, 0.11, 0.49)	(0.6, 0, 0.4)	(0.2, 0, 0.8)	P
10	(0.2, 0.11, 0.69)	(0, 0, 1)	(0.6, 0, 0.4)	(0.2, 0, 0.8)	P
11	(0, 0.33, 0.67)	(0, 0, 1)	(0.6, 0, 0.4)	(0, 0.33, 0.67)	P
12	(1, 0, 0)	(0, 0, 1)	(0, 0.33, 0.67)	(0, 0.33, 0.67)	P
13	(1, 0, 0)	(0, 0.33, 0.67)	(0.6, 0, 0.4)	(0.2, 0, 0.8)	P
14	(0.2, 0.11, 0.69)	(0, 0, 1)	(0, 0.33, 0.67)	(0, 0.33, 0.67)	N

have a description of the problem (Table 1) in terms of intuitionistic fuzzy sets (see Tables 3, 4), i.e., $(\mu(.), \nu(.), \pi(.))$; for instance, $(0, 0.33, 0.67)$ is for "sunny".

Now we will verify if the proposed measure of knowledge K (17) gives reasonable results from the point of view of pointing out the attributes which gain most information.

Let us remind that in the original solution given by Quinlan [9] (leading to the minimal tree), the order from the point of view of the most informative attributes was the following:

Outlook, Humidity, Windy, Temperature.

If we order the attributes taking into account entropy (14)–(16), the most informative attributes are pointed out by the biggest values in Table 5:

Humidity, Outlook, Windy, Temperature.

i.e., the order of the attributes is different (Humidity replaced Outlook – this order would not result in the smallest tree).

Table 5: Evaluation of the attributes of the “Saturday Morning” data by entropy

Attribute	Outlook	Temperature	Humidity	Windy
Entropy	0.56	0.813	0.535	0.744
1-Entropy	0.44	0.19	0.47	0.26

On the other hand, if we order the attributes taking into account an average hesitation margin $\overline{\pi}$ only, the most informative attributes are (Table 6):

Outlook, Humidity, Temperature, Windy.

i.e., again, the order of the attributes is changed (Temperature replaced Windy).

Table 6: Evaluation of the attributes of the “Saturday Morning” data by $\overline{\pi}$

Attribute	Outlook	Temperature	Humidity	Windy
$\overline{\pi}$	0.453	0.72	0.535	0.735

But when we apply the knowledge measure K (17), the results are the same as Quinlan's ones (Table 7), i.e.: Outlook, Humidity, Windy, Temperature.

Table 7: Evaluation of the attributes of the “Saturday Morning” data by K

Attribute	Outlook	Temperature	Humidity	Windy
K	0.49	0.23	0.47	0.26

4 Conclusions

We have proposed a new measure of knowledge for A-IFSs. The new measure keeps the advantages of the entropy measure (reflecting the relationship of the positive and negative knowledge) but additionally stresses as well the influence of the amount of the lacking information (expressed by the hesitation margins). The measure was constructed to be useful from the point of view of decision making.

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