18th Int. Conf. on IFSs, Sofia, 10–11 May 2014
Notes on Intuitionistic Fuzzy Sets
ISSN 1310–4926
Vol. 20, 2014, No. 2, 94–99

Discussion on the threshold values in the InterCriteria Decision Making approach

Vassia Atanassova[1], Deyan Mavrov[2], Lyubka Doukovska[3] and Krassimir Atanassov[4]

[1] Institute of Biophysics and Biomedical Engineering *and*
Institute of Information and Communication Technologies
Bulgarian Academy of Sciences
e-mail: vassia.atanassova@gmail.com

[2] Prof. Asen Zlatarov University
Burgas–8000, Bulgaria
e-mail: dg@mavrov.eu

[3] Institute of Information and Communication Technologies
Bulgarian Academy of Sciences
e-mail: doukovska@iit.bas.bg

[4] Institute of Biophysics and Biomedical Engineering
Bulgarian Academy of Sciences
e-mail: krat@bas.bg

Abstract: Here we discuss an important aspect of the InterCriteria Decision Making approach, related to the possibilities for defining the intuitionistic fuzzy threshold values that help discriminate between the positive consonance, the negative consonance and the dissonance between the criteria. In our previous research, we used to set thee thresholds to predefined values in the [0; 1]-interval, changing with a step of 0.1, 0.05, or 0.025, according to the desired precision, but we have observed some flaws in this approach, and suggest new, more accurate, ways to determine these values.
We use as example data sets, derived from the World Economic Forum's annual Global Competitiveness Reports, for the 28 European Union Member States between 2008 and 2014 year.
Keywords: Global Competitiveness Index, Index matrix, Intercriteria decision making, Intuitionistic fuzzy sets, Multicriteria decision making, Correlation.
AMS Classification: 03E72.

1 Introduction

In a series of paper (see [1, 2, 3]) we have discussed the newly proposed InterCriteria Decision Making (ICDM) approach, that is utilized on the concepts of index matrices and intuitionistic fuzzy sets. It is aimed to support a decision maker, who disposes of datasets of evaluations or measurements of multiple objects against multiple criteria, to more profoundly understand the nature of the criteria involved and discover on this basis existing correlations in between the criteria themselves. Theoretically, the ICDM approach has been presented in details in [1], and in [2, 3] it was further discussed in the light of its application to data about EU Member States' competitiveness in the period 2008–2014, as obtained from the World Economic Forum's annual Global Competitiveness Reports.

In the ICDM approach, we have (at least one) matrix of evaluations or measurements of m evaluated objects against n evaluating criteria, and from these we obtain a respective $n \times n$ matrix giving the discovered correlations between the evaluating criteria in the form of intuitionistic fuzzy pairs, or, which is the same but more practical, two $n \times n$ matrices giving in separate views the membership-values (a μ-matrix) and the non-membership pairs (a ν-matrix). Once having these, we are interested to see which of the criteria are in *positive consonance* (situation of definitively correlating criteria), in *negative consonance* (situation of definitively non-correlating criteria), or in *dissonance* (situation of uncertainty, when no definitive conclusion can be made). In order to categorize all the values of the resultant $n(n-1)/2$ pairs of criteria, we need to define two thresholds, α and β, for the positive and for the negative consonance, respectively.

2 Discussion and Problem statement

So far, the question about the precise ways to determine these two threshold values has not been elaborated in details, yet it deserves our attention. It is a significant issue, because for a decision maker who is in a situation to prioritize, it will be important to understand which are, say, the k most strongly correlating among the n criteria, and only invest further efforts on working with these. However, our observations so far show that choosing the 'right' values of the thresholds α and β is a rather problem- and data-specific issue, and there is hardly a universal 'recipe' for choosing them.

When it comes to prioritization, it is also worth commenting whether the values in the μ-matrix (where the values are compared against the α threshold) or the values in the ν-matrix (where compared against the β threshold) are to be given priority. In our understanding, it is more natural and usual to give priority to the values in the μ-matrix, because these indicate the positive consonance, i.e. degree of *presence* of correlation between criteria. Of course, certain situations may require us to give priority to the negative consonance, which would be a situation of looking for *absence* of correlation between these. This again is rather problem-dependent, and due to the analogy, we will only concentrate here on the case when we give priority to the positive consonance.

In our papers [2, 3], where the emphasis was on other aspects of the ICDM's application, we discussed the rather simplistic case when the threshold values α and β were values on the [0; 1]-scale, changing with a predefined precision step of 0.1, 0.05, or 0.025. Moreover, in this simplistic case, the values of the thresholds α and β in the different pairs of numbers were always summing up to 1.00, like (0.85; 0.15), (0.825; 0.175), (0.8; 0.2), etc. Thus, we could notice that this is not enough discriminative: with this setting, the β threshold produces too many pairs of criteria in negative consonance, far beyond the number of pairs in positive consonance produced by the α respective threshold. This is another argument why we need to give priority to either the positive or the negative consonance: they may simply produce inconsistent, differently ordered pairs of criteria. Hence, if we are in situation to be interested in the *presence* of correlation, we have to be primary guided by the values in the μ-matrix, as compared to the α threshold, and only take for secondary reference the data from the ν-matrix, as compared to the β threshold.

Here is an example. As we summarized in Table 4 in [2], when $\alpha = 0.85$, the outlined pairs in positive consonance are only two: '1. Institutions – 9. Technological readiness' and '11. Business sophistication – 12. Innovation'. When $\beta = 1 - \alpha = 0.15$, the outlined pairs in negative consonance are 19. When $(\alpha, \beta) = (0.80; 0.20)$, these numbers are respectively 11 and 29; and so forth step, using a step of 0.05, until (0.65; 0.35), when these numbers are respectively 39 and 51.

In all cases, when $\alpha + \beta = 1$, the results for the negative consonance are much more than these for the positive one. The anticipated reason for this is in the presence of certain uncertainty. So, it would be interesting to detect for a given value of α, what shall be the value of β, so that the produced sets of positive and negative consonance pairs be as close and consistent as possible (they can hardly be identical).

Following this line of reasoning, we will restate the problem in the following way:

If the decision maker wishes to focus only on the k most positively correlated criteria out of the totality of n criteria, what values of the thresholds α and β shall he/she select?

In the particular case, when we discuss the EU Member States' competitiveness, this problem statement is in line with World Economic Forum's address to state policy makers to 'identify and strengthen the transformative forces that will drive future economic growth', [4], i.e. to concentrate their efforts and political measures where the greatest 'reverberation' would occur.

3 Proposed solution

With the so formulated problem statement, we propose an algorithm for identifying (shortlisting) the k most positively correlated criteria out of the totality of n criteria. We repeat the consideration from above, that here we will give priority to the positive consonance, presumably being the more often case in practical decision making with the ICDM approach. The reverse follows by analogy.

Algorithm

1.1. For each criterion C_i, $i = 1, \ldots, n$, we find $\max_{j, j \neq i} \mu(C_i, C_j)$, i.e. the maximum of the discovered correlations of C_i with all the rest criteria C_j, $j = 1, \ldots, n$, $i \neq j$. Thus we obtain for each criterion, which is its top-correlating value.

1.2. We create a table like the one shown on Table 1.

Column (1)	Column (2)
C_1	$\max_{j,j\neq 1} \mu(C_1, C_j)$
C_2	$\max_{j,j\neq 2} \mu(C_2, C_j)$
…	…
C_i	$\max_{j,j\neq i} \mu(C_i, C_j)$
…	…
C_n	$\max_{j,j\neq n} \mu(C_n, C_j)$

Table 1.

1.3. We sort the whole Table 1 by Column (2), thus ordering the top-correlating values for all individual criteria, and obtaining the desired order.
1.4. We shortlist the first k criteria from Column (1) in the resultant sorted table.
1.5. The sought value of the threshold α is then the respective value in Column (2) on k-th place top down, in the resultant sorted table.

We will only make a small comment. If for the two distinct numbers i, $p \in \{1, \ldots, n\}$ it holds that

$$\max_{j,j\neq i} \mu(C_i, C_j) = \max_{q,q\neq p} \mu(C_p, C_q)$$

this means that criteria C_i and C_p are 'twin criteria' and together enter the set of criteria being in positive consonance. This only may turn out to be important in the situation when the decision maker aims to select k out of n most strongly correlated criteria, and in the sorted table (after Step 1.3.) criterion C_i is on k-th place and criterion C_p which is its 'twin' is listed $(k + 1)$-st. In such a situation, the decision maker should treat both C_i and C_p equally, and either include both of them in his/her shortlist, or ignore both of them during his/her further decision making process. The same holds also if three or more criteria exhibit identical maximal values, this case reduces to the case of two 'twins'.

We can apply the same algorithm to the ν-matrix, though it is expected that there will be differences in the ranking of all n criteria, and the shortlisted k. It is needed to be done in order to exactly define the working pair of threshold (α, β), although the information coming from the comparisons of the values in the ν-matrix against threshold β only has auxiliary role (when we have chosen to focus on *positive* consonance).

Although the algorithm gives easily the order of appearance of the unique correlating criteria, it is also easy and worth checking the number of consonance pairs that are formed for every number of correlating criteria. This number can give us certain understanding of the level of interconnectedness between the involved criteria.

Example

The proposed algorithm is approbated with data from the two extreme years from the discussed above datasets of EU Member States' competitiveness, as evaluated by the World Economic

Forum. We made the calculations for both the positive and the negative consonance, in order to compare the results on this plane, as well as in time.

Number of correlating criteria	Number of pairs of correlating criteria	Criteria ordered by positive consonance	True when $\alpha \geq$
2	1	11	0.86
		12	
4	2	1	0.844
		2	
5	3	6	0.833
6	5	8	0.828
7	6	9	0.823
8	14	5	0.796
9	18	4	0.780
10	37	3	0.693
11	41	7	0.664
12	45	10	0.648

Number of correlating criteria	Number of pairs of correlating criteria	Criteria ordered by negative consonance	True when $\beta \leq$
2	1	1	0.077
		6	
4	2	11	0.079
		12	
5	3	8	0.09
6	4	9	0.095
8	5	4	0.108
		5	
9	8	2	0.114
11	35	3	0.204
		7	
12	54	10	0.307

Table 2. Results for year 2008–2009.

Number of correlating criteria	Number of pairs of correlating criteria	Criteria ordered by positive consonance	True when $\alpha \geq$
2	1	11	0.873
		12	
4	2	1	0.854
		9	
5	3	5	0.847
6	11	2	0.804
7	13	6	0.788
9	20	7	0.749
		8	
10	25	4	0.730
11	37	3	0.675
12	39	10	0.661

Number of correlating criteria	Number of pairs of correlating criteria	Criteria ordered by negative consonance	True when $\beta \leq$
2	1	11	0.071
		12	
4	2	1	0.077
		6	
5	3	5	0.079
6	4	9	0.09
8	13	2	0.135
		7	
9	17	4	0.143
10	19	8	0.146
11	38	3	0.251
12	45	10	0.286

Table 3. Results for year 2013–2014.

We can observe that despite the anticipated differences in the order of ranking of the criteria, certain trends and dependences exist, not only in time, as we already showed in great detail in [3], but also between the respective sub-tables for the positive and the negative consonance. In year 2008–2009, three out of four top-ranked criteria (1, 11, 12) and the three bottom-ranked criteria (3, 7, 10) are the same; in the middle criteria 8 and 9 follow the same order; with the greatest shift observed with criterion 2. In year 2013–2014, again the three top-ranked criteria (1, 11, 12) are identical, and the five bottom-ranked criteria (3, 4, 7, 8, 10) are the same, although in slightly different order. This means that, with slight differences, which stem from inherent uncertainty, we may expect rather close results from the analysis of the discussed data sets will occur, no matter if we give priority to the positive, or to the negative consonance. In both cases we obtain outline of general trends that deserve the attention of economy analysts and national policy makers.

Conclusion

The present work aims to initiate discussions about how the threshold values in the InterCriteria Decision Making approach are defined, in order to be theoretically most precise and practically most useful. With the algorithm proposed here, we give decision makers the possibility to define how many out of the total number of involved criteria they would like to work with, and thus determine the values of the thresholds of positive and negative consonance, in order to shortlist the desired number of criteria and the values of the correlation pairs they form.

In addition, the proposed algorithm directly connects to our idea from [2] for presenting correlating criteria in the form of graph. It is a promising idea to study which of the vertices are more tightly or completely connected with others, which are more loosely connected, and which are isolated or connected by just one arc. Thus, the visual representation of the ICDM results as graph structures may give us extra information about the nature of the criteria involved in the decision making process.

Acknowledgements

The research work reported in the paper is partly supported by the project AComIn “Advanced Computing for Innovation”, grant 316087, funded by the FP7 Capacity Programme (Research Potential of Convergence Regions).

References

[1] Atanassov, K., D. Mavrov, V. Atanassova (2013). Intercriteria decision making. A new approach for multicriteria decision making, based on index matrices and intuitionistic fuzzy sets. *Proc. of 12th International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets*, 11 Oct. 2013, Warsaw, Poland (in press).

[2] Atanassova, V., L. Doukovska, K. Atanassov, D. Mavrov (2014) Intercriteria Decision Making Approach to EU Member States Competitiveness Analysis. *Proc. of 4th Int. Symp. on Business Modelling and Software Design*, Luxembourg, 24–26 June 2014 (in press).

[3] Atanassova, V., L. Doukovska, D. Mavrov, K. Atanassov, Intercriteria Decision Making Approach to EU Member States Competitiveness Analysis: Temporal and Threshold Analysis. *Proc. of 7th Int. IEEE Conference on Intelligent Systems*, Warsaw, 24–26 Sept. 2014 (submitted).

[4] World Economic Forum (2008, 2013). The Global Competitiveness Reports. `http://www.weforum.org/issues/global-competitiveness`.