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Arcs in intuitionistic fuzzy graphs

M. G. Karunambigai[1], R. Parvathi[2] and R. Buvaneswari[3]

[1] Department of Mathematics, Sri Vasavi College,
Erode – 638 316, Tamilnadu, India
e-mail: gkaruns@yahoo.co.in

[2] Department of Mathematics, Vellalar College for Women,
Erode – 638 012, Tamilnadu, India
e-mail: paarvathis@rediffmail.com

[3] Department of Mathematics, Sankara College of Science and Commerce,
Coimbatore – 641 035, Tamilnadu, India
e-mail: buvanaamohan@gmail.com

Abstract: The structure of an Intuitionistic Fuzzy Graph (IFG) depends mainly on its arcs, as in crisp graphs. In an IFG, the arcs are classified into α-strong, β-strong and δ-weak, based on its strength. These arcs are used to study the structure of complete IFG and constant IFG. Their properties have also been studied.
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1 Introduction

Intuitionistic Fuzzy Graph theory was introduced by Krassimir T Atanassov in [1]. In [7], M.G. Karunambigai and R. Parvathi introduced intuitionistic fuzzy graph as a special case of Atanassov's IFG. In [10], these concepts had been applied to find the shortest path in networks using dynamic programming problem approach. Further in [10], some important operations on IFGs are defined and their properties are studied. Constant Intuitionistic Fuzzy Graph was introduced by M.G. Karunambigai, R. Parvathi, and R. Buvaneswari in [8].

In [9]. the authors classified strong arcs into two types namely α-strong, β-strong and introduced two other types of arcs in fuzzy graphs which are not strong and are termed as δ, δ^* arcs.

In graph theory, arc analysis is not very important as all arcs are strong in the sense of [4]. But in IFG, it is very important to identify the nature of arcs and no such analysis on arcs is available in the literature. Also, as far as the applications are concerned, the classification of arcs highlights the importance of each arc, which will be improving the efficiency of the system especially in problems involving networks.

In this paper, depending on the strength of connectedness between two nodes, the authors extended the study of α-strong, β-strong and δ-weak with suitable illustrations. Necessary and sufficient conditions for their equivalence is studied here. The paper is organised as follows:

Section 2 contains preliminaries and in section 3, we introduce the concept of α-strong, β-strong, and δ-weak arcs. In this section, we emphasis that the connectivity of arcs cannot be determined simply by examining the weights of arcs and also we examine the relationship between a strong path and a strongest path in an IFG. We show that an arc (v_i, v_j) of G is an IF bridge if and only if it is α-strong. It is seen that complete IFGs have no δ-arcs and has atmost one α-strong arc. We also analyse the connectivity of arcs in constant IFGs.

2 Preliminaries

In this section, some basic definitions and theorems which are used in constructing the properties relating to IFGs are given.

Definition 2.1. [10] *Minmax Intuitionistic Fuzzy Graph* (IFG) is of the form $G = (V, E)$, where (i) $V = \{v_1, v_2, \ldots v_n\}$ such that $\mu_1 : V \to [0, 1]$ and $\nu_1 : V \to [0, 1]$ denote the degrees of membership and non - membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$, for every $v_i \in V$ $(i = 1, 2, \ldots, n)$.
(ii) $E \subset V \times V$ where $\mu_2 : V \times V \to [0, 1]$ and $\nu_2 : V \times V \to [0, 1]$ are such that

$$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$$

$$\nu_2(v_i, v_j) \leq \max[\nu_1(v_i), \nu_1(v_j)]$$

and $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.
Here the triple $(v_i, \mu_{1i}, \nu_{1i})$ denotes the degree of membership and degree of non - membership of the vertex v_i. The triple $(e_{ij}, \mu_{2ij}, \nu_{2ij})$ denotes the degree of membership and degree of non - membership of the edge relation $e_{ij} = (v_i, v_j)$ on $V \times V$.

Notation: Here after an IFG, $G = (V, E)$ means a Minmax IFG $G = (V, E)$.

Note 1. When $\mu_{2ij} = \nu_{2ij} = 0$ for some i and j, then there is no edge between v_i and v_j. Otherwise there exists an edge between v_i and v_j.

Definition 2.2. [10] An IFG, $G = \langle V, E \rangle$ is said to be a *semi-μ strong* IFG if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ for every i and j.

Definition 2.3. [10] An IFG, $G = \langle V, E \rangle$ is said to be a *semi-ν strong* IFG if $\nu_{2ij} = \max(\nu_{1i}, \nu_{1j})$ for every i and j.

Definition 2.4. [10] An IFG, $G = \langle V, E\rangle$ is said to be *strong* IFG if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\nu_{2ij} = \max(\nu_{1i}, \nu_{1j})$ for all $(v_i, v_j) \in E$.

Definition 2.5. [10] An IFG, $G = \langle V, E\rangle$ is said to be a *complete-μ strong* IFG if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\nu_{2ij} < \max(\nu_{1i}, \nu_{1j})$ for all i and j.

Definition 2.6. [10] An IFG, $G = \langle V, E\rangle$ is said to be a *complete-ν strong* IFG if $\mu_{2ij} < \min(\mu_{1i}, \mu_{1j})$ and $\nu_{2ij} = \max(\nu_{1i}, \nu_{1j})$ for all i and j.

Definition 2.7. [10] An IFG, $G = \langle V, E\rangle$ is said to be a *complete* IFG if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\nu_{2ij} = \max(\nu_{1i}, \nu_{1j})$ for every $v_i, v_j \in V$

Definition 2.8. [8] Let $G = ((\mu_1, \nu_1), (\mu_2, \nu_2))$ be an IFG. The μ- degree of a vertex v_1 is

$$d_\mu(v_i) = \sum_{(v_i,v_j)\in E} \mu_2(v_i, v_j)$$

The ν- degree of a vertex v_1 is

$$d_\nu(v_i) = \sum_{(v_i,v_j)\in E} \nu_2(v_i, v_j)$$

The degree of a vertex is

$$d(v_i) = \left[\sum_{v_i,v_j\in E} (\mu_2(v_i, v_j)), \sum_{(v_i,v_j)\in E} (\nu_2(v_i, v_j))\right] \text{ and } \mu_2(v_i, v_j) = \nu_2(v_i, v_j) = 0 \text{ for } (v_i, v_j) \notin E.$$

Definition 2.9. [8] The minimum μ-degree is $\delta_\mu(G) = \wedge \{d_\mu(v_i)/v_i \in V\}$
The minimum ν-degree is $\delta_\nu(G) = \wedge \{d_\nu(v_i)/v_i \in V\}$
The minimum degree of G is $\delta(G) = \wedge \{d_\mu(v_i), d_\nu(v_i)/v_i \in V\}$
The maximum μ-degree is $\Delta_\mu(G) = \vee \{d_\mu(v_i)/v_i \in V\}$
The maximum ν-degree is $\Delta_\nu(G) = \vee \{d_\nu(v_i)/v_i \in V\}$
The maximum degree of G is $\Delta(G) = \vee \{d_\mu(v_i), d_\nu(v_i)/v_i \in V\}$

Definition 2.10. [8] Let $G : [(\mu_{1i}, \nu_{1i}), (\mu_{2ij}, \nu_{2ij})]$ be an IFG on $G^* = (V, E)$. If $d_\mu(v_i) = k_i$ and $d_\nu(v_j) = k_j$ for all $v_i, v_j \in V$ i.e the graph is called as (k_i, k_j)-IFG (or) constant IFG of degree (k_i, k_j)

Definition 2.11. [8] Let G be an IFG. The total degree of a vertex $v \in V$ is defined as

$$td(v) = \left[\sum_{v_1v_2\in E} d_{\mu_2}(v) + \mu_1(v), \sum_{v_1v_2\in E} d_{\nu_2}(v) + \nu_1(v)\right]$$

If each vertex of G has the same total degree (r_1, r_2), then G is said to be an *IFG of total degree* (r_1, r_2) or a (r_1, r_2)-totally constant IFG.

3 Types of arcs in IFGs and its properties

Definition 3.1. A path P in an IFG sequence of distinct vertices $v_1, v_2, ..., v_n$ for all $(i, j = 1, 2, ..., n)$ such that either one of the following conditions is satisfied.

i) $\mu_{2ij} > 0$ and $\nu_{2ij} = 0$ for some i and j.
ii) $\mu_{2ij} = 0$ and $\nu_{2ij} > 0$ for some i and j.
iii) $\mu_{2ij} > 0$ and $\nu_{2ij} > 0$ for some i and j.

Definition 3.2. The $\mu - strength$ of a path $P = v_1, v_2, ..., v_n$ is defined as min $\{\mu_{2ij}\}$ for all $(i, j = 1, 2, ...n)$ and it is denoted by S_μ.

Definition 3.3. The $\nu - strength$ of a path $P = v_1, v_2, ..., v_n$ is defined as max $\{\nu_{2ij}\}$ for all $(i, j = 1, 2, ...n)$ and it is denoted by S_ν.

Note 2. If same edge possess both the values (S_μ, S_ν), then it is the strength of the path P and is denoted by S_P.

Definition 3.4. If $v_i, v_j \in V \subseteq G$, the μ - strength of connectedness between two nodes v_i and v_j is $CONN_{\mu(G)}(v_i, v_j) = \max\{S_\mu\}$ and ν - strength of connectedness between two nodes v_i and v_j is $CONN_{\nu(G)}(v_i, v_j) = \min\{S_\nu\}$ of all possible paths between v_i and v_j.

Note 3. $CONN_{\mu(G)-(v_i,v_j)}(v_i, v_j), CONN_{\nu(G)-(v_i,v_j)}(v_i, v_j)$ is the strength of connectedness between v_i and v_j in the IFG obtained from G by deleting the arc (v_i, v_j).

Definition 3.5. An arc (v_i, v_j) is said to be a bridge in G, if either $CONN_{\mu(G)-(v_i,v_j)}(v_i, v_j) < CONN_{\mu(G)}(v_i, v_j)$ and

$$CONN_{\nu(G)-(v_i,v_j)}(v_i, v_j) \geq CONN_{\nu(G)}(v_i, v_j)$$

or

$$CONN_{\mu(G)-(v_i,v_j)}(v_i, v_j) \leq CONN_{\mu(G)}(v_i, v_j)$$

and

$$CONN_{\nu(G)-(v_i,v_j)}(v_i, v_j) > CONN_{\nu(G)}(v_i, v_j)$$

for some $v_i, v_j \in V$.
In other words, deleting an edge (v_i, v_j) reduces the strength of connectedness between some pair of vertices (or) (v_i, v_j) is a bridge if there exist vertices v_i, v_j such that (v_i, v_j) is an edge of every strongest path from v_i to v_j.

Definition 3.6. A vertex v_i is said to be a *cut-vertex* in G if deleting a vertex v_i reduces the strength of connectedness between some pair of vertices or v_i is a cut vertex if and only if there exists v_iv_j such that v_i is a vertex of every strongest path from v_i to v_j. In other words, $CONN_{\mu(G)-(v_i,v_j)}(v_i, v_j) \leq CONN_{\mu(G)}(v_i, v_j)$ and

$$CONN_{\nu(G)-(v_i,v_j)}(v_i, v_j) < CONN_{\nu(G)}(v_i, v_j)$$

or

$$CONN_{\mu(G)-(v_i,v_j)}(v_i,v_j) < CONN_{\mu(G)}(v_i,v_j)$$

and

$$CONN_{\nu(G)-(v_i,v_j)}(v_i,v_j) \leq CONN_{\nu(G)}(v_i,v_j)$$

, for some $v_i, v_j \in V$.

Definition 3.7. An arc (v_i, v_j) is said to be a strong arc if $\mu_{2ij} \geq CONN_{\mu(G)}(v_i,v_j)$ and $\nu_{2ij} \leq CONN_{\nu(G)}(v_i,v_j)$ for every $v_i, v_j \in V$.

Definition 3.8. An arc (v_i, v_j) is said to be the weakest arc if $\mu_{2ij} < CONN_{\mu(G)}(v_i,v_j)$ and $\nu_{2ij} > CONN_{\nu(G)}(v_i,v_j)$ for every $v_i, v_j \in V$.

Definition 3.9. In an IFG $G = (V, E)$, a path P between any two nodes is called the strongest path if its strength equals the strength of connectedness $CONN_{\mu(G)}(v_i,v_j)$ and $CONN_{\nu(G)}(v_i,v_j)$ and both the values lie in the same edge.

Definition 3.10. A $v_i - v_j$ path P in an IFG $G = (V, E)$ is called a strong path if P contains only strong arcs.

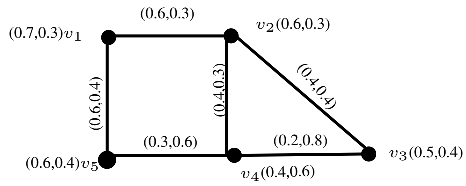

Figure 3.1:

Example 3.1. In Figure 3.1, $(v_1, v_2), (v_1, v_5), (v_2, v_3), (v_2, v_4)$ are strong arcs, $(v_3, v_4), (v_4, v_5)$ are weakest arcs. Depending on the strength of connectedness between the nodes v_1 and v_3, the path $P = v_1v_2v_3$ is strong path and it is also the strongest path.

Depending on the strength of arcs (v_i, v_j) in an IFG, we define the following three different connectivity of arcs.

Definition 3.11. An arc (v_i, v_j) in G is called α-strong if $\mu_{2ij} > CONN_{\mu(G)-(v_i,v_j)}(v_i,v_j)$ and $\nu 2ij < CONN_{\nu(G)-(v_i,v_j)}(v_i,v_j)$.

Definition 3.12. An arc (v_i, v_j) in G is called β-strong if $\mu_{2ij} = CONN_{\mu(G)-(v_i,v_j)}(v_i,v_j)$ and $\nu_{2ij} = CONN_{\nu(G)-(v_i,v_j)}(v_i,v_j)$.

Definition 3.13. An arc (v_i, v_j) in G is called δ-weak if $\mu_{2ij} < CONN_{\mu(G)-(v_i,v_j)}(v_i,v_j)$ and $\nu_{2ij} > CONN_{\nu(G)-(v_i,v_j)}(v_i,v_j)$.

Example 3.2. In Figure 3.2, the arcs $(v_1, v_4), (v_2, v_3), (v_4, v_5)$ are α-strong, (v_1, v_3), (v_3, v_4) are β-strong, (v_1, v_2), (v_3, v_5) are δ-weak.

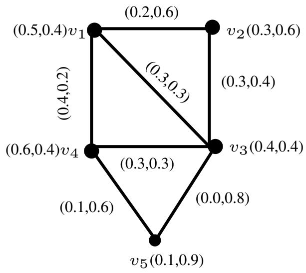

Figure 3.2: Connectivity of arcs in an IFG

Definition 3.14. A path in an IFG $G = (V, E)$ is called an α-strong path if all its arcs are α-strong and also a path is β-strong path if all its arcs are β-strong.

Example 3.3. In Figure 3.2, the path $v_1v_4v_5$ is an α-strong path and the path $v_1v_3v_4$ is a β-strong path.

Now we discuss the connectivity of arcs of the strongest path in G.

Remark 3.1. The strongest path may contain all types of arcs.
In Figure 3.2, the strength of the path $v_1v_2v_3v_4v_5$ is $(0.1, 0.6)$, which is the strongest path between the nodes v_1 and v_5 and it contains all types of arcs, namely α-strong, β-strong, and δ-weak.

Remark 3.2. A strong path contains only α-strong or β-strong arcs, but no δ-weak arcs.
In Figure 3.2, the strong path between the nodes v_1 and v_5 is $v_1v_3v_4v_5$. It contains only α-strong and β-strong arcs.

Remark 3.3. It is to be noted that the strongest path without δ-weak arc is a strong path; for, it contains only α-strong or β-strong arcs. In Figure 3.2, the path $v_1v_3v_4v_5$ is the strongest path without δ-arc between the nodes v_1 and v_5, but it is strong path too.

Proposition 3.1. Let $G = (V, E)$ be an IFG. A strong path P in G from the vertices v_i to v_j in V, is the strongest v_iv_j path in the following cases:
(i) if P contains only α-strong arcs,
(ii) if P is the unique strong v_iv_j path,
(iii) if all v_iv_j paths in G are of equal strength.

Proof. (i) Given that $G = (V, E)$ be an IFG. Let P be a strong v_iv_j path in G contains only α-strong arcs. If possible suppose that P is not the strongest v_iv_j path. Let Q be the strongest v_iv_j path in G. Then PUQ will contain at least one cycle C in which every arc of $C - P$ will have strength greater than strength of P. Thus a weakest arc of C is an arc of P and let (u, v) be such an arc of C. Let C' be the uv path in C, not containing the arc (u, v). Then,

$$\mu_2(u, v) < \text{strength of } C' \leq CONN_{\mu(G)-(v_i,v_j)}(v_i, v_j)$$
$$\nu_2(u, v) > \text{strength of } C' \geq CONN_{\nu(G)-(v_i,v_j)}(v_i, v_j)$$

which implies that (u, v) is not α-strong , a contradiction. Thus P is the strongest v_iv_j path.

(ii) Let $G = (V, E)$ be an IFG. Let P be the unique strong v_iv_j path in G. If possible suppose that P is not the strongest v_iv_j path. Let Q be the strongest $v_iv_j$path in G. Then, strength of $Q >$ strength of P for every arc (u, v) in Q, $\mu_2(u, v) > \mu_2(v_i^{'}, v_j^{'})$ and $\nu_2(u, v) < \nu_2(v_i^{'}, v_j^{'})$ where $(v_i^{'}, v_j^{'})$ is a weakest arc of P.

Claim. Q is a strong v_iv_j path. For, otherwise, if there exists an arc (u, v) in Q which is a δ-arc, then

$$\mu_2(u, v) < CONN_{\mu(G)-(v_i,v_j)}(v_i, v_j) \leq CONN_{\mu(G)}(v_i, v_j)$$
$$\nu_2(u, v) > CONN_{\nu(G)-(v_i,v_j)}(v_i, v_j) \geq CONN_{\nu(G)}(v_i, v_j)$$

and hence

$$\mu_2(u, v) < CONN_{\mu(G)}(v_i, v_j)$$
$$\nu_2(u, v) > CONN_{\nu(G)}(v_i, v_j)$$

Then there exists a path from u to v in G whose strength is greater than $\mu_2(u, v)$ and less than $\nu_2(u, v)$. Let it be $P^{'}$. Let w be the last node after u, common to Q and $P^{'}$ in the uw subpath of $P^{'}$ and $w^{'}$ be the first node before v, common to Q and $P^{'}$ in the $w^{'}v$ subpath of $P^{'}$. (If $P^{'}$ and $Q$are disjoint uv paths then $w = u$ and $w^{'} = v$). Then the path $P^{'}$ consisting of the xw path of Q, $ww^{'}$ path of $P^{'}$, and $w^{'}v_j$ path of Q is an v_iv_j path in G such that strength of $P^{'} >$ strength of Q, contradiction to the assumption that Q is the strongest v_iv_j path in G. Thus (u, v) cannot be a δ-arc, and hence Q is a strong v_iv_j path in G.

Thus we have another strong path from v_i to v_j , other than P, which is a contradiction to the assumption that P is the unique strong v_iv_j path in G. Hence P should be the strongest v_iv_j path in G.

(iii) If every path from v_i to v_j have the same strength, then each such path is the strongest v_iv_j path. In particular, a strong v_iv_j path is the strongest v_iv_j path.

□

In the following theorem, we present a necessary and sufficient condition for IF bridges.

Let G be an IFG. Then an arc (v_i, v_j) of G is an IF bridge iff it is α-strong.

Proof. Let G be an IFG. Let (v_i, v_j) be an IF bridge. By the definition of bridge, we have
$CONN_{\mu(G)-(v_i,v_j)}(v_i, v_j) \leq CONN_{\mu(G)}(v_i, v_j)$, then, $CONN_{\mu(G)}(v_i, v_j) = \mu_{2ij}$, $\mu_{2ij} > CONN_{\mu(G)-(v_i,v_j)}(v_i, v_j)$
and $CONN_{\nu(G)-(v_i,v_j)}(v_i, v_j) > CONN_{\nu(G)}(v_i, v_j)$, then, $CONN_{\nu(G)}(v_i, v_j) = \nu_{2ij}$, $\nu_{2ij} < CONN_{\nu(G)-(v_i,v_j)}(v_i, v_j)$
which shows that (v_i, v_j) is α-strong.

Conversely suppose that (v_i, v_j) is α-strong. Then by definition, it follows that v_iv_j is the unique strongest path from v_i to v_j and the removal of (v_i, v_j) will reduce the strength of connectedness between v_i and v_j. Thus (v_i, v_j) is IF bridge. Note that if an arc (v_i, v_j) in G is an IF bridge, then

$$CONN_{\mu(G)}(v_i, v_j) = \mu_{2ij}$$
$$CONN_{\nu(G)}(v_i, v_j) = \nu_{2ij}$$

The converse of the above theorem need not be true. $\square$

Example 3.4. Let $G = (V, E)$ be an IFG and $V = \{v_1, v_2, v_3\}$, E=$\{(v_1, v_2), (v_1, v_3), (v_2, v_3)\}$. Here $(v_1, v_3), (v_1, v_2)$ are α-strong and these are bridges of G.

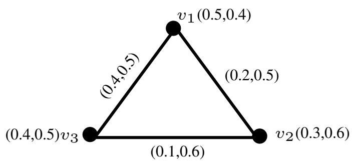

Figure 3.3: Arcs in IF bridge

Lemma 3.2. A complete IFG has no δ-arcs.

Proof. Let G be a complete IFG. If possible assume that G contains a δ-arc (v_i, v_j), then

$$\mu_{2ij} < CONN_{\mu(G)-(v_i,v_j)}(v_i, v_j)$$
$$\nu_{2ij} > CONN_{\nu(G)-(v_i,v_j)}(v_i, v_j)$$

That is, there exists a stronger path P other than the arc (v_i, v_j) from v_i to v_j in G.

Let $\mu_2(v_1, v_2) = p1$, $\nu_2(v_1, v_2) = p2$. The strength of the path P be $(q1, q2)$. Then, $p1 < q1, p2 > q2$.

Let v_3 be the first node in P after v_1. Then $\mu_2(v_1, v_3) > p1$ and $\nu_2(v_1, v_3) < p2$

Similarly, Let v_4 be the last in P before v_2, then $\mu_2(v_2, v_4) > p1$ and $\nu_2(v_2, v_4) < p2$

Since, $\mu_2(v_1, v_2) = p1$, $\nu_2(v_1, v_2) = p2$, atleast one of $\mu_1(v_1)$ or $\mu_1(v_2)$ and $\nu_1(v_1)$ or $\nu_1(v_2)$ should be the $p1$ and $p2$. Now G is being a complete IFG, it gives the contradiction, which completes the proof. $\square$

Example 3.5. *Consider an IFG, $G = (V, E)$ such that $V = \{v_1, v_2, v_3, v_4\}$. This complete IFG has no δ-arcs.*

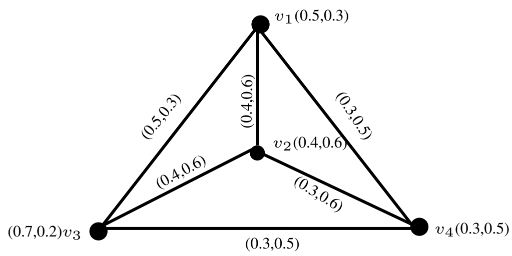

Figure 3.4: Arcs in complete IFG

Lemma 3.3. There exists at most one α-strong arc in a complete IFG.

Example 3.6. *In Figure 3.4, the arc (v_1, v_3) is at most one α-strong in a complete IFG.*

Theorem 3.4. Let G be a complete IFG. Then there exists β-strong paths between any two nodes of G.

Example 3.7. *In Figure 3.4, there exists the path 1: $v_1v_2v_3v_4$, Path 2: $v_1v_2v_4$ are β-strong paths between the nodes v_1 and v_4.*

Theorem 3.5. *Let G be a complete IFG without α-strong arcs. Let P be any v_iv_j path in G. Then the following two conditions are equivalent.*

(i) P is a strong v_iv_j path

(ii) P is the strongest v_iv_j path

Proof. (i)$\Rightarrow$ (ii)

Let G be a complete IFG without a α-strong arcs and let P be any v_iv_j path in G. Assume that P is a strong v_iv_j path. Then by definition, all arcs in G are β-strong arcs.

$$CONN_{\mu(G)-(v_i,v_j)}(v_i, v_j) = \mu_{2ij} = \mu\text{-strength of } P$$
$$CONN_{\nu(G)-(v_i,v_j)}(v_i, v_j) = \nu_{2ij} = \nu\text{-strength of } P$$

Now since G is complete,

$$CONN_{\mu(G)}(v_i, v_j) = \mu_{2ij}$$
$$CONN_{\nu(G)}(v_i, v_j) = \nu_{2ij}$$

From the above

$$CONN_{\mu(G)}(v_i, v_j) = CONN_{\nu(G)}(v_i, v_j) = \text{Strength of } P$$

which implies that P is the strongest path.

(ii) $\Rightarrow$ (i)

Let P be the strongest v_iv_j path in G. Let the path P contains only β-strong arcs and hence is a strong v_iv_j path which completes the proof. $\square$

Theorem 3.6. Let $G = (V, E)$ be an IFG. The strength of connectedness of G is $(CONN_{\mu(G)}(v_i, v_j),$ $CONN_{\nu(G)}(v_i, v_j))$. If G is a constant IFG where underlying graph is an even cycle, then G contains alternatively α-strong and β-strong arcs.

Theorem 3.7. Let $G = (V, E)$ be an IFG. G is constant IFG iff either (μ_{2ij}, ν_{2ij}) constant or alternate edges have same membership values and non-membership values.

Theorem 3.8. For a constant IFG,

$$S_\mu = CONN_{\mu(G)}(v_i, v_j),\ S_\nu = CONN_{\nu(G)}(v_i, v_j)$$

Proof. Let $G = (V, E)$ be a constant IFG.

Case 1: (μ_{2ij}, ν_{2ij}) is a constant function. Then,

$$S_\mu = \text{min } (\mu_{2ij}) = \text{max } (S_\mu) = CONN_{\mu(G)}(v_i, v_j)$$
$$S_\nu = \text{max } (\nu_{2ij}) = min(S_\nu) = CONN_{\nu(G)}(v_i, v_j)$$

Case 2: Alternate edges have same membership and non-membership values.

Let $\mu_{2ij} = a$ and $\nu_{2ij} = b$ for (v_i, v_j) be an edge in G and let $\mu_{2(i+1)(j+1)} = c$ and $\nu_{2(i+1)(j+1)} = d$ for (v_{i+1}, v_{j+1}) in G and $0 \leq a, b, c, d \leq 1$.

Subcase 2.1: Let $a < c$ and $b < d$. Then,

$$S_\mu = a \text{ for all } (v_i, v_j) \in P. \ S_\nu = d \text{ for all } (v_i, v_j) \in P.$$

Therefore,

$$CONN_{\mu(G)}(v_i, v_j) = \max\ (S_\mu) = a \text{ for all } (v_i, v_j) \in P.$$
$$CONN_{\nu(G)}(v_i, v_j) = \min\ (S_\nu) = d \text{ for all } (v_i, v_j) \in P.$$

Subcase 2.2: Let $a > c$ and $b > d$.

$$S_\mu = c \text{ for all } (v_i, v_j) \in P. \ S_\nu = b \text{ for all } (v_i, v_j) \in P.$$

Therefore,

$$CONN_{\mu(G)}(v_i, v_j) = \max\ (S_\mu) = c \text{ for all } (v_i, v_j) \in P.$$
$$CONN_{\nu(G)}(v_i, v_j) = \min\ (S_\nu) = b \text{ for all } (v_i, v_j) \in P.$$

Hence, for an IFG $G = (V, E)$,

$$S_\mu = CONN_{\mu(G)}(v_i, v_j) \text{ and } S_\nu = CONN_{\nu(G)}(v_i, v_j) \text{ for all } (v_i, v_j) \in P.$$

□

4 Conclusion

The arcs in an IFG are very rich both in theoretical developments and applications. In this paper, we have seen that the arcs are classified into α-strong, β-strong and δ-weak, based on their strength. Some interesting properties are also studied. The authors further proposed to work on the properties of the types of arcs.

References

[1] Atanassov, K. *Intuitionistic Fuzzy Sets: Theory and Applications*, Springer Physica-Verlag, Berlin, 1999.

[2] Atanassov, K. On intuitionistic fuzzy graphs and intuitionistic fuzzy relations, *Proceedings of the VI IFSA World Congress*, Sao Paulo, Brazil, July 1995, Vol.1, 551–554.

[3] Atanassov, K., A. Shannon, On a generalization of intuitionistic fuzzy graphs, *Notes on Intuitionistic Fuzzy Sets*, Vol. 12, 2006, No. 1, 24–29.

[4] Bhutani, K. R., A. Rosenfeld, Strong arcs in fuzzy graphs, *Information Sciences*, Vol.152, 2003, 319–322.

[5] Bhutani, K. R., A. Rosenfeld, Fuzzy end nodes in fuzzy graphs, *Information Sciences*, Vol. 152, 2003, 323–326.

[6] Bhutani, K. R., A. Rosenfeld, Geodesics in fuzzy graphs, *Electronic Notes in Discrete Mathematics*, Vol. 15, 2003, 51–54.

[7] Karunambigai, M. G., R. Parvathi, Intuitionistic Fuzzy Graphs, *Proceedings of 9th Fuzzy Days International Conference on Computational Intelligence*, *Advances in soft computing: Computational Intelligence,Theory and Applications*, Springer-Verlag, Vol. 20, 2006, 139–150.

[8] Karunambigai, M. G., R. Parvathi, R. Buvaneswari, Constant IFG, *Notes on Intuitionistic Fuzzy Sets*, Vol. 17, 2011, No. 1, 37–47.

[9] Mathew, S., M. S. Sunitha, Types of arcs in a fuzzy graph, *Information Sciences*, Vol. 179, 2009, 1760–1768.

[10] Parvathi, R., M. G. Karunambigai, K. Atanassov, Operations on Intuitionistic Fuzzy Graphs, *Proceedings of IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, August 2009, 1396–1401.

[11] Sameena, K., M. S. Sunitha, Strong arcs and maximum spanning trees in fuzzy graphs, *International Journal of Mathematical Sciences*, Vol. 5, 2006, 17–20.

[12] Sameena, K., M. S. Sunitha, *Distance in fuzzy graphs*, Ph.D Thesis, National Institute of Technology, Calicut, India, 2008.

[13] Shannon, A., K. Atanassov, A first step to a theory of the intutionistic fuzzy graphs, *Proceedings of the 1st Workshop on Fuzzy Based Expert Systems (D.Lakov, Ed.)*, Sofia, September 28-30, 1994, 59–61.