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Intuitionistic Fuzzy Optimization*

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Abstract A new concept of the optimization problem under uncertainty is proposed and treated in the paper. It is an extension of fuzzy optimization and an application of the intuitionistic fuzzy set concept. Some optimization problems are defined in an intuitionistic fuzzy environment. An approach to solving such problems is proposed.

Key words: fuzzy optimization, intuitionistic fuzzy sets, Bellman-Zadeh's approach

1. Introduction

Deterministic optimization problems are well studied, but they are so strict and in many cases they did not represent exactly the real problem [1]. Usually, it is difficult to describe the constraints of an optimization problem [2]. Practically, a small violation of a given constraint is admissible and it can lead to a more efficient solution of the real problem [2]. Objective formulation represents, in fact, a subjective estimation of a possible effect of a given value of the objective function. In the last two decades optimization problems have been investigated in the sense of fuzzy set theory [3-5]. Fuzzy optimization problems are more flexible and allow to find solutions which are more respective to the real problem. One of the poorly studied problems in this field is definition of membership degrees [6]. The principles of fuzzy optimization problems are critically studied in [7,8]. However, the authors investigate mainly the transformations and the solution procedures [9-12]

On the other hand, fuzzy set theory has been widely developed and various modifications and generalizations have appeared. One of them, are intuitionistic fuzzy sets [13]. They consider not only the degree of membership to a given set, but also the degree of rejection such that the sum of both values is less than one [14]. Applying this concept it is possible to reformulate optimization problem by using degrees of rejection of constraints and values of the objective which are non-admissible. The degrees of acceptance and of rejection can be arbitrary (the sum of both have to be less than one). An approach to solving such intuitionistic fuzzy optimization (IFO) problems is proposed in the paper.

2. Formulation of IFO problem

In general, an optimization problem includes objective(s) and constraints. In fuzzy optimization problems (fuzzy mathematical programming [2], fuzzy optimal control [15], linear programming with fuzzy parameters [5] etc.) the objective(s) and/or constraints are

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represented by fuzzy sets. These fuzzy sets explain the degree of satisfaction of the respective condition and are expressed by their membership functions [1]. IFO problem is formulated by analogy via intuitionistic fuzzy sets over the objective(s) and constraints:

To maximize the degree of acceptance of intuitionistic fuzzy objective(s) AND constraints AND to minimize the degree of rejection of intuitionistic fuzzy objective(s) AND constraints:

$$\max_x \{\mu_i(x)\} \qquad x \in R^n \qquad i = 1,\dots,p+q \tag{1}$$

$$\min_x \{\nu_i(x)\} \qquad i = 1,\dots,p+q$$

subject to

$$\nu_i(x) \geq 0 \qquad i = 1,\dots,p+q$$

$$\mu_i(x) \geq \nu_i(x) \qquad i = 1,\dots,p+q$$

$$\mu_i(x) + \nu_i(x) \leq 1 \qquad i = 1,\dots,p+q$$

where x denotes unknowns
$\mu_i(x)$ denotes degree of membership of x to the i-th intuitionistic fuzzy set
$\nu_i(x)$ denotes degree of rejection of x from the i-th intuitionistic fuzzy set
p denotes the number of objectives
q denotes the number of constraints

Example

Let us consider the following simple transportation problem:

Loads from 3 ports have to be divided between 4 markets. Expenses for a delivery from the i-th port to the j-th market (in thousands dollars) are given in the respective cells of the Table 1. The demands and capacity of loads in every port and market are given (in tons) in the last column (row) of the Table 1. An optimal transportation plan x ($x \in R^n$) which minimizes the costs have to be determined.

	Market 1	Market 2	Market 3	Market 4	Capacity
Port 1	10	7	4	1	400
Port 2	2	7	10	6	150
Port 3	8	5	3	2	300
Demand	200	**200**	100	**350**	

Table 1

Practically, the demands of markets are determined on the basis of sale forecasting. If the prognosis for the Market 4 is *about* 350 [t] the following intuitionistic fuzzy set seems to be a more realistic description:

$$\mu_4 = \frac{1}{1+0.01(x_{14}+x_{24}+x_{34}-350)^2} \tag{2}$$

$$\nu_4 = \frac{(x_{14}+x_{24}+x_{34}-350)^2}{500+(x_{14}+x_{24}+x_{34}-350)^2}$$

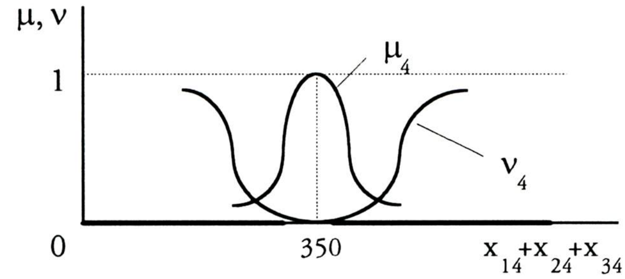

Fig.1

It means that a most strong condition (degree of rejection is also defined which determines the worst case. In general, it can be not simply a complement to the degree of acceptance. The degrees of acceptance (μ_4) of values of the demand in the Market 4 increases rapidly than the rejection (ν_4) decreases such that their sum is less than 1.

By analogy, the rest intuitionistic fuzzy sets can be defined. Three of them determine the demand of the Market 1, Market 2 and Market 3 and the other three sets determine the capacity of the Port 1, Port 2 and Port 3 taking into account subjective estimation of acceptance of various values of the demand and the capacity. In the considered problem the prognosis for the Market 2 is *about* 200 [t]:

$$\mu_2 = \frac{1}{1+0.01(x_{14}+x_{24}+x_{34}-200)^2} \tag{3}$$

$$\nu_2 = \frac{(x_{14}+x_{24}+x_{34}-200)^2}{500+(x_{14}+x_{24}+x_{34}-200)^2}$$

The demand of Market 1 and Market 3 and the capacity of all ports we suppose to be given with crisp sets. The intuitionistic fuzzy objective can be determined by degrees of acceptance (μ_0) and rejection (ν_0) of the cost function as follows:

$$\mu_0 = \begin{cases} 1 & \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} > 2500 \\ \dfrac{2500 - \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij}}{500} & 2000 \le \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} \le 2500 \\ 0 & \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} < 2000 \end{cases} \tag{4}$$

$$\nu_0 = \begin{cases} 1 & \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} > 2500 \\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} - 2200)^2}{90000} & 2200 \le \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} \le 2500 \\ 0 & \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} < 2200 \end{cases}$$

where $c = \begin{vmatrix} 10 & 7 & 4 & 1 \\ 2 & 7 & 10 & 6 \\ 8 & 5 & 3 & 2 \end{vmatrix}$

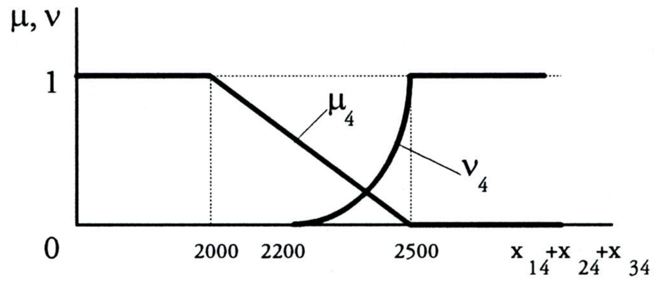

Fig.2

In fact, the set <μ_0,ν_0> is fuzzy set for $\sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} < 2000$ and $\sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} > 2500$ and intuitionistic fuzzy set within this range:

$$\mu_4 + \nu_4 = 1 \quad \text{for } \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} > 2500 \text{ and } \quad \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} < 2000$$

$$\mu_4 + \nu_4 < 1 \quad \text{for } 2000 \le \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} \le 2500$$

The optimism dominates in this variant. The opposite case is also possible when the pessimism dominates. For example:

$$\mu_0 = \begin{cases} 1 & \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} > 2500 \\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} - 2000)^2}{250000} & 2000 \le \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} \le 2500 \\ 0 & \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} < 2000 \end{cases} \tag{5}$$

$$\nu_0 = \begin{cases} 1 & \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} > 2500 \\ \dfrac{2500 - \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij}}{300} & 2200 \le \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} \le 2500 \\ 0 & \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} < 2200 \end{cases}$$

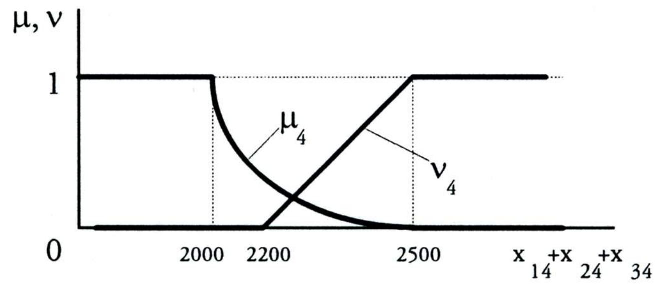

Fig.3

It is possible that a part of constraints and objective(s) are intuitionistic fuzzy and other are fuzzy or crisp. This IFO problem is formulated as follows:

$$\{\mu_0(x),\mu_2(x),\mu_4(x)\} \dashrightarrow \max \qquad (6)$$

$$\{\nu_0(x),\nu_2(x),\nu_4(x)\} \dashrightarrow \min$$

subject to
$$\mu_k(x) + \nu_k(x) \leq 1 \qquad k = \{0,2,4\}$$
$$\nu_k(x) \geq 0 \qquad k = \{0,2,4\}$$
$$\mu_k(x) \geq \nu_k(x)$$
$$x_{11}+x_{21}+x_{31}=200$$
$$x_{13}+x_{23}+x_{33}=100$$
$$x_{11}+x_{12}+x_{13}+x_{14}=400$$
$$x_{21}+x_{22}+x_{23}+x_{24}=150$$
$$x_{31}+x_{32}+x_{33}+x_{34}=300$$
$$x_{ij} > 0 \qquad i=1,\ldots,3;\ j=1,\ldots,4$$

3. An Approach to solving IFO problems

IFO problem such as fuzzy optimization problems can be represented as a two-stage process which includes aggregation of constraints and objective(s) and defuzzification (maximization of aggregated function) [7,8]. Usually applied Bellman-Zadeh's approach [16] for fuzzy optimization problem solving realizes min-aggregator. Conjunction of intuitionistic fuzzy sets is defined as [13]:

$$G \cap C = \{ <x, \mu_G(x)\cap\mu_C(x), \nu_G(x)\cup\nu_C(x)>, x\in R^n\} \qquad (7)$$

where G denotes an intuitionistic fuzzy objective (gain)

C denotes an intuitionistic fuzzy constarint

This operator can be easily generalized and applied to IFO problem:

$$D = \{<x, \mu_D(x), \nu_D(x)>, x\in R^n\} \qquad (8)$$

$$\mu_D(x) = \bigcap_{i=1}^{p+q} \mu_i(x)$$

$$\nu_D(x) = \bigcup_{i=1}^{p+q} \nu_i(x)$$

where D denotes intuitionistic fuzzy set of the decision

If min-aggregator is used for conjunction and max-operator for disjunction, IFO problem can be transformed to the following crisp (non-fuzzy) optimization problem which can be easily solved numerically by simnplex or gradient technique:

$$(\alpha-\beta) \dashrightarrow \max \qquad (9)$$

subject to
$$\alpha \leq \mu_i(x) \qquad i=1,\ldots,p+q$$
$$\beta \geq \nu_i(x) \qquad i=1,\ldots,p+q$$
$$\alpha \geq \beta$$
$$\beta \geq 0$$
$$\alpha + \beta \leq 1$$

where $\alpha = \min_{i=1}^{p+q} \mu_i(x)$

$$\beta = \max_{i=1}^{p+q} \nu_i(x)$$

Example

IFO problem (5) is transformed to the following problem:

$$(\alpha-\beta) \text{ ---> max} \tag{10}$$

subject to
$$\alpha \leq \frac{2500 - \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij}}{500}$$

$$\alpha \leq \frac{1}{1+0.01(x_{14}+x_{24}+x_{34}-200)^2}$$

$$\alpha \leq \frac{1}{1+0.01(x_{14}+x_{24}+x_{34}-350)^2}$$

$$\beta \geq \frac{(\sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} -2500)^2}{250000}$$

$$\beta \geq \frac{(x_{14}+x_{24}+x_{34}-350)^2}{500+(x_{14}+x_{24}+x_{34}-350)^2}$$

$$\beta \geq \frac{(x_{14}+x_{24}+x_{34}-200)^2}{500+(x_{14}+x_{24}+x_{34}-200)^2}$$

$$\beta \geq 0$$
$$\alpha \geq \beta$$
$$\alpha + \beta \leq 1$$
$$x_{11}+x_{21}+x_{31}=200$$
$$x_{13}+x_{23}+x_{33}=100$$
$$x_{11}+x_{12}+x_{13}+x_{14}=400$$
$$x_{21}+x_{22}+x_{23}+x_{24}=150$$
$$x_{31}+x_{32}+x_{33}+x_{34}=300$$
$$x_{ij}>0 \qquad i=1,\dots,3;\ j=1,\dots,4$$

The intuitionistic fuzzy optimal transportation plan is:

	Market 1	Market 2	Market 3	Market 4	Capacity
Port 1			31.8	368.2	400
Port 2	150				150
Port 3	50	181.8	68.2		300
Demand	200	**181.8**	100	**368.2**	

Table 2

It leads to costs of \$2 309000. This solution satisfies the objective with degree <0.382;0.382>. The solutions of analogous crisp LP problem and FLP problem are (resp.) costs of \$ 2 400 000 and of \$ 2 330 000 and plans given in Table 3 (the solutions of LP problem are in the left corner of the cells and solutions of FLP problem in the rigth corners). The degree of satisfaction of fuzzy objective in FLP is 0.34.

	Market 1		Market 2		Market 3		Market 4		Capacity
Port 1					50	36	350	364	400
Port 2	150	150							150
Port 3	50	50	200	186	50	64			300
Demand	200		**200**	**186**	100		**350**	**364**	

Table 3

It is easy to see that the costs in IFO problem are lowest with 4% and 1% (resp.).

4. Conclusion

A new concept to optimization problem in an intuitionistic fuzzy environment is introduced in the paper. It can be considered as an extension of fuzzy optimization and as an application of intuitionistic fuzzy sets. An approach to solving IFO problems and illustrative exmple are proposed.

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