Optimal weighting method for interval-valued intuitionistic fuzzy opinions

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Abstract: In this work, we propose a method to achieve consensus in a group decision making situation, where the opinions are described by interval-valued intuitionistic fuzzy sets. Optimality is achieved by minimizing weighed incoherencies. An illustrative example is proposed.

Keywords: Optimal weighing, Intuitionistic fuzzy set, Interval-valued intuitionistic fuzzy set.

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1 Introduction

Since unanimity is rarely achieved in group decision making, a certain level of consensus might be acceptable. The achieved consensus must take into consideration human uncertainty, to do so, we model the expressed opinions by interval-valued intuitionistic fuzzy numbers. In the rest of this manuscript the needed background for fuzzy logic is presented in Section 2, while Section 3 encompasses the used algorithm with an illustrative example.
2 Preliminaries

In classical sets, each element either belongs to a certain set or not at all, while in fuzzy set theory a certain degree of membership is tolerated [13]. Let $X$ be a set and $F$ be a fuzzy set in $X$, where $F$ is defined as follows:

$$F = \{ (x, \mu_F(x)) \mid x \in X \},$$

where $\mu_F(x)$ is the degree of membership of $x$ in $F$ in the unity interval:

$$\mu_F : X \rightarrow [0, 1].$$

Atanassov [1, 2] extended the notion of fuzzy sets to intuitionistic fuzzy sets (IFS). An intuitionistic fuzzy set $A$ is defined as follows:

$$A = \{ (x, \mu_A(x), \nu_A(x)) \mid x \in X \},$$

where $\mu_A(x)$ and $\nu_A(x)$ are respectively the membership function and the non-membership function, with the following conditions:

$$\mu_A : X \rightarrow [0, 1], \nu_A : X \rightarrow [0, 1]$$

$$\mu_A(x) + \nu_A(x) \leq 1 \ \forall x \in X.$$

The hesitancy function can be computed by the following formula:

$$\pi_A(x) = 1 - [\mu_A(x) + \nu_A(x)] \ \forall x \in X.$$

The fuzzy sets were presented in order to permit human uncertainty, while it is counterintuitive to demand an exact membership function and non-membership function. In that sense Atanassov and Gargov [4] extended the IFS to interval-valued intuitionistic fuzzy sets (IVIFS) fulfilling the following:

$$A = \{ (x, M_A(x), N_A(x)) \mid x \in X \},$$

where $M_A(x) \subset [0, 1]$ and $N_A(x) \subset [0, 1]$ are respectively the membership interval and the non-membership interval, and for these two intervals it holds that [4]:

$$\text{sup} M_A(x) + \text{sup} N_A(x) \leq 1.$$

For convenience, we note an interval-valued fuzzy number as $\beta = ([a, b], [c, d])$ where $a = \text{inf} M_\beta$, $b = \text{sup} M_\beta$, $c = \text{inf} N_\beta$ and $d = \text{sup} N_\beta$ are interval numbers.

Let $\beta_i = ([a_{\beta_i}, b_{\beta_i}], [c_{\beta_i}, d_{\beta_i}])$ be a collection of interval-valued intuitionistic fuzzy numbers, the main aggregation operators are the interval-valued intuitionistic fuzzy weighting averaging $IIFWA$, and the interval-valued intuitionistic fuzzy weighting geometric $IIFWG$ [11], hence the aggregated value according to $IIFWA$ is:

$$IIFWA_w(\beta_1, \beta_2, \ldots, \beta_n) = ([a, b], [c, d]),$$

where

$$a = 1 - \prod_{i=1}^{n}(1 - a_{\beta_i}), \ b = 1 - \prod_{i=1}^{n}(1 - b_{\beta_i}), \ c = 1 - \prod_{i=1}^{n} c_{\beta_i}, \ d = 1 - \prod_{i=1}^{n} d_{\beta_i}$$

and $w_i$ are the weights of the respective $\beta_i$.

The main question is how to attribute the correct weight to each decision.
3 Proposed method

Several methods exist in the literature to attribute the correct weights [5, 7, 8, 12, 14]. Here we propose to follow the procedure proposed in [7] to the IVIFS. The desired consensus is achieved by minimizing the following function:

\[
\min_{M \times \mathbb{R}^4} \sum_{i=1}^{n} w_i^m \ast \left( c - S(\beta_i, \beta) \right),
\]

where \( M = \left\{ W = (w_1, w_2, \ldots, w_n), \sum_{i=1}^{n} w_i \geq 0 \right\} \), \( m \) is a positive integer \((m > 1)\), \( S(\beta_i, \beta) \) is the similarity between the \( i \)-th decision and the consensus, \( c \) is a real number \((c > 1)\).

Several methods have been proposed to compute similarity from a distance [6, 9, 10], here we adopt the Hamming distance for IVIFS [3], and derive the similarity as by Santini and Jain [9] to ease computation. Hence, the distance between two IVIFS \( \beta_1 \) and \( \beta_2 \) is:

\[
D(\beta_1, \beta_2) = \frac{1}{2} \left( |a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2| \right).
\]

3.1 Algorithm

Step 1: Each expert \( E_i : 1 \leq i \leq n \) assesses each alternative using an IVIFS.

Step 2: Set the initial aggregation weights such that \( 0 \leq w_i^{(0)} \leq 1 \) and \( \sum_{i=1}^{n} w_i = 1 \). The iterations are labeled \( l = 0, 2, \ldots \).

Step 3: Compute the aggregated consensus at Step \( l \):

\[
\beta^l = IIWA(\beta_i).
\]

Step 4: Let \( W^l = \left( w_1^{(l)}, w_2^{(l)}, \ldots, w_n^{(l)} \right) \). Compute \( W^{l+1} \) as follows:

\[
W^{l+1} = \frac{\left( 1 / (c - S(\beta^l, \beta_j)) \right)^{1/(m-1)}}{\sum_{j=1}^{n} \left( 1 / (c - S((\beta^l, \beta_j)) \right)^{1/(m-1)}}.
\]

Step 5: If \( \| W^{l+1} - W^l \| > \varepsilon \), set \( l = l + 1 \) and go to Step 3. Else Stop.

3.2 Illustrative example

Let three experts assess an alternative as follows: \( \beta_1 = \left( [0.22, 0.31]; [0.23, 0.54] \right) \), \( \beta_2 = \left( [0.04, 0.21]; [0.35, 0.46] \right) \) and \( \beta_3 = \left( [0.25, 0.27]; [0.23, 0.4] \right) \).

We choose \( m = 2 \), \( c = 1.5 \) and \( W^0 = (1, 0, 0) \). Table 1 resumes the evolution of weights in each iteration.
<table>
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<tr>
<th>Iteration</th>
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<th>Expert 2</th>
<th>Expert 3</th>
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<td>6</td>
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<td>0.323968922812703</td>
<td>0.339877914618833</td>
</tr>
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</table>

Table 1. Results of each iteration

4 Conclusion

In this work, we adapted Lees algorithm to achieve group consensus in the interval-valued intuitionistic fuzzy context. We restricted ourselves to the interval-valued intuitionistic fuzzy weighting averaging operator to merge opinions, used the hamming metric to compute their distances and derived similarities as a distance dual. In future research, we will investigate different combinations of aggregation operators, similarities and distances that may be more appropriate in such situations.

References


