Notes on Intuitionistic Fuzzy Sets
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Hesitant intuitionistic fuzzy linguistic term sets

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Abstract: Dealing with uncertainty is a challenging problem, and different tools have been proposed in the literature to deal with it. Intuitionistic fuzzy sets was presented to manage situations in which experts have some membership and non-membership value to assess an alternative. Hesitant fuzzy linguistic term sets was used to handle such situations in which experts hesitate between several possible linguistic values or interval to assess an alternative and variable in qualitative settings. In this paper, the concept of an hesitant intuitionistic fuzzy linguistic term set is introduced to provide a linguistic and computational basis to manage the situations in which experts assess an alternative in possible linguistic interval and impossible linguistic interval. Distance measure is defined between any two elements of hesitant intuitionistic fuzzy linguistic term set. Technique for order preference by similarity to ideal solution is proposed in hesitant intuitionistic fuzzy linguistic term set setting for multi-criteria group decision making. An example is given to elaborate the proposed method for the selection of the best alternative as well as rank the alternatives from the best to worst.
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1 Introduction

Due to the inherent vagueness and uncertainty of human preferences, the best expression of decision makers comes in natural language. Thus using linguistic variables is much more realistic then using numerical values. Fuzzy set theory have been successfully applied to handle vague, imperfect and imprecise information [22]. Ordinary fuzzy set theory is limited for the modelling of decision problems in which two or more sources of vagueness appear simultaneously. To overcome this situation extensions of fuzzy set are given like intuitionistic fuzzy set; hesitant fuzzy set; hesitant fuzzy linguistic term set. Atanassov [1, 2] introduced the concept of intuitionistic fuzzy set (IFS) characterized by a membership function and a non-membership function, which is more suitable for dealing with fuzziness and uncertainty than the ordinary fuzzy set. The IFS is highly useful in depicting uncertainty and vagueness of an object, and thus can be used as a powerful tool to express data information under various fuzzy environments. Recently, the intuitionistic fuzzy set has been widely applied to decision making problems [4, 5, 9, 11]. Torra [19] introduced an extension for fuzzy set to manage those situations in which several values are possible for the definition of a membership function of a fuzzy set. Hesitant fuzzy set (HFS) is defined in terms of a function that returns a set of membership values for each element in the domain. Zadeh [23] proposed the concept of linguistic variable and used it in approximate reasoning . Mendel [14] used the term computing with words for the calculation work on linguistic variables and applied it for judgment making. Afterward Martinez et al. [13], used the concept of computing with words in multi-criteria decision making problems, models and applications . The relationship between decision making and computing with words is also discussed in [16]. They focused on symbolic linguistic computing models that have been used in linguistic decision making. Furthermore, the concept of hesitant fuzzy linguistic term set (HFLTS) is introduced by Rodríguez et al. [17] and this concept is based on the HFS and fuzzy linguistic approach. A group decision making model based on HFLTS was given by Rodríguez et al. [18].

Technique for order preference by similarity to ideal solution (TOPSIS) is a useful technique for the selection of the best alternative and also for the ranking of alternatives. Hwang and Yoon [10] developed TOPSIS to multi-attribute/multi-criteria decision making (MADM/MCDM) problems. TOPSIS is extended to fuzzy environment [6, 7, 8, 20]. Xu and Chen [21] used fuzzy TOPSIS for multiple attribute group decision making. Rashid et al. [15] proposed a generalized interval valued trapezoidal fuzzy number based TOPSIS that reflected subjective judgment and objective information in real life. The weights of criteria and performance rating values of criteria were linguistic variables expressed as generalized interval valued trapezoidal fuzzy numbers for the selection of suitable robot. Beg and Rashid [4] extended fuzzy TOPSIS and used it for multi-criteria trapezoidal valued intuitionistic fuzzy decision making. In [3] TOPSIS is further modified for HFLTS to solve the multi-criteria group decision making problems. HFLTS is limited for the modelling of decision problems in which two or more sources of vagueness appear simultaneously. In fuzzy decision making environment, decision makers may hesitate to choose appropriate linguistic term or linguistic interval to assess alternatives. Similarly, they may feel some hesitation to decide that some linguistic term or linguistic interval is not possible to assess the alternative. Thus to manage this type of situation, we introduce the concept of hesitant intuitionistic fuzzy linguistic term set in this paper. hesitant intuitionistic fuzzy linguistic term set is characterized by a membership function and a non-membership function, which is more suitable for dealing with fuzziness and uncertainty than the HFLTS. In this paper, first we introduce the notion of hesitant intuitionistic fuzzy linguistic term set (HIFLTS) and then we extend fuzzy TOPSIS for hesitant intuitionistic fuzzy linguistic term sets with the opinion of finite decision makers about the criteria of alternatives. We proposed a method for aggregation of the experts' opinion on different criteria for alternatives, where the opinion of the experts are represented by hesitant intuitionistic fuzzy linguistic term sets.

This article is organized as follows: In Section 2, some basic preliminary concepts are discussed. In Section 3, we introduce the notion of hesitant intuitionistic fuzzy linguistic term set and distance between any two elements of hesitant intuitionistic fuzzy linguistic term set. In Section 4, we proposed a modified TOPSIS for HIFLTS. In Section 5, an example is given to show the practicality and feasibility of the modified TOPSIS by the ranking of alternatives. In Section 6, conclusion is given.

2 Preliminaries

Next we review some basic concepts, necessary to understand our proposal.

Let X be a crisp universe of generic elements, a *fuzzy set* B in the universe X is a mapping from X to $[0, 1]$. For any $x \in X$, the value $B(x)$ is called the *degree of membership of* x in B. Atanassov [1] defined the concept of intuitionistic fuzzy set by generalizing the concept of fuzzy set.

Definition 1 [1] Let X be a universe of discourse, an intuitionistic fuzzy set in X is an expression A given by $A = \{(x, t_A(x), f_A(x))|x \in X\}$, where $t_A : X \to [0, 1]$, $f_A : X \to [0, 1]$ with the condition: $0 \leq t_A(x) + f_A(x) \leq 1$, for all x in X. The numbers $t_A(x)$ and $f_A(x)$ represent the degree of membership and the degree of non-membership of the element x in the set A, respectively.

If $t_A(x) + f_A(x) = 1$, for all $x \in X$. Then the intuitionistic fuzzy set A is reduced to a fuzzy set. The concept of Hesitant Fuzzy Linguistic Term Set (HFLTS) was introduced by Rodríguez et al. [17] and this concept is based on the HFS and fuzzy linguistic approach.

Some definitions, basic operations and computations performed on HFLTS due to [17] are given below.

Definition 2 [17] Let S be a linguistic term set, $S = \{s_0, ..., s_g\}$, an HFLTS, H_S, is an ordered finite subset of the consecutive linguistic terms of S.

Let S be a linguistic term set, $S = \{s_0, ..., s_g\}$, we then define the empty HFLTS and the full HFLTS for a linguistic variable ϑ as follows.

1) empty HFLTS: $H_S(\vartheta) = \{\ \}$,

2) full HFLTS: $H_S(\vartheta) = S$.

Any other HFLTS is formed with at least one linguistic term in S.

Example 1 Let S be a linguistic term set (Fig. 1), $S = \{s_0$: Extremely Poor (EP), s_1 : Very Poor (VP), s_2 : Poor (P), s_3 : Medium (M), s_4 : Good (G), s_5 : Very Good (VG), s_6 : Extremely Good (EG)$\}$, a different HFLTS might be $H_S(\vartheta) = \{s_1$: Very Poor, s_2 : Poor, s_3 : Medium, s_4 : Good$\}$ *and* $H_S(\vartheta) = \{s_3$: Medium, s_4 : Good, s_5 : Very Good$\}$.

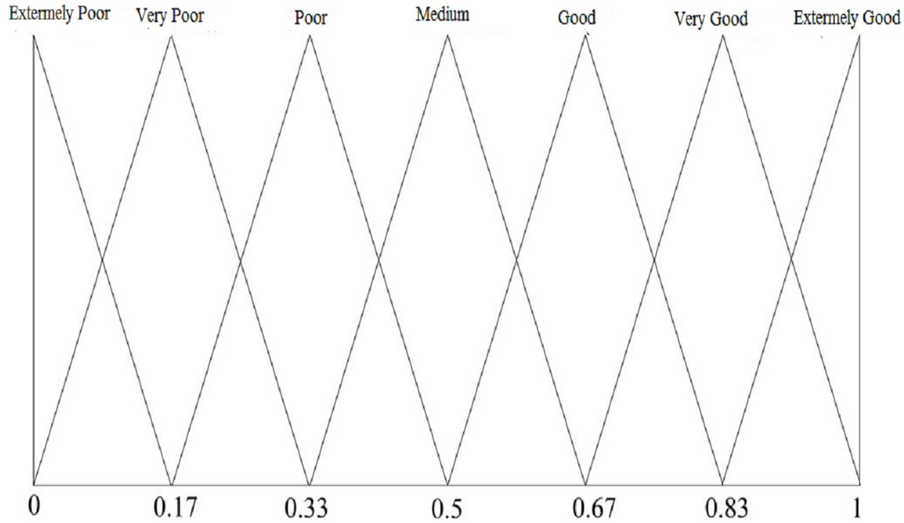

Figure 1: Set of seven terms with its semantics.

3 Hesitant intuitionistic fuzzy linguistic term sets

First we proposed the concept of hesitant intuitionistic fuzzy linguistic term set (HIFLTS). Some of the definitions, operations and computations performed on HIFLTS are also given.

Definition 3 Let S be an ordered finite set of linguistic terms, $S = \{s_0, ..., s_g\}$, A is an ordered finite subset of the consecutive linguistic terms of S. Then the 'max' and 'min' operators on set A are defined as:

1) $\max(A) = \max(s_i) = s_j$, $s_i \in A$ and $s_i \leq s_j \quad \forall i$;

2) $\min(A) = \min(s_i) = s_j$, $s_i \in A$ and $s_i \geq s_j \quad \forall i$.

Definition 4 An HIFLTS on X are functions h and h' that when applied to X return ordered finite subsets of the consecutive linguistic term set, $S = \{s_0, ..., s_g\}$, which can be represented as the following mathematical symbol:

$$E = \{(x, h(x), h'(x)) | x \in X\},$$

where $h(x)$ and $h'(x)$ are subsets of the consecutive linguistic terms of S, denoting the possible membership degrees and non-membership degrees of the element $x \in X$ to the set E with the conditions that $\max(h(x)) + \min(h'(x)) \leq s_g$ and $\min(h(x)) + \max(h'(x)) \leq s_g$.

For convenience, $(h(x), h'(x))$ an hesitant intuitionistic fuzzy linguistic term element (HIFLTE). Any other HIFLTS is formed with at least one linguistic term in S.

Example 2 Let S be a linguistic term set, $S = \{s_0$: Extremely Poor (EP), s_1 : Very Poor (VP), s_2 : Poor (P), s_3 : Medium (M), s_4 : Good (G), s_5 : Very Good (VG), s_6 : Extremely Good (EG)$\}$, an HIFLTS is $A = \{(x_1$, (s_1 : Very Poor, s_2 : Poor, s_3 : Medium), (s_3 : Medium, s_4 : Good)), (x_2, (s_4 : Good, s_5 : Very Good), (s_1 : Very Poor, s_2 : Poor))$\}$.

It is easy to check the conditions of HIFLTS for A.

Envelope of hesitant fuzzy linguistic term set is applied in multi-criteria decision making methods [3, 12]. Motivated by the concept of envelope of hesitant fuzzy linguistic term set in [17], we introduce the envelope of hesitant intuitionistic fuzzy linguistic term set.

Definition 5 The envelope of an HIFLTS A, is defined as:

$$env(A) = \{(x_{1,}[\min(h(x_1)), \max(h(x_1))], [\min(h'(x_1)), \max(h'(x_1))]) | x_i \in A)\}.$$

For convenience, env(A(x))=$[\min(h(x)), \max(h(x))], [\min(h'(x)), \max(h'(x))]$ is an envelope of hesitant intuitionistic fuzzy linguistic term element (EHIFLTE).

Example 3 Let S be a linguistic term set (Fig. 1), $S = \{s_0$: Extremely Poor (EP), s_1 : Very Poor (VP), s_2 : Poor (P), s_3 : Medium (M), s_4 : Good (G), s_5 : Very Good (VG), s_6 : Extremely Good (EG)$\}$, an HIFLTS might be $A = \{(x_1$, $(s_1$: Very Poor, s_2 : Poor, s_3 : Medium*),* $(s_3$: Medium, s_4 : Good*)*$)$, $(x_2$, *(*s_4 : Good, s_5 : Very Good*), (*s_1 : Very Poor, s_2 : Poor*))*$\}$. Its envelope can be calculated as:

$\min(h(x_1)) = \min(s_1$: Very Poor, s_2 : Poor, s_3 : Medium)=s$_1$;
$\max(h(x_1)) = \max(s_1$: Very Poor, s_2 : Poor, s_3 : Medium)=s$_3$;
$\min(h'(x_1)) = \min(s_3$: Medium, s_4 : Good*)*$= s_3$;
$\max(h'(x_1)) = \max(s_3$: Medium, s_4 : Good*)*$= s_4$;
$\min(h(x_2)) = \min(s_4$: Good, s_5 : Very Good)=s$_4$;
$\max(h(x_2)) = \max(s_4$: Good, s_5 : Very Good)=s$_5$;
$\min(h'(x_2)) = \min(s_1$: Very Poor, s_2 : Poor*)*$= s_1$;
$\max(h'(x_2)) = \max(s_1$: Very Poor, s_2 : Poor*)*$= s_2$;

Then

$$\begin{aligned} env(A) &= \{(x_1, env(A(x_1))), (x_2, env(A(x_2)))\}. \\ &= \{(x_1, [s_1, s_3], [s_3, s_4]), (x_2, [s_4, s_5], [s_1, s_2])\}. \end{aligned}$$

Consequently, envelope of HIFLTS gives the complete information of HIFLTS. If we know the envelope of HIFLTS and linguistic term set then we can write HIFLTS.
We proposed the distance measure between any two elements of HIFLTS which can be calculated with the help of envelope of that elements.

Definition 6 Let $A(x)$ and $A(y)$ be the two elements of HIFLTS and $env(A(x)) = ([s_{p_x}, s_{q_x}]$, $[s_{p'_x}, s_{q'_x}])$ and $env(A(y)) = ([s_{p_y}, s_{q_y}], [s_{p'_y}, s_{q'_y}])$, then distance '$d$' between $A(x)$ and $A(y)$ is defined as:

$$d(A(x), A(y)) = |p_x - p_y| + |q_x - q_y| + |p'_x - p'_y| + |q'_x - q'_y|.$$

It is easy to show that this distance 'd' satisfies the following properties.

1. $d(A(x), A(y)) = 0$ if and only if $A(x) = A(y)$;

2. $d(A(x), A(y)) = d(A(y), A(x))$.

4 TOPSIS for HIFLTS

In this section we give steps for the complete construction of TOPSIS in HIFLTS.

Step 1. Let $\tilde{X}^l = [(H^l_{S_{ij}}, H'^l_{S_{ij}})]_{m\times n}$ be a fuzzy decision matrix for the MCDM problem and the following notations are used to depict the considered problems:

$M = \{m_1, m_2, \ldots, m_K\}$ is the set of the decision makers or experts involved in the decision making process;

$P = \{P_1, P_2, \ldots, P_m\}$ is the set of the considered alternatives;

$Q = \{Q_1, Q_2, \ldots, Q_n\}$ is the set of the criteria used for evaluating the alternatives.

Performance of alternative P_i with respect to decision maker m_l and criterion Q_j is denoted as HIFLTE $H^l_{S_{ij}}$, in a group decision environment with K persons.

Step 2. We calculate the one decision matrix X by aggregating the opinions of DMs ($\tilde{X}^1$, $\tilde{X}^2$, $\cdots$, $\tilde{X}^K$);

$X = [x_{ij}]$, where $x_{ij} = ([s_{p_{ij}}, s_{q_{ij}}], [s_{p'_{ij}}, s_{q'_{ij}}])$ where

$$s_{p_{ij}} = \min\left\{\min_{l=1}^{K}(\max H^l_{S_{ij}}), \max_{l=1}^{K}(\min H^l_{S_{ij}})\right\},$$

$$s_{q_{ij}} = \max\left\{\min_{l=1}^{K}(\max H^l_{S_{ij}}), \max_{l=1}^{K}(\min H^l_{S_{ij}})\right\},$$

$$s_{p'_{ij}} = \min\left\{\min_{l=1}^{K}(\max H'^l_{S_{ij}}), \max_{l=1}^{K}(\min H'^l_{S_{ij}})\right\},$$

and

$$s_{q'_{ij}} = \max\left\{\min_{l=1}^{K}(\max H'^l_{S_{ij}}), \max_{l=1}^{K}(\min H'^l_{S_{ij}})\right\}.$$

Performance of alternative P_i with respect to criterion Q_j is denoted as x_{ij}, in an aggregated matrix X.

The final aggregation result calculated in this step is also HIFLTS. For the clear illustration of this fact, we have to prove that $s_{p_{ij}} + s_{q'_{ij}} \leq s_g$ and $s_{q_{ij}} + s_{p'_{ij}} \leq s_g$. It is known that $[H^l_{S_{ij}}, H'^l_{S_{ij}}]$ is an HIFLTS for every l expert, i alternative and j criteria. By the conditions of HIFLTS $\min H^l_{S_{ij}} + \max H'^l_{S_{ij}} \leq s_g$ and $\max H^l_{S_{ij}} + \min H'^l_{S_{ij}} \leq s_g$. So the above mention simple construction of $s_{p_{ij}}$, $s_{q_{ij}}$, $s_{p'_{ij}}$ and $s_{q'_{ij}}$ guarantee that the x_{ij} is an HIFLTS.

Step 3. Let Ω_b be a collection of benefit criteria (i.e., the larger Q_j, the greater preference) and Ω_c be a collection of cost criteria (i.e., the smaller Q_j, the greater preference). The HIFLTS positive-ideal solution (HIFLTS-PIS), denoted as $\tilde{A}^+ = (\tilde{V}_1^+ \;\; \tilde{V}_2^+ \;\; \ldots \;\; \tilde{V}_n^+)$, and the HIFLTS negative-ideal solution (HIFLTS-NIS), denoted as $\tilde{A}^- = (\tilde{V}_1^- \;\; \tilde{V}_2^- \;\; \ldots \;\; \tilde{V}_n^-)$, are defined as follows:

$$P^+ = \Big[\Big[\Big(\Big(\max_{l=1}^{K}\Big(\max_i \min H^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_b, \Big(\min_{l=1}^{K}\Big(\min_i \min H^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_Q\Big),$$
$$\Big(\Big(\max_{l=1}^{K}\Big(\max_i \max H^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_b, \Big(\min_{l=1}^{K}\Big(\min_i \max H^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_c\Big)\Big],$$
$$\Big[\Big(\Big(\max_{l=1}^{K}\Big(\max_i \min H'^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_c, \Big(\min_{l=1}^{K}\Big(\min_i \min H'^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_b\Big),$$
$$\Big(\Big(\max_{l=1}^{K}\Big(\max_i \max H'^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_c, \Big(\min_{l=1}^{K}\Big(\min_i \max H'^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_b\Big)\Big]\Big]$$
$$i = 1, 2, \ldots, m,$$

$$P^+ = (\tilde{V}^+_1\ \ \tilde{V}^+_2\ \ \ldots\ \ \tilde{V}^+_n)$$

where $\tilde{V}^+_j = ([v^+_{p_j}, v^+_{q_j}], [v^+_{p'_j}, v^+_{q'_j}])\ (j = 1, 2, \ldots, n)$.

$$P^- = \Big[\Big[\Big(\Big(\max_{l=1}^{K}\Big(\max_i \min H^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_c, \Big(\min_{l=1}^{K}\Big(\min_i \min H^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_b\Big),$$
$$\Big(\Big(\max_{l=1}^{K}\Big(\max_i \max H^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_c, \Big(\min_{l=1}^{K}\Big(\min_i \max H^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_b\Big)\Big],$$
$$\Big[\Big(\Big(\max_{l=1}^{K}\Big(\max_i \min H'^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_b, \Big(\min_{l=1}^{K}\Big(\min_i \min H'^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_c\Big),$$
$$\Big(\Big(\max_{l=1}^{K}\Big(\max_i \max H'^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_b, \Big(\min_{l=1}^{K}\Big(\min_i \max H'^l_{S_{ij}}\Big)\Big)\ |j \in \Omega_c\Big)\Big]\Big]$$
$$i = 1, 2, \ldots, m,$$

$$P^- = (\tilde{V}^-_1\ \ \tilde{V}^-_2\ \ \ldots\ \ \tilde{V}^-_n)$$

where $\tilde{V}^-_j = ([v^-_{p_j}, v^-_{q_j}], [v^-_{p'_j}, v^-_{q'_j}])\ (j = 1, 2, \ldots, n)$.

Step 4. Construct positive ideal separation matrix (D^+) and negative ideal separation matrix (D^-) which are defined as follows:

$$D^+ = \begin{bmatrix} d(x_{11}, \tilde{V}^+_1) & + & d(x_{12}, \tilde{V}^+_2) & + & \cdots & + & d(x_{1n}, \tilde{V}^+_n) \\ d(x_{21}, \tilde{V}^+_1) & + & d(x_{22}, \tilde{V}^+_2) & + & \cdots & + & d(x_{2n}, \tilde{V}^+_n) \\ \vdots & & \vdots & & \vdots & & \vdots \\ d(x_{m1}, \tilde{V}^+_1) & + & d(x_{m2}, \tilde{V}^+_2) & + & \cdots & + & d(x_{mn}, \tilde{V}^+_n) \end{bmatrix}$$

and

$$D^- = \begin{bmatrix} d(x_{11}, \tilde{V}^-_1) & + & d(x_{12}, \tilde{V}^-_2) & + & \cdots & + & d(x_{1n}, \tilde{V}^-_n) \\ d(x_{21}, \tilde{V}^-_1) & + & d(x_{22}, \tilde{V}^-_2) & + & \cdots & + & d(x_{2n}, \tilde{V}^-_n) \\ \vdots & & \vdots & & \vdots & & \vdots \\ d(x_{m1}, \tilde{V}^-_1) & + & d(x_{m2}, \tilde{V}^-_2) & + & \cdots & + & d(x_{mn}, \tilde{V}^-_n) \end{bmatrix}$$

Step 5. Calculate the relative closeness (RC) of each alternative to the ideal solution as follows:

$$RC(P_i) = \frac{D^-_i}{D^+_i + D^-_i},\ i = 1, 2, \ldots, m,$$

where $D^-_i = \sum_{j=1}^{n} d(x_{ij}, \tilde{V}^-_j)$ and $D^+_i = \sum_{j=1}^{n} d(x_{ij}, \tilde{V}^+_j)$.

Step 6. Rank all the alternatives P_i $(i = 1, 2, \ldots, m)$ according to the closeness coefficient $RC(P_i)$, the greater the value $RC(P_i)$, the better the alternative $P_{i.}$

5 Illustrative example

In this section, we give an example to illustrate the application of method proposed in Section 3, to multi-criteria group decision making problem including uncertain and imprecise data and information. We utilized the proposed method to get the most desirable alternative.

Step 1. A family wants to invest money in the best option. There are five possibilities: P_1 is real estate, P_2 is stock market, P_3 is T-bills, P_4 is national saving scheme, P_5 is insurance company. Family members take a decision by considering the following four criteria: Q_1 is the risk factor; Q_2 is the growth; Q_3 is quick refund, Q_4 is complicated documents requirement. The five possible alternatives P_i $(i = 1, 2, 3, 4, 5)$ are to be evaluated using the HIFLTS by seven family members m_K $(K = 1, 2, \ldots, 7)$, as listed in Table 1-3.

Table 1. Decision matrix ($\tilde{X}^1$) with respect to family members 1,2,3 (m_1, m_2, m_3).

	Q_1	Q_2
P_1	{(M,G,VG),(VP,P)}	{(G,VG),(EP,VP)}
P_2	{(VP,P),(M,G)}	{(M,G,VG),(VP,P)}
P_3	{(G,VG),(EP,VP,P)}	{(M,G),(VP,P)}
P_4	{(VG,EG),(EP,VP)}	{(VP,P),(M,G)}
P_5	{(EG),(EP)}	{(VP,P),(M,G,VG)}
	Q_3	Q_4
P_1	{(VP,P),(M,G)}	{(VP,P),(M,G)}
P_2	{(M,G),(EP,P)}	{(G,VG),(VP,P)}
P_3	{(VG,EG),(EP)}	{(VP,P),(P,M,G)}
P_4	{(VP,P),(M,G)}	{(M,G,VG),(VP,P)}
P_5	{(EP,VP),(P,M)}	{(G,VG),(VP,P)}

Table 2. Decision matrix ($\tilde{X}^2$) with respect to family members 4,5 (m_4, m_5).

	Q_1	Q_2
P_1	{(VP,P),(M,G)}	{(VG,EG),(EP,VP)}
P_2	{(EP,VP),(P,M)}	{(VP,P),(P,M,G)}
P_3	{(M,G),(EP,VP)}	{(VP,P),(M,G)}
P_4	{(VG,EG),(EP)}	{(M,G),(EP,VP,P)}
P_5	{(G,VG),(VP,P)}	{(M,G),(VP,P,M)}
	Q_3	Q_4
P_1	{(EP,VP),(M,G)}	{(M,G),(VP,P)}
P_2	{(G,VG),(EP,VP)}	{(VG,EG),(EP)}
P_3	{(G,VG),(VP,P)}	{(EP,VP),(P,M)}
P_4	{(VP,P),(P,M,G)}	{(G,VG),(VP,P)}
P_5	{(VP,P),(M.G)}	{(VG,EG),(EP)}

Table 3. Decision matrix ($\tilde{X}^3$) with respect to family members 6,7 (m_6, m_7).

	Q_1	Q_2
P_1	{(G,VG),(EP,VP)}	{(VG,EG),(EP)}
P_2	{(M,G),(VP,P,M)}	{(VP,P),(M,G)}
P_3	{(VP,P),(P,M,G)}	{(VG,EG),(EP)}
P_4	{(G,VG),(VP,P)}	{(G,VG),(EP,VP)}
P_5	{(M,G),(EP,VP,P)}	{(VP,P),(P,M,G)}
	Q_3	Q_4
P_1	{(M,G),(VP,P)}	{(EP,VP),(M,G)}
P_2	{(VG,EG),(EP)}	{(M,G),(VP,P,M)}
P_3	{(G,VG),(EP,VP)}	{(EP,VP),(M,G)}
P_4	{(EP,VP,P),(P,M)}	{(M,G,VG),(VP,P)}
P_5	{(P,M),(M,G)}	{(EG),(EP)}

Step 2. The decision matrix X in table 4 is constructed by utilize table 1-3.

Table 4. Decision matrix (X).

	Q_1	Q_2
P_1	([P,G],[VP,M])	([VG,VG],[EP,EP])
P_2	([VP,M],[M,M])	([P,M],[P,M])
P_3	([P,G],[VP,P])	([P,VG],[EP,M])
P_4	([VG,VG],[EP,VP])	([P,G],[VP,M])
P_5	([G,EG],[EP,VP])	([P,M],[M,M])
	Q_3	Q_4
P_1	([VP,M],[P,M])	([VP,M],[P,M])
P_2	([G,VG],[EP,EP])	([G,VG],[EP,VP])
P_3	([VG,VG],[EP,VP])	([VP,VP],[M,M])
P_4	([VP,P],[M,M])	([G,VG],[VP,P])
P_5	([VP,P],[M,M])	([VG,EG],[EP,VP])

Step 3. For cost criteria Q_1, Q_4 and benefit criteria Q_2, Q_3 HIFLTS-PIS P^+ and HIFLTS-NIS P^- is as follows:

$$P^+ = \begin{bmatrix} \text{([EP,VP],[M,G])} & \text{([VG,EG],[EP,EP])} \\ \text{([VG,EG],[EP,EP])} & \text{([EP,VP],[M,G])} \end{bmatrix}$$

$$P^- = \begin{bmatrix} \text{([EG,EG],[EP,EP])} & \text{([VP,P],[M,VG])} \\ \text{([EP,VP],[M,G])} & \text{([EG,EG],[EP,EP])} \end{bmatrix}$$

Step 4. The calculation is shown below for D_{11}^+ and the remaining entries of this matrix filled by similar calculation.

$$\begin{aligned} D_{11}^{+} &= d(x_{11}, \tilde{V}_1^{+}) \\ &= d(([P, G], [VP, M]), ([EP, VP], [M, G])) \\ &= |2-0| + |4-1| + |1-3| + |3-4| \\ &= 2+3+2+1 = 8 \end{aligned}$$

Positive ideal matrix (D^{+}):

$$D^{+} = \begin{bmatrix} 8 & + & 1 & + & 12 & + & 5 \\ 4 & + & 11 & + & 2 & + & 14 \\ 9 & + & 7 & + & 2 & + & 2 \\ 15 & + & 9 & + & 14 & + & 12 \\ 15 & + & 12 & + & 14 & + & 16 \end{bmatrix} = \begin{bmatrix} 26 \\ 31 \\ 20 \\ 50 \\ 57 \end{bmatrix}$$

Negative ideal matrix (D^{-}):

$$D^{-} = \begin{bmatrix} 10 & + & 15 & + & 5 & + & 13 \\ 14 & + & 5 & + & 15 & + & 4 \\ 9 & + & 9 & + & 15 & + & 16 \\ 3 & + & 7 & + & 3 & + & 6 \\ 3 & + & 4 & + & 3 & + & 2 \end{bmatrix} = \begin{bmatrix} 43 \\ 38 \\ 49 \\ 19 \\ 12 \end{bmatrix}$$

Step 5. Relative closeness (RC) of each alternative to the ideal solutions:

$RC(P_1) = 43/(26+43) = 0.6232;$

$RC(P_2) = 38/(31+38) = 0.5507;$

$RC(P_3) = 49/(20+49) = 0.7101;$

$RC(P_4) = 19/(50+19) = 0.2754;$

$RC(P_5) = 12/(57+12) = 0.1739.$

Step 6. Rank all the alternatives P_i $(i = 1, 2, \ldots, 5)$ according to the closeness coefficient $RC(P_i)$:

$$P_3 \succ P_1 \succ P_2 \succ P_4 \succ P_5.$$

Thus the most desirable alternative is P_3. So they will invest in T-bills.

6 Conclusion

It is difficult for decision makers (DMs) to exactly measure opinion in the domain of ordinary fuzzy set theory as well as in intuitionistic fuzzy set theory. DMs assessment in linguistic variable form or the set of consecutive linguistic variables like that HFLTS. But the combination of linguistic variables as HFLTS and intuitionistic fuzzy set provide the best way to show the uncertainty in decision making problems, which concept is named as HIFLTS. The fuzzy multi-criteria group decision analysis provides an effective frame work for ranking of alternatives in terms decision makers assessments with respect to each criteria. Modified fuzzy TOPSIS method is proposed for solving group decision-making problems with the multiple conflicting criteria in HIFLTS. This produces satisfactory results by providing the positive ideal separation and negative ideal separation matrices. The proposed method is different from all the previously known techniques for group decision making due to the fact that the proposed method use HIFLTS and TOPSIS simultaneously.

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