International Workshop on Generalized Nets, Sofia, 9 July 2000, 1-5

Generalized Nets with Decision Making Components

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1 Summary

The present paper defines a class of Generalized Nets with the property that one or more of their transitions depends on an algorithm for decision making. This extension of the class of GNs is proved to be a conservative one. The proof is based on a construction of a Generalized net model of the algorithm for group decision making via individual intuitionistic fuzzy preference relations.

2 Introduction

There exists a variety of situations in modelling with Generalized Nets (GN, see [2]) where an external expert estimation is required to determine the further behaviour of the net. This motivated our attempt to endow Generalized net theory with GNs containing one or more 'decision-making transitions'. Such transitions are featured by the following: i) the set of their input places corresponds to a set of options (alternatives); and ii) their input places would hold tokens whose characteristics represent the degrees of preference of the different options. Thus, basing on some kind of expert estimation, a transition of this kind is used to order the options by their degree of preference.

The choice of a particular decision-making algorithm is beyond the scope of this work, but for the proof of our Theorem 1 we will need a specific one. We use the algorithm for group decision making via individual intuitionistic fuzzy preference relations, proposed in [1] - a developed version of algorithms for group decision making via individual fuzzy preference relations proposed in [3].

3 Group decision making under individual intuitionistic fuzzy preference relations

Suppose we have a set of n options (alternatives) $S = \{s_1, \ldots, s_n\}$ and m experts $I_1, \ldots, I_m$. Each expert $k, k = 1, \ldots, m$, provides his own preferences over S which are represented by his individual intuitionistic fuzzy preference matrix R_k and his individual matrix of degrees of uncertainty π_k:

$$R_k = [r_{ij}^k], i, j = 1, \ldots, n; k = 1, \ldots, m. \tag{1}$$

The elements $0 \leq r_{ij}^k \leq 1$ of R_k are such that the higher the preference of individual k of s_i over s_j the higher r_{ij}^k: from $r_{ij}^k = 0$ indicating a definite preference s_j over s_i, through $r_{ij}^k = 0.5$ indicating indifference between s_i and s_j, to $r_{ij}^k = 1$ indicating a definite preference s_i over s_j.

$$\Pi_k = [\pi_{ij}^k], i, j = 1, \ldots, n; k = 1, \ldots, m. \tag{2}$$

The degrees of uncertainty, $0 \leq \pi_{ij}^k \leq 1$, are such that the higher π_{ij}^k, the higher the hesitation margin of expert k as to the preference between s_i and s_j whose intensity is given by r_{ij}^k.

Thus, the expert's preference is within the range $[r_{ij}^k, r_{ij}^k + \pi_{ij}^k]$. A value of $\pi_{ij}^k = 0$ would mean that the expert I_k has definitely determined his preference.

Moreover, the following condition must be fulfilled:

$r_{ij}^k + r_{ji}^k + \pi_{ji}^k = 1$ for all $i, j = 1, \ldots, n, i \neq j, k = 1, \ldots, m$

The purpose of the decision-making procedure lies in finding a *solution.* Roughly speaking, a solution can be thought of as a set of these options with their degrees of preference that are most acceptable to the majority of experts.

A solution concept with much intuitive appeal is here the core defined as [3]

$$C = \{s_i \in S : \neg\exists s_j \in S \text{ such that } r_{ji}^k > 0.5 \text{ for at least } r \text{ individuals}\}$$

i.e. as a set of undominated options, not defeated by the required majority $r \leq m$.

Next, assuming also a fuzzy majority given as a fuzzy linguistic quantifier, we can define a solution concepts as, e.g.: the fuzzy Q-core [3].

To derive a formal definition, let us first denote:

$$h_{ij}^k = \begin{cases} 1, & if r_{ij}^k < 0.5 \\ 0, & otherwise \end{cases} \tag{3}$$

i.e. $h_{ij}^k = 1$ means that the k-th expert prefers s_j over s_i[1]. (If not otherwise specified, i,j = 1,...,n; k = 1,...,m throughout the paper.)

Then

$$h_j^k = \frac{1}{n-1} \sum_{i=1, i \neq j}^{n} h_{ij}^k \tag{4}$$

is to what extent individual k is not against s_j : from 0 for certainly against, to 1 for certainly not against, through all intermediate values.

Next, using the formula

$$h_j = \frac{1}{m} \sum_{k=1}^{m} h_j^k \tag{5}$$

we calculate the degree to which all experts are not against option s_j.

[1]It may be the case that $h_{ij}^k = h_{ji}^k = 1$. This would mean that neither s_i nor s_j is preferred. In this case it is convenient to assume $h_{ij}^k = h_{ji}^k = 0$, and thus to obtain the necessary matrices $[h_{ij}^k]$.

Then, basing on the matrices Π_k, the values of π_j^k and π_j are calculated by formulas similar to (4) and (5), namely,

$$\pi_j^k = \frac{1}{n-1} \sum_{i=1, i \neq j}^{n} \pi_{ij}^k \tag{6}$$

$$\pi_j = \frac{1}{m} \sum_{k=1}^{m} \pi_j^k \tag{7}$$

Using (5), (7), and

$$h_j' \in [h_j, h_j + \pi_j], \tag{8}$$

we get intervals of possible values showing the least and greatest degree to which all experts are not against option s_j.

Finally, from the equation

$$\nu_Q^j = \mu_Q(h_j') \tag{9}$$

we obtain intervals of values for each option s_j. These intervals show the possible degrees of preference of Q experts.

A particular way of calculating μ_Q must be specified.

For instance, for Q = "most" it may be given as (cf. [3]):

$$\mu_{\text{``}most\text{''}} = \begin{cases} 1, & for \quad x \geq 0.8 \\ 2x - 0.6, & for \quad 0.3 < x < 0.8 \\ 0, & for \quad x \leq 0.3 \end{cases}$$

Informally, the fuzzy Q-core is defined as a set of options, such that Q individuals are not against them (not defeated by Q individuals) [3].

4 Generalized nets with decision making components

Let Σ be the class of all GNs.

Definition. A *decision-making transition* will be called a transition of the kind:

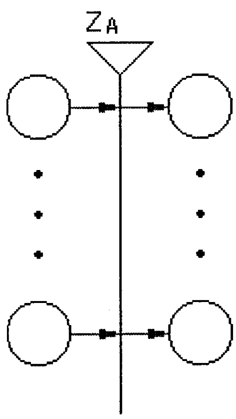

Fig. 1. *A decision-making transition*

It has n input places, corresponding to the n different options, and as many output places – they represent the options arranged in order of preference.

Such transitions will be denoted by Z^{α}, and for the class of all Z^{α} we will write Z^{DM}.

Definition. A GN containing a Z^{α} transition will be called a *Generalized net with a decision making component.* These GNs will be denoted by E^{DM}.

Definition. Σ^{DM} will mean the class of all E^{DM}.

Definition. $D_{m,\alpha} : Z^{DM} \to \Sigma$ is an operator producing an ordinary GN from a decision-making transition, where m is the number of participating experts, and α is a particular algorithm from the collection of all algorithms for decision making and their modifications.

Theorem. Σ^{DM} is a conservative extension of Σ.

Proof. We will prove that every GN from Σ^{DM} is representable by an ordinary GN. To that point, we will construct a GN model of the algorithm described in Section 3.

Let us be given an arbitrary GN E^{DM}, and let Z^{DM} be a decision-making transition. Therefore, the tokens pass from its input to its output places if the algorithm for group decision making via individual intuitionistic fuzzy preference relations succeeds, and obtain as characteristics the results of its application.

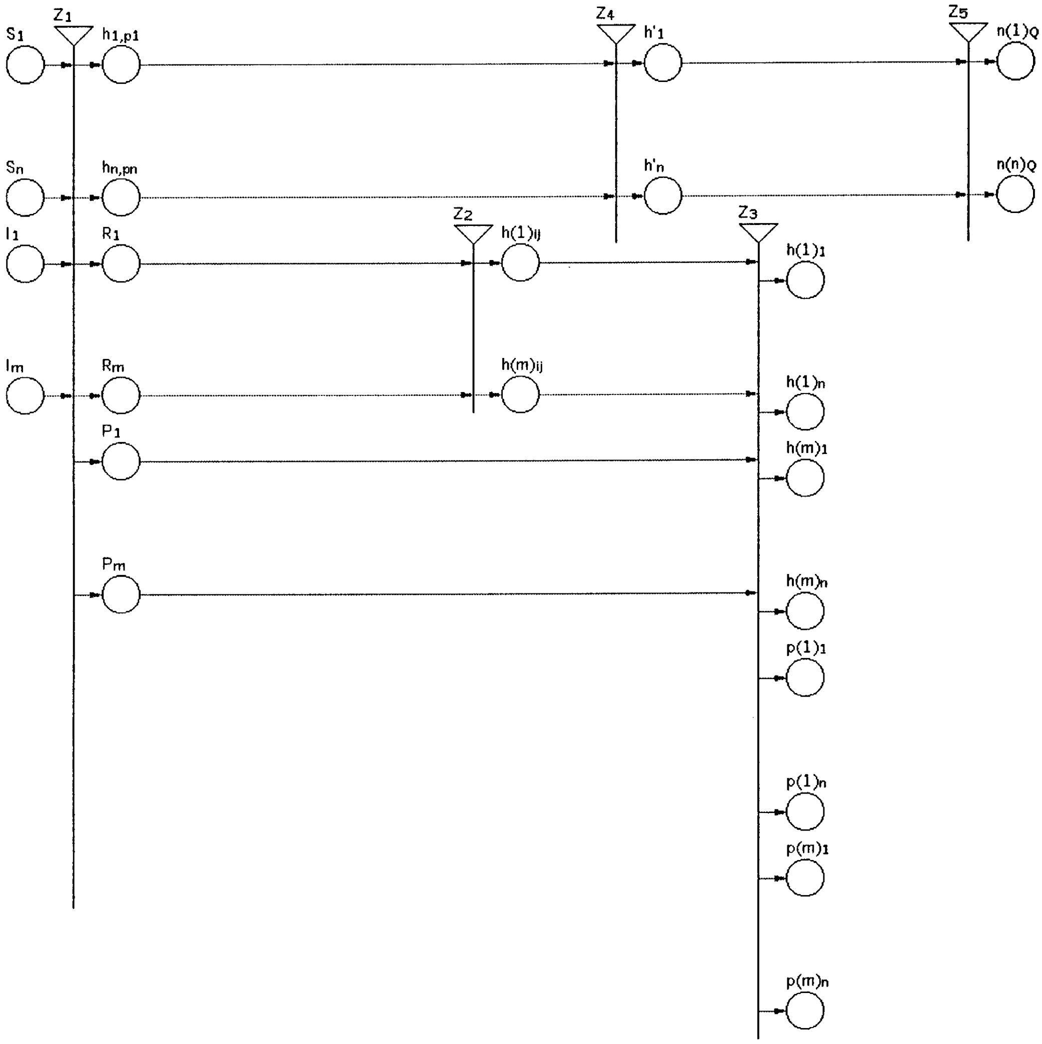

Fig. 2. *The Generalized net E'*

We extend Z^{DM} to a GN E' (Fig. 2) having five transitions. Its input places comprise the input places of the transition Z^{DM} and include m more, meant to host tokens carrying as characteristics data about each of the experts. The output places of E' coincide with the output places of Z^{DM}.

Below we describe the structure of E' by transitions.

Transition Z_1 has $n+m$ input places, corresponding to the options $s_1, \ldots, s_n$ and experts $I_1, \ldots, I_m$. Each expert provides two matrices of the kind as in [1] and [2]. A token in place I_k, $k = 1, \ldots, m$ will split and the two resulting tokens will transfer to places R_k and π_k, taking the respective matrix as characteristics. Tokens from places $s_1, \ldots, s_n$ will transfer to places $\langle h_1, \pi_1 \rangle, \ldots, \langle h_n, \pi_n \rangle$, respectively, only if there is a token in each of the input places of transition Z_3. They will take as characteristics the values calculated by (5) and (7).

Transition Z_2 has m input and m output places. The tokens pass and obtain as characteristics the values calculated by (3).

Transition Z_3 has $2m$ input and $2nm$ output places. A token from place h_{ij}^k, $k = 1, \ldots, m$, splits into n tokens that transfer to $h_1^k, \ldots, h_n^k$, respectively, and obtain as characteristics values calculated by (4). Each token from place π_k, $k = 1, \ldots, m$, splits into n tokens that transfer to $\pi_1^k, \ldots, \pi_n^k$, respectively, and obtain as characteristics values calculated by (6).

Transition Z_4 has n input and n output places. In its output places, the tokens obtain as characteristics values calculated by (8).

Transition Z_5: The output places are output places of the GN E' and coincide with the output places of transition Z^{DM}. The tokens in them obtain as characteristics an interval of values for each option s_i, $i = 1, \ldots, n$. These intervals show the possible degrees of preference of Q experts.

Thus we demonstrated that a GN with a decision-making component Z^α can be represented in terms of classical GNs, by replacing Z^α with the GN E' developed above.

The operator $D_{m,\alpha}$ allows one to employ other decision making algorithms as well. Therefore, for each α from that collection, Z^α can be represented by a GN. This proves that every GN from Z^{DM} is representable by a GN and therefore Z^{DM} is a conservative extension of Σ, which completes the proof.

5 References

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