Notes on Intuitionistic Fuzzy Sets
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Modified and generalized correlation coefficient between intuitionistic fuzzy sets with applications

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Abstract: Intuitionistic fuzzy set (IFS) is a very interesting soft computing technique use to tackle/handle imprecisions embedded in multi-criteria decision-making (MCDM) problems. Correlation coefficient has proven to be an important measuring operator in an intuitionistic fuzzy setting with regard to its applications in solving MCDM problems. In this paper, Xu et al.'s method of correlation coefficient between IFSs is modified because it fails the axiomatic properties of correlation coefficient between IFSs, and hence generalized for a better output. That is, this paper is aimed at modifying and generalizing the triparametric correlation coefficient for IFSs proposed by Xu et al. with applications to some MCDM problems. Some numerical examples are supplied to authenticate the superiority of this new correlation coefficient for IFSs over some similar existing correlation coefficient measures. Subsequently, some MCDM problems such as medical diagnosis and pattern recognition problems represented in intuitionistic fuzzy pairs are determined with the aid of the novel correlation coefficient. An intuitionistic fuzzy clustering algorithm based on this novel correlation coefficient with applications could be an interesting research for future work.

Keywords: Fuzzy set, Intuitionistic fuzzy set, Correlation coefficient, Multi-criteria decision making.
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1 Introduction

MCDM deals with decisions involving the choice of a best alternative from several potential candidates in a decision, subject to several criteria or attributes that may be concrete or vague.

MCDM methods are used to help decision-makers make their decision according to their preferences to enhance best choice among the alternatives, in cases where there is more than one conflicting criterion [24]. Nonetheless, the ideas of fuzziness and imprecision posed a great challenge in MCDM problems.

In a quest to arrest the challenge posed by fuzziness and imprecision, Zadeh [34] introduced the theory of fuzzy sets. Fuzzy set constitutes a membership degree, μ which ascribes to each element of universe of discourse, a number from the closed interval, $[0, 1]$ to specify the degree of belongingness to the set under deliberation. However, fuzzy set theory could not accurately tackle the imprecisions/fuzziness embedded in decision-making process. As a result, various generalizations of fuzzy sets such as intuitionistic fuzzy sets (IFSs), Pythagorean fuzzy sets, etc. were presented. Atanassov [1] proposed the notion of IFSs by combining membership degree, μ and non-membership degree, ν with hesitation margin, π such that their sum is one with an additional property that $\mu + \nu \leq 1$. IFS offers a framework which reasonably restricts fuzziness and imprecision and hence, very appropriate in modelling many real-life problems [3, 4, 6, 7, 10–16, 21, 25, 26, 30, 31].

Correlation coefficient is of paramount important in sciences and engineering. In correlation analysis, the joint relationship of two variables can be certified with the help of a measure of interdependency of the two variables. In statistics, the values of the correlation coefficient are in the range, $[-1, 1]$ in which the value of the correlation coefficient is 1 when two variables have linear relation in the same direction (two variables either increase or decrease). In contrast, the value of the correlation coefficient is -1 when two variables are linearly inverted (one variable increases, while the other decreases). The notion of correlation coefficient was first studied in fuzzy context in [8, 9] and supplemented in [5]. For better application in MCDM problems, correlation coefficient was introduced in intuitionistic fuzzy context [18]. Since correlation coefficient for IFSs is very productive, some new versions of it were proposed and applied to MCDM problems [19, 20, 22, 23, 27–29, 32, 33]. Garg [17] listed some limitations of correlation coefficients for IFSs studied in [27, 32, 33], all with triparametric approach. The limitations prompted this present study. Thus, this study modifies and generalizes a correlation coefficient between IFSs in [33] for better result.

The motivation for the paper is to introduce a new correlation coefficient for IFSs that modifies and generalizes the one studied in [33] with better interpretation and output when apply to MCDM problems. Precisely, this paper presents an axiomatic definition of correlation coefficient for IFSs, modifies the correlation coefficient for IFSs in [33] because it fails the axiomatic properties of correlation coefficient, generalizes the modified version and authenticates its advantage and illustrates its applications in some selected MCDM problems. The remaining parts of the paper are thus; Section 2 provides some preliminaries on IFSs, while Section 3 covers the notion of correlation coefficient for IFSs with numerical illustrations and presents a number of results. Section 4 discusses the application of the introduced method in some MCDM problems such as medical diagnosis and pattern recognition problems captured in intuitionistic fuzzy pairs. Finally, Section 5 concludes the paper and provides direction for further studies.

2 Preliminaries

Some relevant notions like fuzzy sets and IFSs are recall for reference and completeness. Suppose S is a non-empty set that is fixed, then the following definitions follow.

Definition 2.1. [34] A fuzzy set $\tilde{M}$ of S which is characterized by a membership function $\mu_{\tilde{M}} : S \to [0, 1]$ is of the form

$$\tilde{M} = \{\langle s, \mu_{\tilde{M}}(s)\rangle \mid s \in S\}. \tag{1}$$

Definition 2.2. [1] An IFS M of S is an object having the form

$$M = \{\langle \frac{\mu_M(s), \nu_M(s)}{s}\rangle \mid s \in S\} \tag{2}$$

or

$$M = \{\langle s, \mu_M(s), \nu_M(s)\rangle | s \in S\}, \tag{3}$$

where the functions

$$\mu_M(s) : S \to [0, 1] \text{ and } \nu_M(s) : S \to [0, 1] \tag{4}$$

are the degree of membership and the degree of non-membership, respectively of the element $s \in S$ to M, and for every $s \in S$,

$$0 \leq \mu_M(s) + \nu_M(s) \leq 1. \tag{5}$$

For each M of S,

$$\pi_M(s) = 1 - \mu_M(s) - \nu_M(s) \tag{6}$$

is the intuitionistic fuzzy set index or hesitation margin of s in S. The hesitation margin $\pi_M(s)$ is the degree of non-determinacy of $s \in S$, to M and $\pi_M(s) \in [0, 1]$. The hesitation margin is the function that states lack of knowledge of whether $s \in S$ or $s \notin S$. Thus,

$$\mu_M(s) + \nu_M(s) + \pi_M(s) = 1. \tag{7}$$

Example 2.3. Let $S = \{s_1, s_2, s_3\}$ be a fixed universe of discourse and

$$M = \{\langle \frac{0.7, 0.2}{s_1}\rangle, \langle \frac{0.5, 0.3}{s_2}\rangle, \langle \frac{0.8, 0.2}{s_3}\rangle\}$$

be an intuitionistic fuzzy set of S. Then, the indexes of the elements s_1, s_2, s_3 to M are

$$\pi_M(s_1) = 0.1,\ \pi_M(s_2) = 0.2 \text{ and } \pi_M(s_3) = 0.0.$$

Definition 2.4. [2] Suppose $M, N \in IFS(S)$, where $IFS(S)$ denotes the set of all IFSs of S. Then, we have the following:

(i) $\bar{M} = \{\langle s, \nu_M(s), \mu_M(s)\rangle | s \in S\}$.

(ii) $M \cup N = \{\langle s, \max(\mu_M(s), \mu_N(s)), \min(\nu_M(s), \nu_N(s))\rangle | x \in S\}$.

(iii) $M \cap N = \{\langle s, \min(\mu_M(s), \mu_N(s)), \max(\nu_M(s), \nu_N(s))\rangle | s \in S\}$.

(iv) $M \oplus N = \{\langle s, \mu_M(s) + \mu_N(s) - \mu_M(s)\mu_N(s), \nu_M(s)\nu_N(s)\rangle | s \in S\}$.

(v) $M \otimes N = \{\langle s, \mu_M(s)\mu_N(s), \nu_M(s) + \nu_N(s) - \nu_M(s)\nu_N(s)\rangle | s \in S\}$.

Definition 2.5. [2] Let M and N be IFSs of S. Then,

$$M = N \Leftrightarrow \mu_M(s) = \mu_N(s) \text{ and } \nu_M(s) = \nu_N(s)\ \forall s \in S,$$

and

$$M \subseteq N \Leftrightarrow \mu_M(s) \leq \mu_N(s) \text{ and } \nu_M(s) \geq \nu_N(s)\ \forall s \in S.$$

We say $M \subset N \Leftrightarrow M \subseteq N$ and $M \neq N$. Also, M and N are comparable to each other if $M \subseteq N$ and $N \subseteq M$.

Definition 2.6. [2] Suppose $M \in IFS(S)$. Then, the level/ground set or support of M is defined by

$$M_* = \{s \in S | \mu_M(s) > 0,\ \nu_M(s) < 1\},$$

and the set M^* is defined by

$$M^* = \{s \in S | \mu_M(s) \geq 0,\ \nu_M(s) \leq 1\}.$$

Certainly, M_* and M^* are subsets of S.

Definition 2.7. Intuitionistic fuzzy pairs (IFPs) or intuitionistic fuzzy values (IFVs) is an object in the form $\langle x, y\rangle$, where $x, y \in [0, 1]$, and $x + y \leq 1$. IFPs are used for the evaluation of objects or processes and which components (x and y) are interpreted as degrees of membership and non-membership or degrees of validity and non-validity or degrees of correctness and non-correctness.

Example 2.8. Suppose M is a IFS of $S = \{s_1, s_2, s_3, s_4, s_5\}$. Assume

$$M = \{\langle \frac{0.5, 0.4}{s_1}\rangle, \langle \frac{0.7, 0.2}{s_3}\rangle, \langle \frac{0.7, 0.2}{s_5}\rangle\},$$

which is rewritten as

$$M = \{\langle \frac{0.5, 0.4}{s_1}\rangle, \langle \frac{0.0, 1.0}{s_2}\rangle, \langle \frac{0.7, 0.2}{s_3}\rangle, \langle \frac{0.0, 1.0}{s_4}\rangle, \langle \frac{0.7, 0.2}{s_5}\rangle\}.$$

Then,

$$M_* = \{s_1, s_3, s_5\}$$

and

$$M^* = \{s_1, s_2, s_3, s_4, s_5\} = S.$$

IFPs are

$$s_1 = \langle 0.5, 0.4\rangle,\ s_2 = \langle 0.0, 1.0\rangle,\ s_3 = \langle 0.7, 0.2\rangle,$$
$$s_4 = \langle 0.0, 1.0\rangle,\ s_5 = \langle 0.7, 0.2\rangle.$$

3 Correlation coefficients for intuitionistic fuzzy sets

In this section, some existing similar/triparametric correlation coefficient measures for IFSs are presented. Subsequently, the new triparametric correlation coefficient measure for IFSs is given. The comparative analysis of the existing ones and the proposed method is carried out to ascertain the reasonability of the new method.

Foremostly, the axiomatic definition of correlation coefficient for IFSs is given thus.

Definition 3.1. Let M and N be IFSs of a nonempty set S. Then, the correlation coefficient denoted by $\mathcal{K}(M,N)$ is a measuring function

$$\mathcal{K} : IFS \times IFS \to [0,1]$$

satisfying the following axioms;

(i) $\mathcal{K}(M,N) \in [0,1]$,

(ii) $\mathcal{K}(M,N) = \mathcal{K}(N,M)$,

(iii) $\mathcal{K}(M,N) = 1$ if and only if $M = N$.

3.1 Correlation coefficient for intuitionistic fuzzy sets in [18]

The notion of correlation coefficient in intuitionistic fuzzy environment was initiated in [18], and defined thus:

$$\mathcal{K}_1(M,N) = \frac{\mathcal{C}(M,N)}{\sqrt{\mathcal{T}(M)}\sqrt{\mathcal{T}(N)}}, \tag{8}$$

where M and N are IFSs of a nonempty set S, and $\mathcal{C}(M,N)$ is the correlation of IFSs, $\mathcal{T}(M)$ and $\mathcal{T}(N)$ are informational energies of M and N, respectively defined as follows:

$$\mathcal{C}(M,N) = \sum_{i=1}^{n}[\mu_M(s_i)\mu_N(s_i) + \nu_M(s_i)\nu_N(s_i)], \tag{9}$$

$$\mathcal{T}(M) = \sum_{i=1}^{n}[\mu_M^2(s_i) + \nu_M^2(s_i)] \tag{10}$$

and

$$\mathcal{T}(N) = \sum_{i=1}^{n}[\mu_N^2(s_i) + \nu_N^2(s_i)]. \tag{11}$$

3.2 Xu et al. [33] correlation coefficient for intuitionistic fuzzy sets

By modifying the method in [18], Xu et al. [33] proposed the following correlation coefficient for IFSs:

$$\mathcal{K}_2(M,N) = \frac{\mathcal{C}(M,N)}{\max[\sqrt{\mathcal{T}(M)}, \sqrt{\mathcal{T}(N)}]}, \tag{12}$$

where M and N are IFSs of a nonempty set S, and $\mathcal{C}(M,N)$ is the correlation of IFSs, $\mathcal{T}(M)$ and $\mathcal{T}(N)$ are informational energies of M and N, respectively defined as follows:

$$\mathcal{C}(M,N)=\sum_{i=1}^{n}[\mu_M(s_i)\mu_N(s_i)+\nu_M(s_i)\nu_N(s_i)+\pi_M(s_i)\pi_N(s_i)], \tag{13}$$

$$\mathcal{T}(M)=\sum_{i=1}^{n}[\mu_M^2(s_i)+\nu_M^2(s_i)+\pi_M^2(s_i)] \tag{14}$$

and

$$\mathcal{T}(N)=\sum_{i=1}^{n}[\mu_N^2(s_i)+\nu_N^2(s_i)+\pi_N^2(s_i)]. \tag{15}$$

3.3 Garg [17] correlation coefficients in intuitionistic fuzzy environment

After Garg [17] pinpointed the limitations in [18, 33], the following correlation coefficient was introduced which we present in intuitionistic fuzzy setting as follows:

$$\mathcal{K}_3(M,N)=\frac{\mathcal{C}(M,N)}{\max[\mathcal{T}(M),\mathcal{T}(N)]}, \tag{16}$$

where M and N are IFSs of a nonempty set S, and

$$\mathcal{C}(M,N)=\sum_{i=1}^{n}[\mu_M^2(s_i)\mu_N^2(s_i)+\nu_M^2(s_i)\nu_N^2(s_i)+\pi_M^2(s_i)\pi_N^2(s_i)], \tag{17}$$

$$\mathcal{T}(M)=\sum_{i=1}^{n}[\mu_M^4(s_i)+\nu_M^4(s_i)+\pi_M^4(s_i)] \tag{18}$$

and

$$\mathcal{T}(N)=\sum_{i=1}^{n}[\mu_N^4(s_i)+\nu_N^4(s_i)+\pi_N^4(s_i)]. \tag{19}$$

3.4 New correlation coefficient for intuitionistic fuzzy sets

By synthesizing the correlation coefficients for IFSs in [33], a new correlation coefficient for IFSs that generalizes the one in [33] is introduced.

Definition 3.2. Let $M,N\in IFS(S)$ for $S=\{s_1,s_2,...,s_n\}$. Then, the generalized informational energies and correlation for M and N are given as:

$$\mathcal{T}(M)=\sum_{i=1}^{n}[\mu_M^k(s_i)+\nu_M^k(s_i)+\pi_M^k(s_i)],$$

$$\mathcal{T}(N)=\sum_{i=1}^{n}[\mu_N^k(s_i)+\nu_N^k(s_i)+\pi_N^k(s_i)]$$

and

$$\mathcal{C}(M,N)=\sum_{i=1}^{n}[\mu_M^{\frac{k}{2}}(s_i)\mu_N^{\frac{k}{2}}(s_i)+\nu_M^{\frac{k}{2}}(s_i)\nu_N^{\frac{k}{2}}(s_i)+\pi_M^{\frac{k}{2}}(s_i)\pi_N^{\frac{k}{2}}(s_i)],$$

where $k=2n-1$ for $n=1,2$.

Remark 3.3. Let M and N be IFSs of S. Then $\mathcal{T}(M)=\mathcal{T}(\bar{M})$ and $\mathcal{C}(M,N)=\mathcal{C}(N,M)$.

Remark 3.4. Let M and N be IFSs of S. Then, the following statements are equivalent:

(i) $\mathcal{C}(M,N)=\mathcal{C}(\bar{M},\bar{N})$.

(ii) $\mathcal{C}(\bar{M},\bar{N})=\mathcal{C}(N,M)$.

Proposition 3.5. *Suppose M and N are IFSs of S. If $M=N$, then*

(i) $\mathcal{C}(M,N)=\mathcal{T}(M)$ *or* $\mathcal{C}(M,N)=\mathcal{T}(N)$*,*

(ii) $\mathcal{C}(M,N)=\max[\mathcal{T}(M),\mathcal{T}(N)]$*,*

(iii) $\dfrac{\mathcal{C}(M,N)}{\max[\mathcal{T}(M),\mathcal{T}(N)]}=1$.

Proof. Assume that $M=N$. Then

(i)

$$\begin{aligned}\mathcal{C}(M,N) &= \sum_{i=1}^{n}[\mu_M^{\frac{k}{2}}(s_i)\mu_N^{\frac{k}{2}}(s_i)+\nu_M^{\frac{k}{2}}(s_i)\nu_N^{\frac{k}{2}}(s_i)+\pi_M^{\frac{k}{2}}(s_i)\pi_N^{\frac{k}{2}}(s_i)]\\ &= \sum_{i=1}^{n}[\mu_M^{k}(s_i)+\nu_M^{k}(s_i)+\pi_M^{k}(s_i)]\\ &= \mathcal{T}(M).\end{aligned}$$

The second alternative is straightforward.

(ii) $\mathcal{C}(M,N)=\mathcal{C}(M,M)=\mathcal{T}(M)$. Also, $\max[\mathcal{T}(M),\mathcal{T}(N)]=\max[\mathcal{T}(M),\mathcal{T}(M)]=\mathcal{T}(M)$.

(iii) It follows from (ii) that $\dfrac{\mathcal{C}(M,N)}{\max[\mathcal{T}(M),\mathcal{T}(N)]}=1$. □

3.4.1 Limitation of Xu et al. [33] Correlation coefficient

The correlation coefficient proposed in [33] is not a reliable measure because it fails condition (iii) in Definition 3.1, that is, $\mathcal{K}(M,N)=1$ if and only if $M=N$. To see this, recall Xu et al. [33] correlation coefficient as follows:

$$\mathcal{K}_2(M,N)=\frac{\mathcal{C}(M,N)}{\max[\sqrt{\mathcal{T}(M)},\sqrt{\mathcal{T}(N)}]}.$$

If $M = N$ then $\mathcal{T}(M) = \mathcal{T}(N)$. So,

$$\begin{aligned}
\mathcal{K}_2(M,N) &= \frac{\mathcal{C}(M,N)}{\max[\sqrt{\mathcal{T}(M)}, \sqrt{\mathcal{T}(N)}]} \\
&= \frac{\mathcal{C}(M,M)}{\max[\sqrt{\mathcal{T}(M)}, \sqrt{\mathcal{T}(M)}]} \\
&= \frac{\mathcal{C}(M,M)}{\sqrt{\mathcal{T}(M)}} = \frac{\mathcal{T}(M)}{\sqrt{\mathcal{T}(M)}} \\
&= \sqrt{\mathcal{T}(M)} \neq 1.
\end{aligned}$$

Thus, $\mathcal{K}_2$ is not reliable. Hence, we proposed a new correlation coefficient measure for IFSs as follows:

Definition 3.6. Let M and N be IFSs of S for $S = \{s_1, s_2, ..., s_n\}$. Then, the new correlation coefficient measure for M and N is

$$\mathcal{K}(M,N) = \frac{\mathcal{C}(M,N)}{\max[\mathcal{T}(M), \mathcal{T}(N)]}, \tag{20}$$

where $\mathcal{C}(M,N)$, $\mathcal{T}(M)$ and $\mathcal{T}(N)$ are as defined in Definition 3.2.

Thus, (20) could also be written as

$$\mathcal{K}(M,N) = \frac{\mathcal{C}(M,N)}{\max[\mathcal{C}(M,M), \mathcal{C}(N,N)]}. \tag{21}$$

Theorem 3.7. *Suppose M and N are IFSs of S. Then, the function $\mathcal{K}(M,N)$ is a correlation coefficient of M and N.*

Proof. To show that $\mathcal{K}(M,N)$ is a correlation coefficient between M and N, we verify the conditions in Definition 3.1. Firstly, $\mathcal{K}(M,N) \in [0,1]$ implies $0 \leq \mathcal{K}(M,N) \leq 1$. That is, $\mathcal{K}(M,N) \geq 0$ and $\mathcal{K}(M,N) \leq 1$. The first inequality is trivial since $\mathcal{C}(M,N) \geq 0$ and $[\mathcal{T}(M), \mathcal{T}(N)] > 0$. Next, we show that $\mathcal{K}(M,N) \leq 1$. To establish this fact, let us assume following:

$$\sum_{i=1}^{n} \mu_M^k(s_i) = a, \quad \sum_{i=1}^{n} \mu_N^k(s_i) = b,$$

$$\sum_{i=1}^{n} \nu_M^k(s_i) = c, \quad \sum_{i=1}^{n} \nu_N^k(s_i) = d,$$

$$\sum_{i=1}^{n} \pi_M^k(s_i) = e, \quad \sum_{i=1}^{n} \pi_N^k(s_i) = f.$$

But $\mathcal{K}(M,N) = \dfrac{\mathcal{C}(M,N)}{\max[\mathcal{T}(M), \mathcal{T}(N)]}$. Using the Cauchy-Schwarz's inequality, we have

$$
\begin{aligned}
\mathcal{K}(M,N) &= \frac{\sum_{i=1}^{n}[\mu_M^{\frac{k}{2}}(s_i)\mu_N^{\frac{k}{2}}(s_i)+\nu_M^{\frac{k}{2}}(s_i)\nu_N^{\frac{k}{2}}(s_i)+\pi_M^{\frac{k}{2}}(s_i)\pi_N^{\frac{k}{2}}(s_i)]}{\max[\sum_{i=1}^{n}(\mu_M^k(s_i)+\nu_M^k(s_i)+\pi_M^k(s_i)),\sum_{i=1}^{n}(\mu_N^k(s_i)+\nu_N^k(s_i)+\pi_N^k(s_i))]} \\
&= \frac{\sum_{i=1}^{n}\mu_M^{\frac{k}{2}}(s_i)\mu_N^{\frac{k}{2}}(s_i)+\sum_{i=1}^{n}\nu_M^{\frac{k}{2}}(s_i)\nu_N^{\frac{k}{2}}(s_i)+\sum_{i=1}^{n}\pi_M^{\frac{k}{2}}(s_i)\pi_N^{\frac{k}{2}}(s_i)}{\max[(\sum_{i=1}^{n}\mu_M^k(s_i)+\sum_{i=1}^{n}\nu_M^k(s_i)+\sum_{i=1}^{n}\pi_M^k(s_i)),(\sum_{i=1}^{n}\mu_N^k(s_i)+\sum_{i=1}^{n}\nu_N^k(s_i)+\sum_{i=1}^{n}\pi_N^k(s_i))]} \\
&\leq \frac{[\sum_{i=1}^{n}\mu_M^k(s_i)\sum_{i=1}^{n}\mu_N^k(s_i)]^{\frac{1}{2}}+[\sum_{i=1}^{n}\nu_M^k(s_i)\sum_{i=1}^{n}\nu_N^k(s_i)]^{\frac{1}{2}}+[\sum_{i=1}^{n}\pi_M^k(s_i)\sum_{i=1}^{n}\pi_N^k(s_i)]^{\frac{1}{2}}}{\max[(\sum_{i=1}^{n}\mu_M^k(s_i)+\sum_{i=1}^{n}\nu_M^k(s_i)+\sum_{i=1}^{n}\pi_M^k(s_i)),(\sum_{i=1}^{n}\mu_N^k(s_i)+\sum_{i=1}^{n}\nu_N^k(s_i)+\sum_{i=1}^{n}\pi_N^k(s_i))} \\
&= \frac{(ab)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}+(ef)^{\frac{1}{2}}}{\max[(a+c+e),(b+d+f)]}.
\end{aligned}
$$

However,

$$
\begin{aligned}
\mathcal{K}(M,N)-1 &\leq \frac{(ab)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}+(ef)^{\frac{1}{2}}}{\max[(a+c+e),(b+d+f)]}-1 \\
&= \frac{(ab)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}+(ef)^{\frac{1}{2}}-\max[(a+c+e),(b+d+f)]}{\max[(a+c+e),(b+d+f)]} \\
&= \frac{-\{\max[(a+c+e),(b+d+f)]-[(ab)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}+(ef)^{\frac{1}{2}}]\}}{\max[(a+c+e),(b+d+f)]} \\
&= -\frac{\{\max[(a+c+e),(b+d+f)]-[(ab)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}+(ef)^{\frac{1}{2}}]\}}{\max[(a+c+e),(b+d+f)]} \\
&\leq 0.
\end{aligned}
$$

Thus, $\mathcal{K}(M,N)\leq 1$. Hence, $\mathcal{K}(M,N)\in[0,1]$.

Again, $\mathcal{K}(M,N)=1\Leftrightarrow M=N\Rightarrow$

$$
\mathcal{K}(M,N)=\frac{\mathcal{C}(M,M)}{\max[\mathcal{T}(M),\mathcal{T}(M)]}=\frac{\mathcal{T}(M)}{\mathcal{T}(M)}=1.
$$

Certainly, $\mathcal{K}(M,N)=\mathcal{K}(N,M)$, so details are omitted. Therefore, $\mathcal{K}(M,N)$ is a correlation coefficient between M and N. □

3.5 Numerical illustrations of the new correlation coefficient

We show the reliability of the new correlation coefficient over Garg's maximum approach in [17] and its all inclusiveness via numerical examples.

3.5.1 Example I

Assume there are two IFSs

$$
M=\{\langle\frac{0.3,0.6,0.1}{s_1}\rangle,\frac{0.5,0.3,0.2}{s_2}\rangle,\frac{0.4,0.5,0.1}{s_3}\rangle\}
$$

and

$$
N=\{\langle\frac{0.3,0.6,0.1}{s_1}\rangle,\frac{0.5,0.3162,0.1838}{s_2}\rangle,\frac{0.3873,0.5,0.1127}{s_3}\rangle\}
$$

of S where $S=\{s_1,s_2,s_3\}$.

Now, we find the correlation coefficient between M and N by employing (16) (which is the valid existing maximum approach of measuring correlation coefficient) and the proposed method. Using (16), we get

$$\mathcal{K}_3(M,N) = 0.2120.$$

By using the proposed method, we obtain the following:

For $k = 1$,

$$\mathcal{K}(M,N) = 0.7051.$$

For $k = 3$,

$$\mathcal{K}(M,N) = 0.2982.$$

3.5.2 Example II

Suppose there are two IFSs O and P of a set $S = \{s_1, s_2, s_3\}$ such that

$$O = \{\langle\frac{0.1, 0.2, 0.7}{s_1}\rangle, \langle\frac{0.2, 0.1, 0.7}{s_2}\rangle, \langle\frac{0.29, 0.0, 0.71}{s_3}\rangle\}$$

and

$$P = \{\langle\frac{0.1, 0.3, 0.6}{s_1}\rangle, \langle\frac{0.2, 0.2, 0.6}{s_2}\rangle, \langle\frac{0.29, 0.1, 0.61}{s_3}\rangle\}.$$

Using the proposed method, we obtain the following:

For $k = 1$

$$\mathcal{K}(O,P) = 0.9769.$$

For $k = 3$

$$\mathcal{K}(O,P) = 0.8104.$$

Using (16), we have

$$\mathcal{K}_3(O,P) = 0.7426.$$

Correlation coefficients $\mathcal{K}$	$\mathcal{K}(M,N)$
Garg [17] maximum approach	0.2120
Proposed method for $k = 1$	**0.7051**
Proposed method for $k = 3$	**0.2982**

Table 1: Numerical Output for Example I

Correlation coefficients $\mathcal{K}$	$\mathcal{K}(M,N)$
Garg [17] maximum approach	0.7426
Proposed method for $k = 1$	**0.9769**
Proposed method for $k = 3$	**0.8104**

Table 2: Numerical Output for Example II

3.5.3 Discussion

From Tables 1 and 2, $\mathcal{K}_3$ (Garg [17] maximum approach) shows that the correlation coefficient between M and N, and O and P are 0.2120 and 0.7426. Whereas that of the proposed method yields 0.7051, 0.2982 and 0.9769, 0.8104, respectively for $k = 1, 3$. The proposed method gives a better correlation coefficient when compare to Garg [17] maximum approach. It is observed that Garg [17] maximum approach is equivalent to the proposed method suppose $k = 4$. The correlation coefficient measure for the proposed method decreases as k increases and it is very reliable because it has two alternatives. That is, whenever one alternative fails to give a reasonable measure, another alternative could be employed.

The possibility of $k = 2$ is excluded because it yields inconsistent results in Examples I (0.4398) and II (1.0886). While Example I gives a valid correlation coefficient value within $[0, 1]$, Example II violents the condition.

4 Multi-criteria decision making problems via the proposed method

MCDM deals with decisions that involve the choice of a best preference from several potential alternatives subject to several criteria or attributes that may be concrete or imprecise. MCDM problems in everyday life pose a huge challenge to the decision maker. In this section, some MCDM problems in medical diagnosis and pattern recognition are discussed via the proposed correlation coefficient for IFSs.

4.1 Medical diagnosis problem

Assume a patient P visits a given medical laboratory for diagnosis. Suppose the patient has the following symptoms viz; temperature, headache, stomach pain, cough, and chest pain. That is, the set of symptoms S is

$$S = \{s_1, s_2, s_3, s_4, s_5\},$$

where s_1 = temperature, s_2 = headache, s_3 = stomach pain, s_4 = cough, s_5 = chest pain.

After the sample collected from P was analyzed, we have the following result represented in IFVs:

$$P = \{\frac{\langle 0.8, 0.1, 0.1\rangle}{s_1}, \frac{\langle 0.6, 0.1, 0.3\rangle}{s_2}, \frac{\langle 0.2, 0.8, 0.0\rangle}{s_3}, \frac{\langle 0.6, 0.1, 0.3\rangle}{s_4}, \frac{\langle 0.1, 0.6, 0.3\rangle}{s_5}\}.$$

Suppose the set of diseases D_i (for $i = 1, 2, 3, 4, 5$) which P is suspected to be suffering from are

$$D = \{D_1, D_2, D_3, D_4, D_5\},$$

where D_1 = viral fever, D_2 = malaria fever, D_3 = typhoid fever, D_4 = stomach problem, and D_5 = heart problem.

The diseases D_i are represented by the following IFVs:

$$D_1 = \{\frac{\langle 0.4, 0.0, 0.6\rangle}{s_1}, \frac{\langle 0.3, 0.5, 0.2\rangle}{s_2}, \frac{\langle 0.1, 0.7, 0.2\rangle}{s_3}, \frac{\langle 0.4, 0.3, 0.3\rangle}{s_4}, \frac{\langle 0.1, 0.7, 0.2\rangle}{s_5}\}$$

$$D_2 = \{\frac{\langle 0.7, 0.0, 0.3\rangle}{s_1}, \frac{\langle 0.2, 0.6, 0.2\rangle}{s_2}, \frac{\langle 0.0, 0.9, 0.1\rangle}{s_3}, \frac{\langle 0.7, 0.0, 0.3\rangle}{s_4}, \frac{\langle 0.1, 0.8, 0.1\rangle}{s_5}\}$$

$$D_3 = \{\frac{\langle 0.3, 0.3, 0.4\rangle}{s_1}, \frac{\langle 0.6, 0.2, 0.2\rangle}{s_2}, \frac{\langle 0.3, 0.7, 0.0\rangle}{s_3}, \frac{\langle 0.2, 0.6, 0.2\rangle}{s_4}, \frac{\langle 0.1, 0.9, 0.0\rangle}{s_5}\}$$

$$D_4 = \{\frac{\langle 0.1, 0.7, 0.2\rangle}{s_1}, \frac{\langle 0.2, 0.4, 0.4\rangle}{s_2}, \frac{\langle 0.8, 0.0, 0.2\rangle}{s_3}, \frac{\langle 0.2, 0.7, 0.1\rangle}{s_4}, \frac{\langle 0.2, 0.7, 0.1\rangle}{s_5}\}$$

$$D_5 = \{\frac{\langle 0.1, 0.8, 0.1\rangle}{s_1}, \frac{\langle 0.0, 0.8, 0.2\rangle}{s_2}, \frac{\langle 0.2, 0.8, 0.0\rangle}{s_3}, \frac{\langle 0.2, 0.8, 0.0\rangle}{s_4}, \frac{\langle 0.8, 0.1, 0.1\rangle}{s_5}\}.$$

The goal is to find the disease that the patient P is suffering from by computing the correlation coefficient between P and D_i. By deploying the proposed method for $k = 3$, we obtain the following outputs:

$$\mathcal{K}(P, D_1) = 0.6554,\ \mathcal{K}(P, D_2) = 0.7870,\ \mathcal{K}(P, D_3) = 0.7173,$$
$$\mathcal{K}(P, D_4) = 0.3084,\ \mathcal{K}(P, D_5) = 0.2745.$$

From the computations, one can conclude that the patient P is suffering from malaria fever since

$$\mathcal{K}(P, D_2) > \mathcal{K}(P, D_3) > \mathcal{K}(P, D_1) > \mathcal{K}(P, D_4) > \mathcal{K}(P, D_5).$$

4.2 Pattern recognition problem

Suppose there is a set of some known mineral fields $C = \{C_1, C_2, C_3\}$ represented by the following IFVs in a given finite universe $S = \{s_1, s_2, s_3\}$ as

$$C_1 = \{\frac{\langle 1.0, 0.0, 0.0\rangle}{s_1}, \frac{\langle 0.8, 0.0, 0.2\rangle}{s_2}, \frac{\langle 0.7, 0.1, 0.2\rangle}{s_3}\}$$

$$C_2 = \{\frac{\langle 0.8, 0.1, 0.1\rangle}{s_1}, \frac{\langle 1.0, 0.0, 0.0\rangle}{s_2}, \frac{\langle 0.9, 0.1, 0.0\rangle}{s_3}\}$$

$$C_3 = \{\frac{\langle 0.6, 0.2, 0.2\rangle}{s_1}, \frac{\langle 0.8, 0.0, 0.2\rangle}{s_2}, \frac{\langle 1.0, 0.0, 0.0\rangle}{s_3}\}.$$

Also, consider an unknown mineral field Q represented by IFVs as

$$Q = \{\frac{\langle 0.5, 0.3, 0.2\rangle}{s_1}, \frac{\langle 0.6, 0.2, 0.2\rangle}{s_2}, \frac{\langle 0.8, 0.1, 0.1\rangle}{s_3}\}$$

that is supposed to be classified into any of the above mineral fields.

The aim of this problem is to classify the unknown mineral field Q into one of the classes C_1, C_2 and C_3. Using the proposed correlation coefficient measure $\mathcal{K}$ for $k = 3$, we compute the correlation coefficient from Q to C_i (for $i = 1, 2, 3$) thus:

$$\mathcal{K}(C_1, Q) = 0.5967,\ \mathcal{K}(C_2, Q) = 0.5961,\ \mathcal{K}(C_3, Q) = 0.7095.$$

Hence, from the computational results, it follows that the unknown mineral field Q belongs to the mineral field C_3 since $\mathcal{K}(C_3, Q)$ is the greatest.

5 Conclusion

In this paper, a new correlation coefficient for IFSs which modified and generalized the one in [33] was introduced and characterized. The weakness of the similar existing correlation coefficients for IFSs have also been highlighted in the article. It was proven that the maximum approach of correlation coefficient in [33] is not a reliable measure because it failed the axiomatic description of correlation coefficient for IFSs. Mathematically, it was shown that the new correlation coefficient for IFSs satisfied the axiomatic description of correlation coefficient for IFSs unlike the one in [33]. It was observed that the maximum approach of correlation coefficient in [17] extended to intuitionistic fuzzy settting can be effectively recovered from the new version if $k = 4$. Some numerical illustrations were given to validate the superiority of the new correlation coefficient in situations where the similar existing correlation coefficients for IFSs could not give an appropriate interrelationship. To establish the application of the proposed method, some cases of MCDM problems such as medical diagnosis and classification of mineral fields were discussed. From the study, it was concluded that the new correlation coefficient for IFSs gives a reliable result when compare to the similar existing ones and hence, can appropriately solve MCDM problems effectively. Some novel areas of application could be established in further research using the proposed correlation coefficient for IFSs.

References

[1] Atanassov, K. T. (1986). Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20, 87–96.

[2] Atanassov, K. T. (1994). New operations defined on intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 61, 137–142.

[3] Atanassov, K. T. (1999). *Intuitionistic Fuzzy Sets: Theory and Applications*. Physica-Verlag, Heidelberg, 1999.

[4] Atanassov, K. T. (2012). *On Intuitionistic Fuzzy Sets Theory*. Springer, Berlin.

[5] Chiang, D. A. & Lin, N. P. (1999). Correlation of fuzzy sets, *Fuzzy Sets and Systems*, 102 (2), 221–226.

[6] Davvaz, B. & Sadrabadi, E. H. (2016). An application of intuitionistic fuzzy sets in medicine, *International Journal of Biomathematics*, 9 (3), 1650037 (15 pages).

[7] De, S. K., Biswas, R. & Roy, A. R. (2001). An application of intuitionistic fuzzy sets in medical diagnosis, *Fuzzy Sets and Systems*, 117 (2), 209–213.

[8] Dumitrescu, D. (1977). A definition of an informational energy in fuzzy set theory, *Studia Univ. Babes-Bolyai Mathematics*, 22, 57–59.

[9] Dumitrescu, D. (1978). Fuzzy correlation, *Studia Univ. Babes-Bolyai Mathematics*, 23, 41–44.

[10] Ejegwa, P. A. (2015). Intuitionistic fuzzy sets approach in appointment of positions in an organization via max-min-max rule, *Global Journal of Science Frontier Research: F Mathematics and Decision Science*, 15 (6), 1–6.

[11] Ejegwa, P. A. & Adamu, I. M. (2019). Distances between intuitionistic fuzzy sets of second type with application to diagnostic medicine, *Notes on Intuitionistic Fuzzy Sets*, 25 (3), 53–70.

[12] Ejegwa, P. A., Akubo, A. J. & Joshua, O. M. (2014). Intuitionistic fuzzy set and its application in career determination via normalized Euclidean distance method, *European Scienttific Journal*, 10 (15), 529–536.

[13] Ejegwa, P. A., Akubo, A. J. & Joshua, O. M. (2014). Intuitionistic fuzzzy sets in career determination, *Journal of Information and Computing Science*, 9 (4), 285–288.

[14] Ejegwa, P. A. & Modom, E. S. (2015). Diagnosis of viral hepatitis using new distance measure of intuitionistic fuzzy sets, *International Journal of Fuzzy Mathematical Archive*, 8 (1), 1–7.

[15] Ejegwa, P. A. & Onasanya, B. O. (2019). Improved intuitionistic fuzzy composite relation and its application to medical diagnostic process, *Notes on Intuitionistic Fuzzy Sets*, 25 (1), 43–58.

[16] Ejegwa, P. A. & Onyeke, I. C. (2018). An object oriented approach to the application of intuitionistic fuzzy sets in competency based test evaluation, *Annals of Communications in Mathematics*, 1 (1), 38–47.

[17] Garg, H. (2016). A novel correlation coefficients between Pythagorean fuzzy sets and its applications to decision making processes, *International Journal of Intelligent Systems*, 31 (12), 1234–1252.

[18] Gerstenkorn, T. & Manko, J. (1991). Correlation of intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 44 (1), 39–43.

[19] Hung, W. L. (2001). Using statistical viewpoint in developing correlation of intuitionistic fuzzy sets, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 9 (4), 509–516.

[20] Hung, W. L. & Wu, J. W. (2002). Correlation of intuitionistic fuzzy sets by centroid method, *Information Sciences*, 144 (1), 219–225.

[21] Iqbal, M. N. & Rizwan, U. (in press). Some applications of intuitionistic fuzzy sets using new similarity measure, *Journal of Ambient Intelligence and Humanized Computing*, https://doi.org/10.1007/s12652-019-01516-7.

[22] Liu, B., Shen, Y., Mu, L., Chen, X. & Chen, L. (2016). A new correlation measure of the intuitionistic fuzzy sets, *Journal of Intelligent and Fuzzy Systems*, 30 (2), 1019–1028.

[23] Mitchell, H. B. (2004). A correlation coefficient for intuitionistic fuzzy sets, *International Journal of Intelligent Systems*, 19 (5), 483–490.

[24] Pavan, M. & Todeschini, R. (2009). Multi-criteria decision-making methods, *Computational Chemometric*, 1, 591–629.

[25] Szmidt, E. & Kacprzyk, J. (2001). Intuitionistic fuzzy sets in some medical applications, *Notes on Intuitionistic Fuzzy Sets*, 7 (4), 58–64.

[26] Szmidt, E. & Kacprzyk, J. (2004). Medical diagnostic reasoning using a similarity measure for intuitionistic fuzzy sets, *Notes on Intuitionistic Fuzzy Sets*, 10 (4), 61–69.

[27] Szmidt, E. & Kacprzyk, J. (2010). Correlation of intuitionistic fuzzy sets, In: Hullermeier, E., Kruse, R. and Hoffmann, F. (eds.): *Proc. of IPMU 2010*, LNAI 6178, pp. 169–177, Springer-Verlag Berlin Heidelberg.

[28] Thao, N. X. (2018). A new correlation coefficient of the intuitionistic fuzzy sets and its application, *Journal of Intelligent and Fuzzy Systems*, 35 (2), 1959–1968.

[29] Thao, N. X., Ali, M. & Smarandache, F. (2019). An intuitionistic fuzzy clustering algorithm based on a new correlation coefficient with application in medical diagnosis, *Journal of Intelligent and Fuzzy Systems*, 36 (1), 189–198.

[30] Todorova, L., Atanassov, K. T., Hadjitodorov, S. & Vassilev, P. (2007). On an intuitionistic fuzzy approach for decision-making in medicine (Part 1), *International Electronic Journal of Bioautomation*, 6, 92–101.

[31] Todorova, L., Atanassov, K. T., Hadjitodorov, S. & Vassilev, P. (2007). On an intuitionistic fuzzy approach for decision-making in medicine (Part 2), *International Electronic Journal of Bioautomation*, 7, 64–69.

[32] Xu, Z. (2006). On correlation measures of intuitionistic fuzzy sets, In: Corchado, E. et al. (eds.): *Proc. of IDEAL 2006*, LNCS 4224, pp. 16–24, Springer-Verlag Berlin Heidelberg.

[33] Xu, S., Chen, J. & Wu, J. J. (2008). Cluster algorithm for intuitionistic fuzzy sets, *Information Sciences*, 178, 3775–3790.

[34] Zadeh, L. A. (1965). Fuzzy sets, *Information and Control*, 8, pp. 338–353.