4th IFSCOM, 3–7 May 2017, Mersin, Turkey
Notes on Intuitionistic Fuzzy Sets
Print ISSN 1310–4926, Online ISSN 2367–8283
Vol. 23, 2017, No. 1, 21–30

On separation axioms in temporal intuitionistic fuzzy Šostak topology

Fatih Kutlu

Department of Mathematics
Yüzüncü Yıl University, Van, Turkey
e-mail: fatihkutlu@yyu.edu.tr

Abstract: In this paper, the concepts of temporal and overall intuitionistic fuzzy point are defined and some properties of theirs investigated. Also $(\alpha_{t_0}, \beta_{t_0}) - T_i$ $(i = 0, 1, 2)$ temporal and $(\alpha_t, \beta_t) - T_i$ $(i = 0, 1, 2)$ overall separation axioms are defined for temporal intuitionistic fuzzy topology in Šostak sense.

Keywords: Temporal intuitionistic fuzzy sets, Temporal intuitionistic fuzzy topology, Temporal intuitionistic fuzzy point, Separation axioms, Homeomorphism.
AMS Classification: 47S40, 03E72.

1 Introduction

After Zadeh defined fuzzy set theory in 1965, many different generalization of this theory have been defined in the literature. One of the most remarkable among these generalizations is the intuitionistic fuzzy set theory, which includes the degrees of membership and uncertainty in addition to the membership degree defined in the fuzzy set theory. Intuitionistic fuzzy set (IFS, for short) was introduced by Atanassov in 1983 [2]. This theory has been found quite successful in dealing with imperfect knowledge owing to its membership, non-membership and uncertainty degrees. On the other hand, the concept of fuzzy topological space was defined by Chang in 1968 as a collection of fuzzy sets [4]. Fuzzifying of topology concept was made by Šostak in 1985 [14]. In his definition, openness and closeness of fuzzy sets are graded among 0 and 1. In [5], Çoker and Demirci introduced the concept of intuitionistic fuzzy set in Šostak's sense and gave fundamental definitions and properties of it.

Aiming to define a suitable set theory for handling situation which can change depending on the time, temporal intuitionistic fuzzy set theory is defined by Atanassov in 1991 [1]. In recent years, some fundamental concepts have been defined by several authors. Yılmaz and Çuvalcıoğlu [15] defined level operators on TIFSs. Šostak's mean temporal intuitionistic fuzzy topology and was defined by Kutlu and Bilgin [9]. Several compactness of temporal intuitionistic fuzzy topology in Šostak's sense and was defined by Kutlu, Ramadan and Bilgin [10]. Also distance and similarity, entropy and inclusion measures of TIFSs were defined and investigated some fundamental properties by Kutlu, Atan and Bilgin [8]. On the other hand, the separation axioms in the fuzzy and intuitionistic fuzzy topologies described by Chang and Šostak's sense are discussed in the following articles [7, 12, 13, 14, 15, 16].

The rest of this study is organized as follows. In section 2, basic definitions of intuitionistic fuzzy sets and temporal intuitionistic fuzzy sets are given. In section 3, temporal and overall intuitionistic fuzzy point are defined and given some properties of them. Also concepts of $(\alpha_{t_0}, \beta_{t_0}) - T_i$ $(i = 0, 1, 2)$ temporal and $(\alpha_t, \beta_t) - T_i$ $(i = 0, 1, 2)$ overall separation axioms for temporal intuitionistic fuzzy topology in Šostak sense are defined and investigated some fundamental properties of them.

2 Preliminaries

Definition 2.1 [2] An intuitionistic fuzzy set in a non-empty set X is given by a set of ordered triples $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ where $\mu_A(x) : X \to I$, $\nu_A(x)\rangle : X \to I$, and $I = [0, 1]$, are functions such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$. For $x \in X$, $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and degree of non-membership of x to A, respectively. For each $x \in X$; intuitionistic fuzzy index of x in A is defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$. π_A is the called degree of hesitation or indeterminacy.

By $IFS^{(X)}$, we denote to the set of all intuitionistic fuzzy sets defined on X.

Definition 2.2 [2] Let $A, B \in IFS^{(X)}$. Then,

(i) $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for $\forall x \in X$,

(ii) $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$,

(iii) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$,

(iv) $\bigcap A_i = \{\langle x, \wedge \mu_{A_i}(x), \vee \nu_{A_i}(x)\rangle \mid x \in X\}$,

(v) $\bigcup A_i = \{\langle x, \vee \mu_{A_i}(x), \wedge \nu_{A_i}(x)\rangle \mid x \in X\}$,

(vi) $\underset{\sim}{0} = \{\langle x, 0, 1\rangle \mid x \in X\}$ and $\underset{\sim}{1} = \{\langle x, 1, 0\rangle \mid x \in X\}$.

Definition 2.3 [2, 5]. Let a and b be two real numbers in [0, 1] satisfying the inequality $a + b \leq 1$. Then, the pair $\langle a, b\rangle$ is called an intuitionistic fuzzy pair. Let $\langle a_1, b_1\rangle$ and $\langle a_2, b_2\rangle$ be two intuitionistic fuzzy pair (briefly IF-pair). Then define

(i) $\langle a_1, b_1\rangle \leq \langle a_2, b_2\rangle \Leftrightarrow a_1 \leq a_2$ and $b_1 \geq b_2$,

(ii) $\langle a_1, b_1\rangle = \langle a_2, b_2\rangle \Leftrightarrow a_1 = a_2$ and $b_1 = b_2$,

(iii) If $\{\langle a_i, b_i\rangle ; i \in J\}$ is a family of intuitionistic fuzzy pairs, then $\vee\langle a_i, b_i\rangle = \langle \vee a_i, \wedge b_i\rangle$ and $\wedge\langle a_i, b_i\rangle = \langle \wedge a_i, \vee b_i\rangle$,

(iv) The complement of $\langle a, b\rangle$ is defined by $\overline{\langle a, b\rangle} = \langle b, a\rangle$,

(v) $1^\sim = \langle 1, 0\rangle$ and $0^\sim = \langle 0, 1\rangle$.

Definition 2.5 [1]**.** Let X be a universe and T be a non-empty time-moment set. We call the elements of T "time moments". Based on the definition of IFS, a temporal intuitionistic fuzzy set (TIFS) A is defined as the following:

$$A(T)=\{\langle x,\mu_A(x,t),\nu_A(x,t)\rangle \mid (x,t)\in X\times T\}$$

where:

(a) $A\subseteq X$ is a fixed set

(b) $\mu_A(x,t)+\nu_A(x,t)\le 1$ for every $(x,t)\in X\times T$

(c) $\mu_A(x,t)$ and $\nu_A(x,t)$ are the degrees of membership and non-membership, respectively, of the element $x\in X$ at the time moment $t\in T$

By $TIFS^{(X,T)}$, we denote to the set of all TIFSs over nonempty set X and time-moment set T. For brevity, we write A instead of $A(T)$. The hesitation degree of a TIFS is defined as $\pi_A(x,t)=1-\mu_A(x,t)-\nu_A(x,t)$. Obviously, every ordinary IFS can be regarded as TIFS for which T is a singleton set. All operations and operators on IFS can be defined for TIFSs.

Definition 2.6 [3]. Let

$$A(T')=\{\langle x,\mu_A(x,t),\nu_A(x,t)\rangle \mid (x,t)\in X\times T'\}$$

and

$$B(T'')=\{\langle x,\mu_B(x,t),\nu_B(x,t)\rangle \mid (x,t)\in X\times T''\},$$

where T' and T'' have finite number of distinct time-elements or they are time intervals. Then,

$$A(T')\cap B(T'')=\{\langle x,\min(\bar{\mu}_A(x,t),\bar{\mu}_B(x,t)),\max(\bar{\nu}_A(x,t),\bar{\nu}_B(x,t)\rangle \mid (x,t)\in X\times(T'\cup T'')\},$$

$$A(T')\cup B(T'')=\{\langle x,\max(\bar{\mu}_A(x,t),\bar{\mu}_B(x,t)),\min(\bar{\nu}_A(x,t),\bar{\nu}_B(x,t))\rangle \mid (x,t)\in X\times(T'\cup T'')\}.$$

Also from the definition of subset in IFS theory, subsets of TIFS can be defined in the following way: $A(T')\subseteq B(T'')$ $\Leftrightarrow$ $\bar{\mu}_A(x,t)\le\bar{\mu}_B(x,t)$ and $\bar{\nu}_A(x,t)\ge\bar{\nu}_B(x,t)$ for every $(x,t)\in X\times(T'\cup T'')$, where

$$\bar{\mu}_A(x,t)=\begin{cases}\mu_A(x,t), & if\ t\in T'\\ 0, & if\ t\in T''-T'\end{cases} \quad\text{and}\quad \bar{\mu}_B(x,t)=\begin{cases}\mu_B(x,t), & if\ t\in T''\\ 0, & if\ t\in T'-T''\end{cases},$$

$$\bar{\nu}_A(x,t)=\begin{cases}\nu_A(x,t), & if\ t\in T'\\ 1, & if\ t\in T''-T'\end{cases} \quad\text{and}\quad \bar{\nu}_B(x,t)=\begin{cases}\nu_B(x,t), & if\ t\in T''\\ 1, & if\ t\in T''-T'\end{cases}.$$

It is obviously seen that if $T'=T''$; $\bar{\mu}_A(x,t)=\mu_A(x,t)$, $\bar{\mu}_B(x,t)=\mu_B(x,t)$, $\bar{\nu}_A(x,t)$ $=\nu_A(x,t)$, $\bar{\nu}_B(x,t)=\nu_B(x,t)$.

Let J be an arbitrary index set. Then we define that $T=\bigcup_{i\in J}T_i$, where T_i is a time set for each $i\in J$. Thus, we can extend the definition of union and intersection of TIFSs family $F=\{A_i(T_i)=(x,\mu_{A_i}(x,t),\nu_{A_i}(x,t))\mid x\in X\times T_i, i\in J\}$ as follows:

$$\bigcup_{i\in J}A(T_i)=\{(x,\max_{i\in J}(\bar{\mu}_{A_i}(x,t)),\min_{i\in J}(\bar{\nu}_{A_i}(x,t)):(x,t)\in X\times T),$$

$$\bigcap_{i\in J} A(T_i)=\left\{\left(x,\ \min_{i\in J}\left(\bar{\mu}_{A_i}(x,t)\right),\max_{i\in J}\left(\bar{\nu}_{A_i}(x,t)\right) \mid (x,t)\in X\times T\right)\right.,$$

where

$$\bar{\mu}_{A_j}(x,t)=\begin{cases}\mu_{A_j}(x,t), & if\ t\in T_j\\ 0, & if\ t\in T-T_j\end{cases}, \quad \text{and} \quad \bar{\nu}_{A_j}(x,t)=\begin{cases}\nu_{A_j}(x,t), & if\ t\in T_j\\ 1, & if\ t\in T-T_j\end{cases}.$$

Definition 2.7 [9]. $\underset{\sim}{0}^t$ and $\underset{\sim}{1}^t$ $\in$ $TIFS^{(X,T)}$ are defined as: $\underset{\sim}{0}^t=\{(x,0,1):(x,t)\in X\times T\}$ and $\underset{\sim}{1}^t=\{(x,1,0):(x,t)\in X\times T\}$ for each time moment t, i.e., $\mu_{\underset{\sim}{0}^t}(x,t)=0$, $\nu_{\underset{\sim}{0}^t}(x,t)=1$ and $\mu_{\underset{\sim}{1}^t}(x,t)=1$, $\nu_{\underset{\sim}{1}^t}(x,t)=0$ for each $(x,t)\in X\times T$.

Definition 2.8 [9]. A temporal intuitionistic fuzzy topology in Šostak's sense (briefly, ST-TIFS) on a non-empty set X is an IFF τ_t defined with $\tau_t(A)=\left(\mu_{\tau_t}(A),\nu_{\tau_t}(A)\right)$ on X, satisfying the following axioms for each time moment t:

I. $\tau_t\left(\underset{\sim}{0}^t\right)=1^{\sim}$ and $\tau_t\left(\underset{\sim}{1}^t\right)=1^{\sim}$,

II. $\tau_t(A_1\cap A_2)\geq\tau_t(A_1)\wedge\tau_t(A_2)$ for any sets $A_1,A_2\in TIFS^{(X,T)}$,

III. $\tau_t\left(\bigcup A_i\right)\geq\bigwedge_{i\in J}\left(\tau_t(A_i)\right)$ for $\{A_i \mid i\in J\}\subseteq TIFS^{(X,T)}$.

The pair (X,τ_t) is called temporal intuitionistic fuzzy topological space in Šostak sense. For any $A\in TIFS^{(X,T)}$, the number $\mu_{\tau_t}(A)$ is called instant openness degree of A at time-moment t, while $\nu_{\tau_t}(A)$ is called instant non-openness degree of A at time-moment t.

In this definition, it is worth to note that the instant openness and the instant non-openness degree change with depending on both time and TIFS.

It is worth to note that for singleton time set $(X,\ \tau_t)$ is an intuitionistic fuzzy topology in Šostak's sense.

Theorem 2.9 [9]. Let $(X,\ \tau_t)$ be a ST-TIFS on X and T be a time-moment set. Then $(X,\wedge\tau_t)$ defined by $\wedge\tau_t(A)=\left(\min_{t\in T}\mu_{\tau_t}(A),\max_{t\in T}\nu_{\tau_t}(A)\right)$ is an intuitionistic fuzzy topology on $TIFS^{(X,T)}$ in Šostak's sense.

Definition 2.10 [10]. Let $(X,\ \tau_t)$ and $(Y,\ \phi_t)$ be ST-TIFSs respectively for non-empty sets X, Y, time sets T' and T''. Let $f:X\rightarrow Y$ be a function. Then,

(i) The pre-image of $B\in TIFS^{(Y,T'')}$ under f at time moment t is defined as

$$f^{-1}(B)=\left\{\left(x,\bar{\mu}_B(f(x),t),\bar{\nu}_B(f(x),t)\right)\mid x\in X\right\},$$

where

$$\bar{\mu}_B(f(x),t)=\begin{cases}\mu_B(f(x),t) & , \quad t\in T''\\ 0 & , \quad t\in T'-T''\end{cases}$$

and

$$\bar{\nu}_B(f(x),t)=\begin{cases}\nu_B(f(x),t) & , \quad t\in T''\\ 1 & , \quad t\in T'-T''.\end{cases}$$

(ii) The image of $A\in TIFS^{(X,T')}$ under f at time moment t is defined as

$$f(A)=\left\{\left(y,f(\overline{\mu}_A)(y,t),f_-(\overline{\nu}_A)(y,t)\right)\mid y\in Y\right\},$$

where

$$f(\overline{\mu}_A)(y,t)=\begin{cases} f(\mu_A)(y,t), & t\in T' \\ \quad 0 \quad , & t\in T''-T' \end{cases}$$

and

$$f_-(\overline{\nu}_A)(y,t)=\begin{cases} 1-f(1-\nu_A)(y,t), & t\in T' \\ \quad 1 \quad , & t\in T''-T'. \end{cases}$$

If $T'=T''$, It is clearly understood that $f^{-1}(B)=\left\{\left(x,\mu_B\left(f(x),t\right),\nu_B\left(f(x),t\right)\right)\mid x\in X\right\}$ and $f(A)=\left\{\left(y,f(\mu_A)(y,t),f_-(\nu_A)(y,t)\right)\mid y\in Y\right\}$.

Let (X, τ_t) and (Y, ϕ_t) be ST-TIFSs for non-empty sets X, Y and time set T. If $\tau_t\left(f^{-1}(B)\right)\geq\phi_t(B)$ for $t\in T$ and each $B\in TIFS^{(Y,T)}$, f is called temporal intuitionistic fuzzy continuous function at time moment t. If f is temporal intuitionistic fuzzy continuous function at each time moment, f is called overall intuitionistic fuzzy continuous function.

On the other hand, If $\phi_t\left(f(A)\right)\geq\tau_t(A)$ for $t\in T$ and each $A\in TIFS^{(Y,\,T')}$, f is called temporal intuitionistic fuzzy open function at time moment t. If f is temporal intuitionistic fuzzy open function at each time moment, f is called overall intuitionistic fuzzy open function.

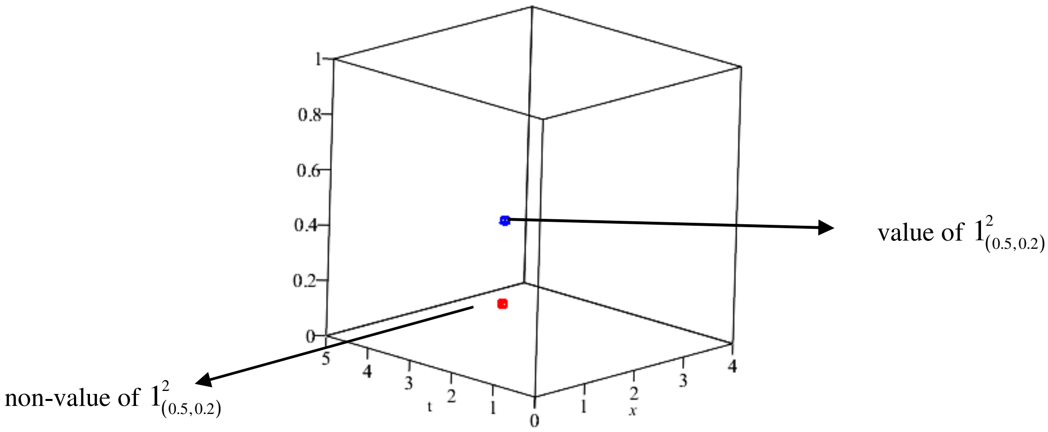
3 Main results

Definition 3.1: Let $\langle\alpha,\beta\rangle\in I_0\otimes I_1$. An temporal intuitionistic fuzzy point (T-IFP) $x^{t_0}_{(\alpha,\beta)}$ at fixed time moment t_0 is an TIFS of X defined by $x^{t_0}_{(\alpha,\beta)}(y)=\begin{cases}\langle\alpha,\beta\rangle, & x=y\\ \langle 0,1\rangle, & x\neq y\end{cases}$. In this, x called the support of $x^{t_0}_{(\alpha,\beta)}$ and α and β are called the value and the non-value of $x_{(\alpha,\beta)}$ at fixed time moment t_0, respectively.

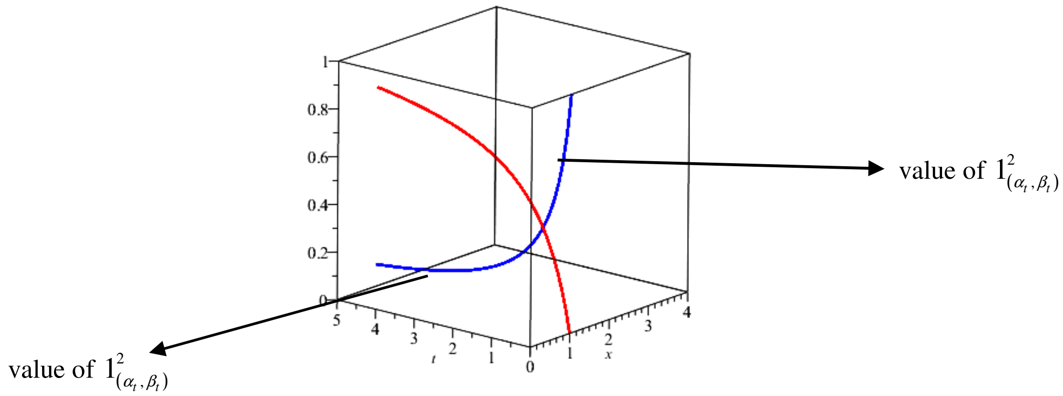
A T-IFP $x^{t_0}_{(\alpha,\beta)}$ is said to belong to an TIFS $A=\left\{\left(x,\mu_A(x,t),\nu_A(x,t)\right)\mid(x,t)\in X\times T\right\}$, denoted by $x^{t_0}_{(\alpha,\beta)}\in A$, if $\mu_A(x,t_0)\geq\alpha$ and $\nu_A(x,t_0)\leq\beta$.

On the other hand, Let $\langle\alpha_t,\beta_t\rangle\in I_0\otimes I_1$ for every time moment $t\in T$. An overall intuitionistic fuzzy point (O-IFP) $x^{t}_{(\alpha_t,\beta_t)}$ at every time moment t is an T-IFS of X defined by $x^{t}_{(\alpha_t,\beta_t)}(y)=\begin{cases}\langle\alpha_t,\beta_t\rangle, & x=y\\ \langle 0,1\rangle, & x\neq y\end{cases}$. In this, x called the support of $x^{t}_{(\alpha,\beta)}$ and α_t and β_t are called the value and the non-value of $x_{(\alpha,\beta)}$ at time moment t, respectively.

Example 3.2: Let X = [0, 4] and T = [0, 5]. Then the T-IFP $1^{2}_{(0.5,0.2)}$ is shown as



On the other hand, the O-IFP $1^t_{(\alpha_t,\beta_t)}$ where $\alpha_t = \dfrac{1}{1+2t}$ and $\beta_t = \dfrac{t}{1+t}$ for every $t \in T$ is shown as



The properties defined for the fuzzy point by Deng in [6] are also preserved for T-IFP as follows:

Proposition 3.3: Let X be non-empty set and T be a time set, the following statements are satisfied for $A, A_i \in TIFS^{(X,T)}$, $\alpha, \alpha_i \in I_0$ and $\beta, \beta_i \in I_1$;

1. $A = \bigcup_{x^t_{(\alpha_t,\beta_t)} \in A} x^t_{(\alpha_t,\beta_t)}$,
2. $x^{t_0}_{(\alpha_{t_0},\beta_{t_0})} \in \bigcup_{i\in J} A_i \Rightarrow x^{t_0}_{(\alpha_{t_0},\beta_{t_0})} \in A_{i_0}$ for $\exists\, i_0 \in J$,
3. $x^{t_0}_{(\alpha_{t_0},\beta_{t_0})} \in \bigcap_{i\in J} A_i \Rightarrow x^{t_0}_{(\alpha_{t_0},\beta_{t_0})} \in A_i$ for $\forall\, i \in J$,
4. $x^{t_0}_{1(\alpha_{t_0},\beta_{t_0})} \in x^{t_0}_{2(\alpha'_{t_0},\beta'_{t_0})} \Leftrightarrow x_1 = x_2$ and $\langle \alpha_{t_0}, \beta_{t_0} \rangle \le \langle \alpha'_{t_0}, \beta'_{t_0} \rangle$,
5. $x^{t_0}_{1(\alpha_{t_0},\beta_{t_0})} \in x^{t_0}_{2(\alpha'_{t_0},\beta'_{t_0})}$ and $x^{t_0}_{2(\alpha'_{t_0},\beta'_{t_0})} \in A_i$ for $\forall i \in J \Rightarrow x^{t_0}_{1(\alpha_{t_0},\beta_{t_0})} \in \bigcap_{i\in J} A_i$,
6. $x^{t_0}_{(\alpha_{t_0},\beta_{t_0})} \in A \Rightarrow x^{t_0}_{(\alpha'_{t_0},\beta'_{t_0})} \in A$ for $\exists \langle \alpha_{t_0}, \beta_{t_0} \rangle > \langle \alpha'_{t_0}, \beta'_{t_0} \rangle$,
7. $A \ne \underset{\sim}{0}^t \Leftrightarrow \exists\, x^{t_0}_{(\alpha_{t_0},\beta_{t_0})} \in A$

Definition 3.4: Let (X, τ_t) be a ST-TIFS on X and T be a time-moment set, $\langle \alpha_{t_0}, \beta_{t_0} \rangle \in I_0 \otimes I_1$ for time moment t_0. Then,

1. ST-TIFS (X, τ_t) is called temporal $(\alpha_{t_0}, \beta_{t_0}) - T_0$ at time moment t_0, if there exist $U \in TIFS^{(X,T)}$ for every distinct T-IFPs $x^{t_0}_{(r,s)}$ and $y^{t_0}_{(r^*,s^*)}$ satisfying these conditions :
   a. $\tau_{t_0}(U) \geq \langle \alpha_{t_0}, \beta_{t_0} \rangle$,
   b. ($x^{t_0}_{(r,s)} \in U$, $y^{t_0}_{(r^*,s^*)} \notin U$) or ($x^{t_0}_{(r,s)} \notin U$, $y^{t_0}_{(r^*,s^*)} \in U$);
2. ST-TIFS (X, τ_t) is called temporal $(\alpha_{t_0}, \beta_{t_0}) - T_1$ at time moment t_0, if there exist $U, V \in TIFS^{(X,T)}$ for every distinct T-IFPs $x^{t_0}_{(r,s)}$ and $y^{t_0}_{(r^*,s^*)}$ satisfying these conditions :
   a. $\tau_{t_0}(U) \geq \langle \alpha_{t_0}, \beta_{t_0} \rangle, \tau_{t_0}(V) \geq \langle \alpha_{t_0}, \beta_{t_0} \rangle$
   b. ($x^{t_0}_{(r,s)} \in U$, $y^{t_0}_{(r^*,s^*)} \notin V$) and ($x^{t_0}_{(r,s)} \notin V$, $y^{t_0}_{(r^*,s^*)} \in U$);
3. ST-TIFS (X, τ_t) is called temporal $(\alpha_{t_0}, \beta_{t_0}) - T_2$ at time moment t_0, if there exist $U, V \in TIFS^{(X,T)}$ for every distinct T-IFPs $x^{t_0}_{(r,s)}$ and $y^{t_0}_{(r^*,s^*)}$ satisfying these conditions :
   a. $\tau_{t_0}(U) \geq \langle \alpha_{t_0}, \beta_{t_0} \rangle, \tau_{t_0}(V) \geq \langle \alpha_{t_0}, \beta_{t_0} \rangle$
   b. $x^{t_0}_{(r,s)} \in U$, $y^{t_0}_{(r^*,s^*)} \in V$ and $U \cap_{t_0} V(x, t_0) = \langle \mu_{U \cap_{t_0} V}(x, t_0), \eta_{U \cap_{t_0} V}(x, t_0) \rangle = \langle 0, 1 \rangle$ for every $(x, t_0) \in X \times T$.

If (X, τ_t) satisfies the conditions of $(\alpha_t, \beta_t) - T_i$ $(i = 0, 1, 2)$ for every time moment $t \in T$, it is called overall $(\alpha_t, \beta_t) - T_i$ $(i = 0, 1, 2)$ ST-TIFS.

Theorem 3.5: Let (X, τ_t) be a ST-TIFS on X and T be a time-moment set. If (X, τ_t) is temporal $(\alpha_{t_0}, \beta_{t_0}) - T_2$, therewithal (X, τ_t) is temporal $(\alpha_{t_0}, \beta_{t_0}) - T_1$. On the other hand, if (X, τ_t) is temporal $(\alpha_{t_0}, \beta_{t_0}) - T_1$ therewithal (X, τ_t) is temporal $(\alpha_{t_0}, \beta_{t_0}) - T_0$.

The statements of Theorem 3.5 are also valid for overall $(\alpha_t, \beta_t) - T_i$ axioms.

Theorem 3.6: Let (X, τ_t) be a ST-TIFS on X and T be a finite time-moment set. If (X, τ_t) is overall $(\alpha_t, \beta_t) - T_i$ $(i = 0, 1, 2)$, there exists an $\langle \alpha, \beta \rangle \in I_0 \times I_1$ such that $(X, \wedge\tau_t)$ defined by $\wedge\tau_t(A) = \left(\min_{t \in T} \mu_{\tau_t}(A), \max_{t \in T} \eta_{\tau_t}(A) \right)$ is $(\alpha, \beta) - T_i$ $(i = 0, 1, 2)$ ST-IFS on $TIFS^{(X,T)}$.

Proof. Let us prove this theorem only for $(\alpha, \beta) - T_2$. Since we accept that (X, τ_t) is overall $(\alpha_t, \beta_t) - T_2$ ST-TIFS, there exist $U_t, V_t \in TIFS^{(X,T)}$ for every distinct T-IFPs $x^t_{(r_t, s_t)}$ and $y^t_{(r_t^*, s_t^*)}$ satisfying following conditions:

a. $\tau_t(U_t) \geq \langle \alpha_t, \beta_t \rangle, \tau_t(V) \geq \langle \alpha_t, \beta_t \rangle$

b. $x^t_{(r_t, s_t)} \in U_t$, $y^t_{(r_t^*, s_t^*)} \in V_t$ and $U_t \cap_t V_t(x, t) = \langle \mu_{U_t \cap V_t}(x, t), \eta_{U_t \cap_t V_t}(x, t) \rangle = \langle 0, 1 \rangle$ for each $(x, t) \in X \times T$.

Let us define $\langle \alpha, \beta \rangle = \left\langle \underset{t \in T}{\wedge} \alpha_t, \underset{t \in T}{\vee} \beta_t \right\rangle$, $U = \bigcap_{t \in T} U_t$ and $V = \bigcap_{t \in T} V_t$. Then, a*. Since $\tau_t(U) \geq \underset{t\in T}{\wedge} \tau_t(U_t) \geq \underset{t\in T}{\wedge}\langle \alpha_t, \beta_t \rangle = \langle \alpha, \beta \rangle$, $\wedge\tau_t(U) \geq \langle \alpha, \beta \rangle$. Similarly, it can be shown that $\wedge\tau_t(V) \geq \langle \alpha, \beta \rangle$.

b*. Since $x^t_{(r_t,s_t)} \in U_t$ for every $t \in T$, $x^t_{(r_t,s_t)} \in U$. Similarly, since $y^t_{(r_t*,s_t*)} \in V_t$ for every $t \in T$, $y^t_{(r_t*,s_t*)} \in V$. After this step, we must show $U \cap V = \tilde{0}$. From the second condition of being $(\alpha_t, \beta_t) - T_2$ ST-TIFS, we get $U_t \cap_t V_t(x,t) = \langle \mu_{U_t \cap V_t}(x,t), \eta_{U_t \cap_t V_t}(x,t) \rangle = \langle 0,1 \rangle$ for every $(x,t) \in X \times T$. Then $U \cap V = \bigcap_{t\in T} U_t \cap \bigcap_{t\in T} V_t$ i.e., $\mu_{U_t \cap V_t}(x,t) = \mu_{\bigcap_{t\in T} U_t \cap \bigcap_{t\in T} V_t}(x,t) = 0$, $\eta_{U_t \cap V_t}(x,t) = \eta_{\bigcap_{t\in T} U_t \cap \bigcap_{t\in T} V_t}(x,t) = 1$. From the two last equations, it is understood that $U \cap V = \tilde{0}$.
From a* and b*, it is understood that $(X, \wedge\tau_t)$ is $(\alpha,\beta) - T_2$ ST-IFS on $TIFS^{(X,T)}$. □

But this theory does not provide for infinite time sets. Let us show this with an example.

Example 3.7: Let (X, τ_t) be a ST-TIFS on X and $T = \{1, 2, 3, \dots\}$ defined as:

$$\mu_{\tau_t}(A) = \begin{cases} 1 & , \quad A \in \{\underset{\sim}{0}^t, \underset{\sim}{1}^t\} \\ \dfrac{1}{1+t} & , \quad \text{otherwise} \end{cases}$$

and

$$\nu_{\tau_t}(A) = \begin{cases} 0 & , \quad A \in \{\underset{\sim}{0}^t, \underset{\sim}{1}^t\} \\ \dfrac{t}{2+t} & , \quad \text{otherwise} \end{cases}.$$

From this definition, (X, τ_t) is $\left(\dfrac{1}{1+t_0}, \dfrac{t_0}{2+t_0}\right) - T_i$ ST-TIFS on $TIFS^{(X,T)}$ at fixed time moment $t_0 \in T$. But $\wedge\tau_t(A) = \langle \mu_{\wedge\tau_t}(A), \nu_{\wedge\tau_t}(A) \rangle = \begin{cases} \langle 1,0 \rangle & , \quad A \in \{\underset{\sim}{0}^t, \underset{\sim}{1}^t\} \\ \langle 0,1 \rangle & , \quad \text{otherwise} \end{cases}$. Then $\wedge\tau_t(A)$ is not $(\alpha, \beta) - T_i$ ST-IFS on $TIFS^{(X,T)}$ for any $\langle \alpha, \beta \rangle \in I_0 \times I_1$.

Definition 3.8: Let (X, τ_t) and (X, φ_t) be ST-TIFSs on X and T time-moment set. If $\tau_{t_0}(A) \leq \varphi_{t_0}(A)$ for each $A \in TIFS^{(X,T)}$ and fixed time moment t_0, (X, φ_t) is called finer ST-TIFS than (X, τ_t) at time moment t_0.

Theorem 3.9: Let (X, τ_t) and (X, φ_t) be ST-TIFSs on X and T time-moment set. We suppose that (X, φ_t) is finer ST-TIFS than (X, τ_t) at time moment t_0. If (X, τ_t) is temporal $(\alpha_{t_0}, \beta_{t_0}) - T_i$ $(i = 0, 1, 2)$ ST-TIFS at time moment t_0, (X, φ_t) is also $(\alpha_{t_0}, \beta_{t_0}) - T_i$ $(i = 0, 1, 2)$ ST-TIFS at time moment t_0.

Proof. Let us prove this theorem only for $(\alpha, \beta) - T_2$. Since we accept (X, τ_t) is temporal $(\alpha_{t_0}, \beta_{t_0}) - T_2$ ST-TIFS, there exist $U_{t_0}, V_{t_0} \in TIFS^{(X,T)}$ for every distinct T-IFPs $x^{t_0}_{(r_{t_0},s_{t_0})}$ and $y^{t_0}_{(r_{t_0}*,s_{t_0}*)}$ satisfying following conditions:

a. $\tau_{t_0}(U_{t_0}) \geq \langle \alpha_{t_0}, \beta_{t_0} \rangle, \tau_{t_0}(V_{t_0}) \geq \langle \alpha_{t_0}, \beta_{t_0} \rangle$

b. $x^{t_0}_{(r_{t_0}, s_{t_0})} \in U_{t_0}$, $y^{t_0}_{(r_{t_0}^*, s_{t_0}^*)} \in V_{t_0}$ and $U_{t_0} \cap_{t_0} V_{t_0}(x,t) = \langle \mu_{U_{t_0} \cap_{t_0} V_{t_0}}(x,t), \nu_{U_{t_0} \cap_{t_0} V_{t_0}}(x,t) \rangle$ $= \langle 0,1 \rangle$ for every $(x,t) \in X \times T$.

Since (X, φ_t) is finer ST-TIFS than (X, τ_t) at time moment t_0. We get the inequalities $\varphi_{t_0}(U_{t_0}) \geq \tau_{t_0}(U_{t_0}) \geq \langle \alpha_{t_0}, \beta_{t_0} \rangle$, $\varphi_{t_0}(V_{t_0}) \geq \tau_{t_0}(V_{t_0}) \geq \langle \alpha_{t_0}, \beta_{t_0} \rangle$. It is clearly understood from the last two inequalities that (X, φ_t) is $(\alpha_{t_0}, \beta_{t_0}) - T_2$ ST-TIFS at time moment t_0. □

Definition 3.10: Let (X, τ_t) and (Y, φ_t) be ST-TIFSs on X, Y and T time-moment set. Then, $f : (X, \tau_t) \to (Y, \varphi_t)$ is called a temporal homeomorphism at time moment t_0, if f is bijective, temporal intuitionistic fuzzy continuous, temporal intuitionistic fuzzy open.

Theorem 3.11: The property temporal $(\alpha_{t_0}, \beta_{t_0}) - T_i$ ($i = 0, 1, 2$) is a topological property.

Proof. Let us prove this theorem only for $(\alpha_{t_0}, \beta_{t_0}) - T_2$. Let $f : (X, \tau_t) \to (Y, \varphi_t)$ be a temporal homeomorphism and (X, τ_t) be a temporal $(\alpha_{t_0}, \beta_{t_0}) - T_2$ ST-TIFS on non-empty sets X, Y and time set T. Since f is a homeomorphism, there exist two distinct T-IFPs $(x_1)^{t_0}_{(r_{t_0}, s_{t_0})}$ $(x_2)^{t_0}_{(r^*_{t_0}, s^*_{t_0})}$ on $TIFS^{(X,T)}$ such that $(x_1)^{t_0}_{(r_{t_0}, s_{t_0})} = f^{-1}\left((y_1)^{t_0}_{(r_{t_0}, s_{t_0})}\right)$ and $(x_2)^{t_0}_{(r_{t_0}, s_{t_0})} = f^{-1}\left((y_2)^{t_0}_{(r_{t_0}, s_{t_0})}\right)$ for every distinct, arbitrary T-IFPs $(y_1)^{t_0}_{(r_{t_0}, s_{t_0})}$, $(y_2)^{t_0}_{(r^*_{t_0}, s^*_{t_0})}$ on $TIFS^{(Y,T)}$. Since (X, τ_t) be a temporal $(\alpha_{t_0}, \beta_{t_0}) - T_2$ ST-TIFS, There exist $U_1, U_2 \in TIFS^{(X,T)}$ such that

a. $\tau_{t_0}(U_1) \geq \langle \alpha_{t_0}, \beta_{t_0} \rangle, \tau_{t_0}(U_2) \geq \langle \alpha_{t_0}, \beta_{t_0} \rangle$

b. $(x_1)^{t_0}_{(r_{t_0}, s_{t_0})} \in U_1$, $(x_2)^{t_0}_{(r_{t_0}^*, s_{t_0}^*)} \in U_2$ and $U_1 \cap_{t_0} U_2(x,t) = \langle \mu_{U_1 \cap_{t_0} U_2}(x,t), \nu_{U_1 \cap_{t_0} U_2}(x,t) \rangle$ $= \langle 0,1 \rangle$ for every $(x,t) \in X \times T$.

Since f is temporal open function, $\tau_{t_0}(f(U_1)) \geq \langle \alpha_{t_0}, \beta_{t_0} \rangle$ and $\tau_{t_0}(f(U_2)) \geq \langle \alpha_{t_0}, \beta_{t_0} \rangle$. On the other hand, since f is bijective, it can be easily seen that $f\left((x_1)^{t_0}_{(r_{t_0}, s_{t_0})}\right) = (y_1)^{t_0}_{(r_{t_0}, s_{t_0})} \in U_1$, $f\left((x_2)^{t_0}_{(r_{t_0}, s_{t_0})}\right) = (y_2)^{t_0}_{(r_{t_0}, s_{t_0})} \in U_2$ and $f(U_1 \cap_{t_0} U_2)(x,t) = \left\langle \mu_{f(U_1 \cap_{t_0} U_2)}(x,t), \eta_{f(U_1 \cap_{t_0} U_2)}(x,t) \right\rangle = \langle 0,1 \rangle$ for every $(x, t) \in X \times T$.

Thence, it is understood that (Y, φ_t) is a temporal $(\alpha_{t_0}, \beta_{t_0}) - T_2$ ST-TIFS. □

Conclusion 3.12: The property overall $(\alpha_{t_0}, \beta_{t_0}) - T_i$ ($i = 0, 1, 2$) is a topological property.

References

[1] Atanassov, K. T. (1991). Temporal intuitionistic fuzzy sets. *Comptes Rendus de l'Academie Bulgare*, 44(7), 5–7.

[2] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87-96.

[3] Atanassov, K. T. (2012). *On Intuitionistic Fuzzy Sets Theory*. Springer, Berlin.

[4] Chang, C. L. (1968). Fuzzy topological spaces. *J. Math Ana. Appl.* 24, 182–190.

[5] Çoker, D., & Demirci, M. (1996). An introduction to intuitionistic topological spaces in Šostak's sense. *BUSEFAL* 67, 67–76.

[6] Deng, Z. (1982). Fuzzy pseudo-metric spaces. *Journal of Mathematical Analysis and Applications* 86(1), 74–95.

[7] Ibedou, I. (2016). Graded fuzzy topological spaces. *Cogent mathematics*, 3:1138574, 13 pages.

[8] Kutlu, F., Atan, Ö., & Bilgin, T. (2016). Distance measure, similarity measure, entropy and inclusion measure for temporal intuitionistic fuzzy sets. *Proceedings of IFSCOM'2016, Book (1),* 130–148.

[9] Kutlu, F. & Bilgin, T. (2015). Temporal intuitionistic fuzzy topology in Šostak's sense. *Notes on Intuitionistic Fuzzy Sets*, 21(2), 63–70.

[10] Kutlu, F., Ramadan, A. A., & Bilgin, T. (2016). On compactness in temporal intuitionistic fuzzy Šostak topology. *Notes on Intuitionistic Fuzzy Sets*, 22(5), 46–62.

[11] Lee, E. P. (2004). Semiopen sets on intuitionistic fuzzy topological spaces in Šostak's sense. *J. Fuzzy Logic and Intelligent Systems*, 14(2), 234–23.

[12] Singh, A. K. & Srivastava, R. (2012). Separation axioms in intuitionistic fuzzy topological spaces. *Advances in Fuzzy Systems*, Article ID 604396, 7 pages, doi:10.1155/2012/604396.

[13] Singh, A. (2009). On T1 separation axioms in I-fuzzy topological spaces. *Applied math. Sci* 3(49), 2421–2425.

[14] Šostak, A. (1985). On a fuzzy topological structure. *Rend Circ. Mat. Palermo Supp*. 11, 89–103.

[15] Yılmaz, S. & Çuvalcıoğlu, G. (2014). On level operators for temporal intuitionistic fuzzy sets. *Notes on Intuitionistic Fuzzy Sets* 20(2), 6–15.

[16] Yue, Y., & Fang, J. (2006). On separation axioms in I-fuzzy topological spaces. *Fuzzy Sets and Systems*, 157(6), 780–793.